Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014

Mathematics
(Project Maths – Phase 3)

Paper 2

Higher Level

Monday 9 June Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Questions 7 to 9.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

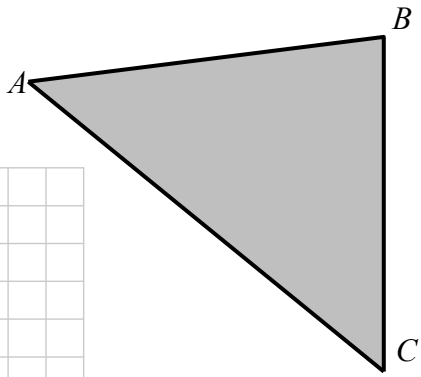
Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 (25 marks)

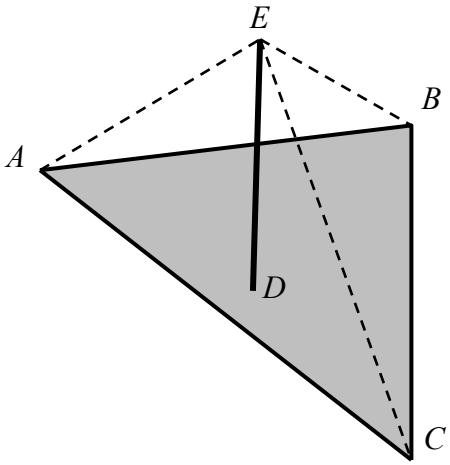
The lengths of the sides of a flat triangular field ACB are, $|AB| = 120$ m, $|BC| = 134$ m and $|AC| = 150$ m.



(a) **(i)** Find $|\angle CBA|$. Give your answer, in degrees, correct to two decimal places.

(ii) Find the area of the triangle ACB correct to the nearest whole number.

(b) A vertical mast, $[DE]$, is fixed at the circumcentre, D, of the triangle. The mast is held in place by three taut cables $[EA]$, $[EB]$ and $[EC]$. Explain why the three cables are equal in length.



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Question 2 **(25 marks)**

(a) Prove that $\cos 2A = \cos^2 A - \sin^2 A$.

(b) The diagram shows part of the circular end of a running track with three running lanes shown. The centre of each of the circular boundaries of the lanes is at O.

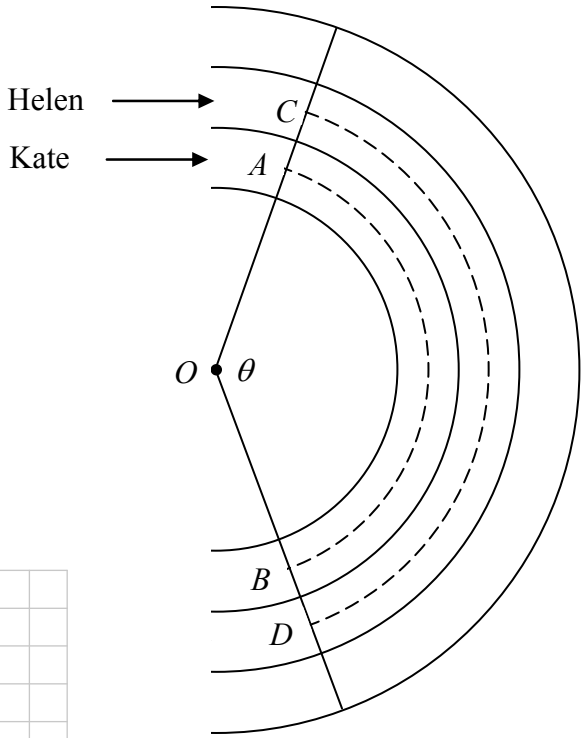
Kate runs in the middle of lane 1, from A to B as shown.

Helen runs in the middle of lane 2, from C to D as shown.

Helen runs 3 m further than Kate.

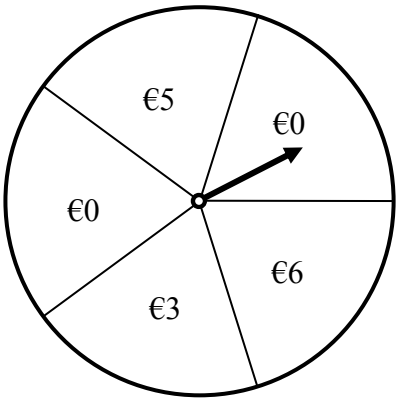
$|\angle AOB| = |\angle COD| = \theta$ radians.

If each lane is 1·2 m wide, find θ.

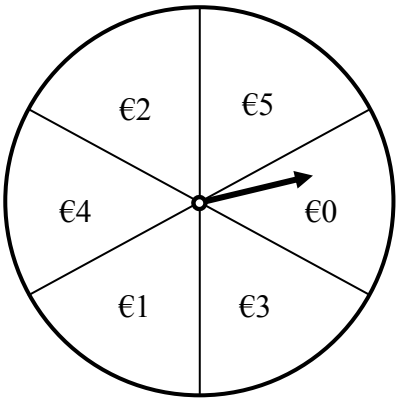


Question 3 **(25 marks)**

Two different games of chance, shown below, can be played at a charity fundraiser. In each game, the player spins an arrow on a wheel and wins the amount shown on the sector that the arrow stops in. Each game is fair in that the arrow is just as likely to stop in one sector as in any other sector on that wheel.



Game *A*



Game *B*

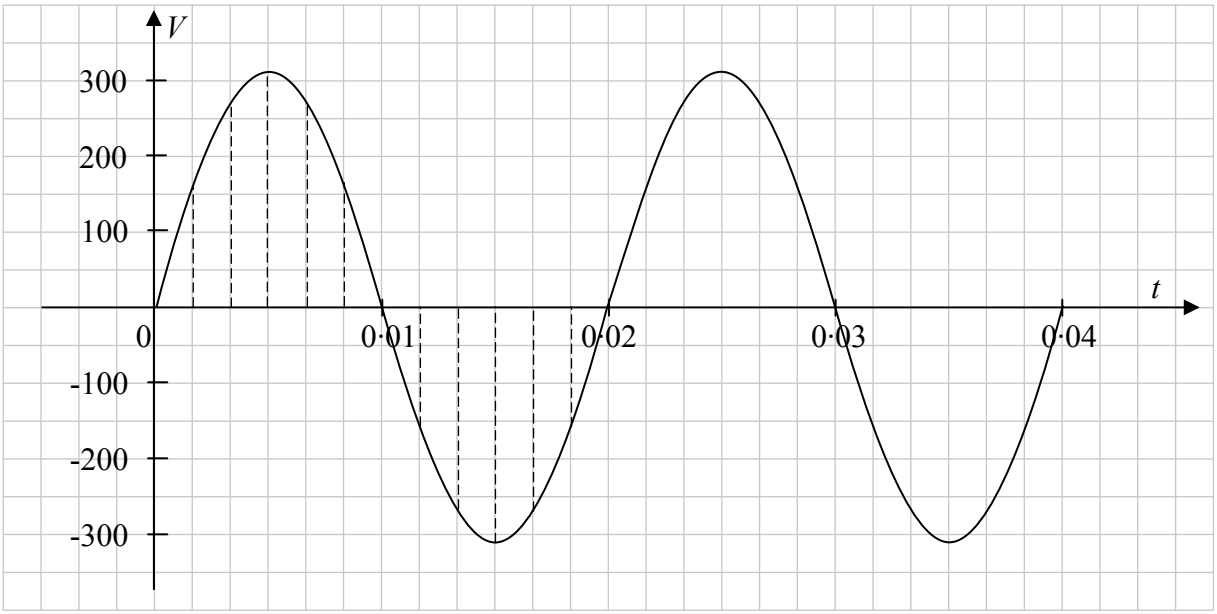
(a) John played Game *A* four times and tells us that he has won a total of €8. In how many different ways could he have done this?

(b) To spin either arrow once, the player pays €3. Which game of chance would you expect to be more successful in raising funds for the charity? Give a reason for your answer.

(c) Mary plays Game *B* six times. Find the probability that the arrow stops in the €4 sector exactly twice.

Question 4 **(25 marks)**

The graph below shows the voltage, V, in an electric circuit as a function of time, t. The voltage is given by the formula $V = 311\sin(100\pi t)$, where V is in volts and t is in seconds.



(a) **(i)** Write down the range of the function.

(ii) How many complete periods are there in one second?

(b) **(i)** The table below gives the voltage, correct to the nearest whole number, at equally spaced intervals from t_1 to t_{12} over one complete period (as shown by the dashed lines on the diagram). Use the entries given in the table and the properties of the function to complete the table.

t	t_1	t_2	t_3	t_4	t_5	$t_6 = 0{\cdot}01$	t_7	t_8	t_9	t_{10}	t_{11}	$t_{12} = 0{\cdot}02$
V	156	269	311									

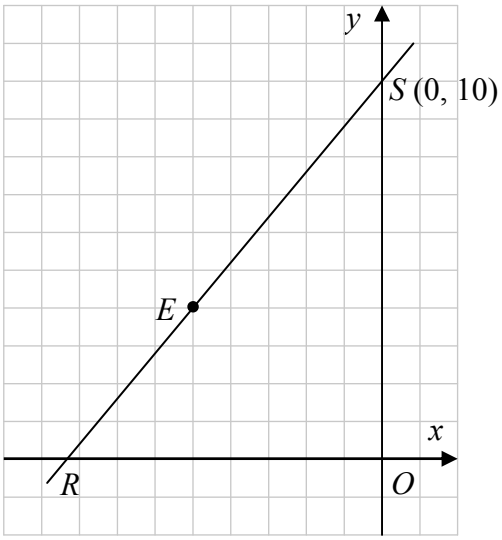
(ii) Using a calculator, or otherwise, calculate the standard deviation, σ, of the twelve values of V in the table, correct to the nearest whole number.

(c) **(i)** The standard deviation, σ, of closely spaced values of any function of the form $V = a\sin(bt)$, over 1 full period, is given by $k\sigma = V_{\max}$, where k is a constant that does not depend on a or b, and $V_{\max}$ is the maximum value of the function. Use the function $V = 311\sin(100\pi t)$ to find an approximate value for k correct to three decimal places.

(ii) Using your answer in part **(c) (i)**, or otherwise, find the value of b required so that the function $V = a\sin(bt)$ has 60 complete periods in one second and the approximate value of a so that it has a standard deviation of 110 volts.

Question 5 **(25 marks)**

The line RS cuts the x-axis at the point R and the y-axis at the point $S(0, 10)$, as shown. The area of the triangle ROS, where O is the origin, is $\frac{125}{3}$.



(a) Find the co-ordinates of R.

(b) Show that the point $E(-5, 4)$ is on the line RS.

(c) A second line $y = mx + c$, where m and c are positive constants, passes through the point E and again makes a triangle of area $\frac{125}{3}$ with the axes. Find the value of m and the value of c.

Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

(a) Prove that, if two triangles ΔABC and $\Delta A'B'C'$ are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$

Diagram:

Given:

To Prove:

Construction:

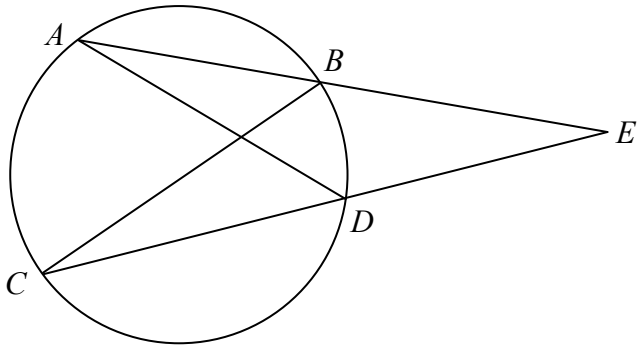
Proof:

(b) Given the line segment $[BC]$, construct, without using a protractor or set square, a point A such that $|\angle ABC| = 60°$. Show your construction lines.

OR

Question 6B

$[AB]$ and $[CD]$ are chords of a circle that intersect externally at E, as shown.



(a) Name two similar triangles in the diagram above and give reasons for your answer.

(b) Prove that $|EA| \cdot |EB| = |EC| \cdot |ED|$.

(c) Given that $|EB| = 6 \cdot 25$, $|ED| = 5 \cdot 94$ and $|CB| = 10$, find $|AD|$.

Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 **(45 marks)**

Table 1 below gives details of the number of males (M) and females (F) aged 15 years and over at work, unemployed, or not in the labour force for each year in the period 2004 to 2013.

Table 1

Labour Force Statistics 2004 to 2013 - Persons aged 15 years and over (000's)

Year	At work			Unemployed			Not in labour force			Total
	M	F	Total	M	F	Total	M	F	Total	
2004	1045.9	738.9	1784.8	79.6	31.6	111.2	457.1	854.2	1311.3	3207.3
2005	1087.3	779.7	1867.0	81.3	33.5	114.8	459.5	846.6	1306.1	3287.9
2006	1139.8	815.1	1954.9	80.6	38.1	118.7	457.6	844.9	1302.5	3376.1
2007	1184.0	865.6	2049.6	84.3	39.2	123.5	472.4	852.7	1325.1	3498.2
2008	1170.9	889.5	2060.4	106.3	41.0	147.3	494.8	872.5	1367.3	3575.0
2009	1039.8	863.5	1903.3	234.0	82.4	316.4	505.6	874.9	1380.5	3600.2
2010	985.1	843.5	1828.6	257.6	98.2	355.8	529.2	884.6	1413.8	3598.2
2011	970.2	843.2	1813.4	260.7	103.4	364.1	540.1	881.5	1421.6	3599.1
2012	949.6	823.8	1773.4	265.2	108.0	373.2	546.5	896.9	1443.4	3590.0
2013	974.4	829.0	1803.4	227.7	102.3	330.0	557.8	895.0	1452.8	3586.2

(*Source: Central Statistics Office http://www.cso.ie*)

(a) Suggest two categories of people, aged 15 years and over, who might not be in the labour force.

(b) Find the median and the interquartile range of the total persons at work over the period.

(c) The following data was obtained from Table 1. The percentages of persons aged 15 years and over at work, unemployed, or not in the labour force for the year 2006 are given below.

		At work	Unemployed	Not in the labour force
Persons aged 15 years and over	2006	57·9%	3·5%	38·6%
	2011			

(i) Complete the table for the year 2011. Give your answers correct to one decimal place.

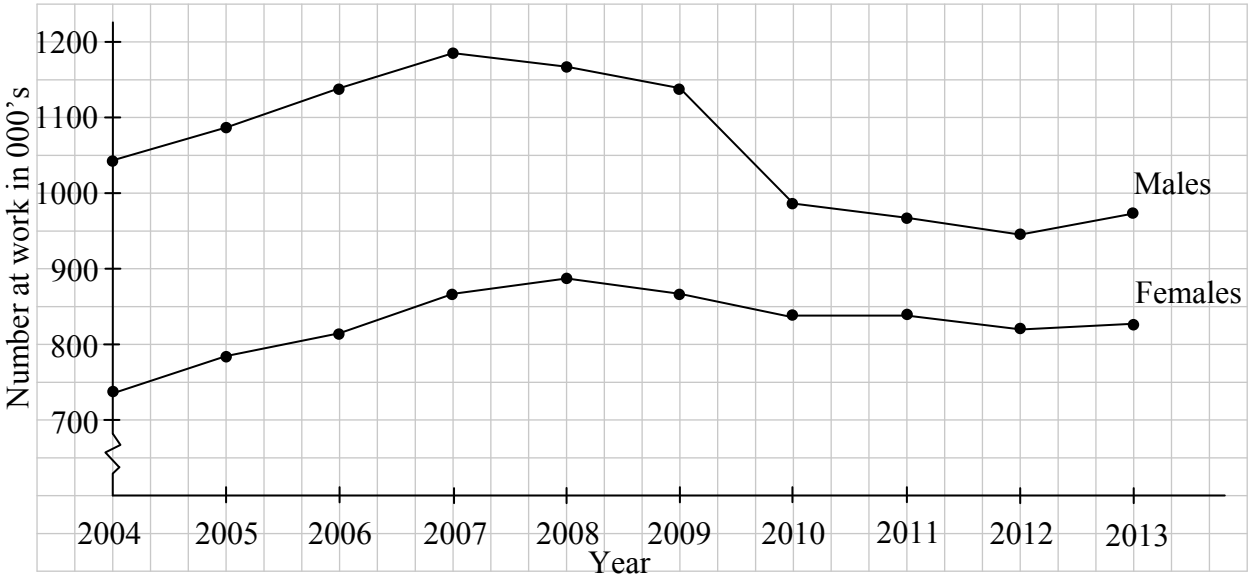
(ii) A census in 2006 showed that there were 864 449 persons in the population aged under 15 years of age. The corresponding number in the 2011 census was 979 590. Assuming that none of these persons are in the labour force, complete the table below to give the percentages of the *total population* at work, unemployed, or not in the labour force for the year 2011.

		At work	Unemployed	Not in the labour force
Total population	2006	46·1%	2·8%	51·1%
	2011			

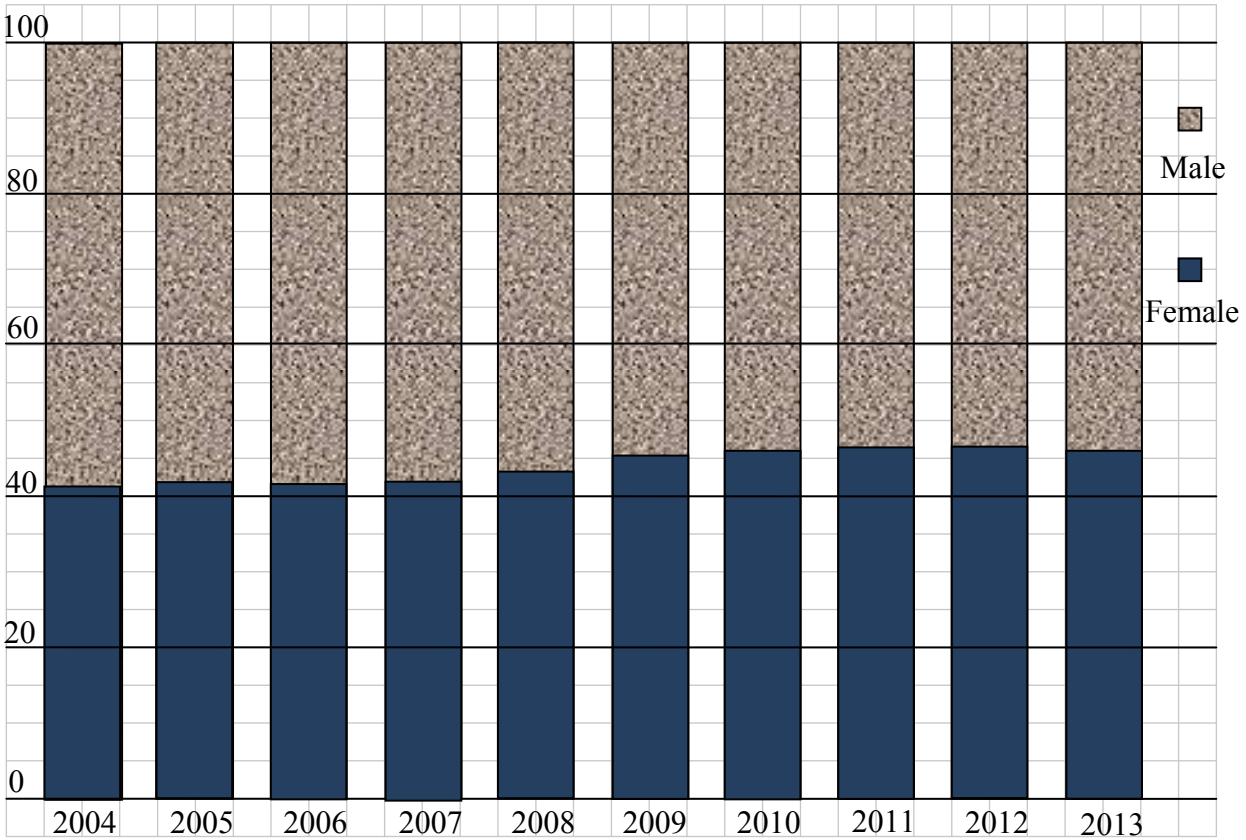
(iii) A commentator states that "The changes reflected in the data from 2006 to 2011 make it more difficult to balance the Government's income and expenditure."
Do you agree with this statement? Give two reasons for your answer based on your calculations above.

(d) Liam and Niamh are analysing the number of males and the number of females at work over the period 2004 to 2013.

Liam draws the following chart, using data from Table 1.



Niamh uses the same data and calculates the number of females at work as a percentage of the total number of persons at work and then draws the following chart.



(i) Having examined both charts, a commentator states "females were affected just as much as males by the downturn in employment." Do you agree or disagree with this statement? Give a reason for your conclusion.

(ii) Which, if any, of the two charts did you find most useful in reaching your conclusion above? Give a reason for your answer.

(iii) Use the data in Table 1, for the years 2012 and 2013 only, to predict the percentage of persons, aged 15 years and over, who will be at work in 2014.

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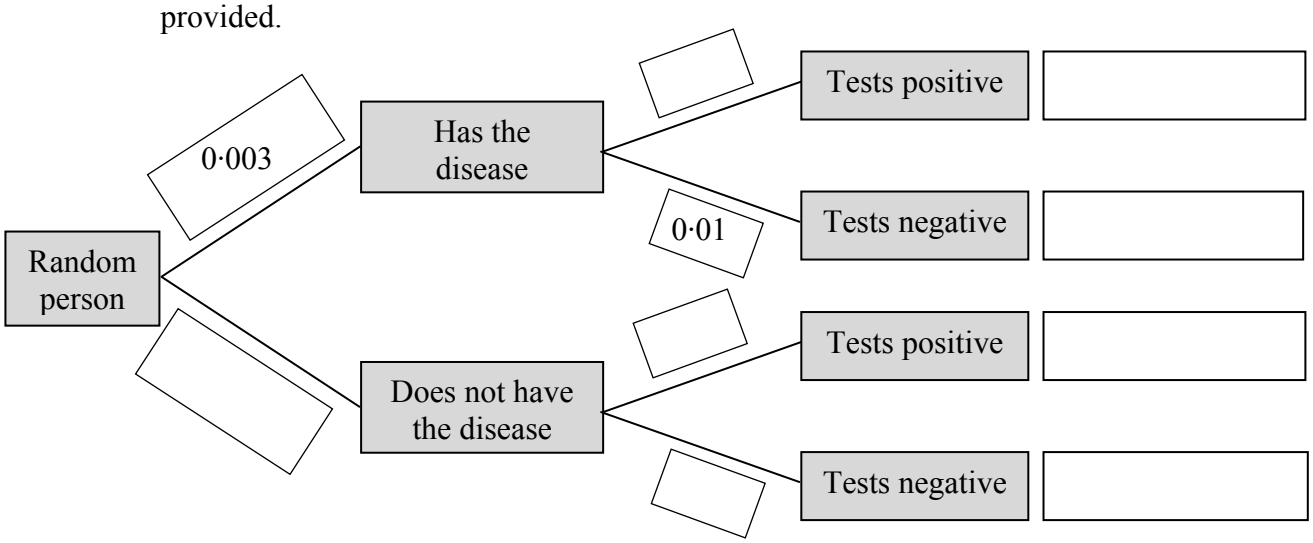
Question 8 **(45 marks)**

Blood tests are sometimes used to indicate if a person has a particular disease. Sometimes such tests give an incorrect result, either indicating the person has the disease when they do not (called a false positive) or indicating that they do not have the disease when they do (called a false negative).

It is estimated that 0·3% of a large population have a particular disease. A test developed to detect the disease gives a false positive in 4% of tests and a false negative in 1% of tests. A person picked at random is tested for the disease.

(a) **(i)** Write the probability associated with each branch of the tree diagram in the blank boxes provided.



(ii) Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.

(iii) A person tests positive for the disease. What is the probability that the person actually has the disease? Give your answer correct to three significant figures.

(iv) The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.

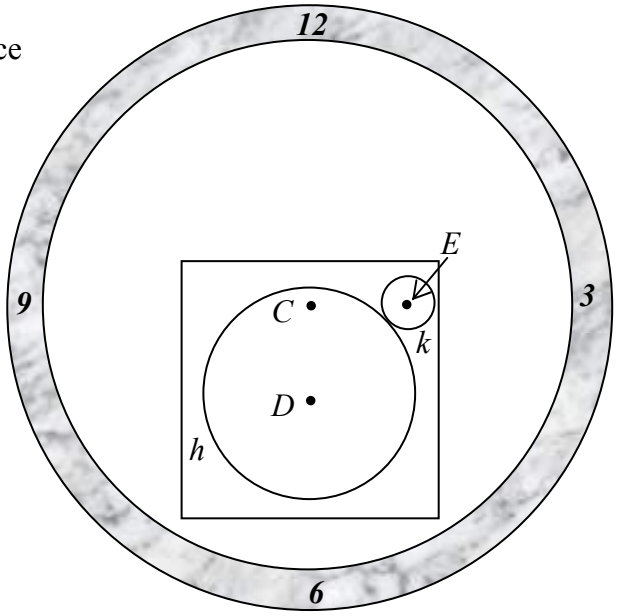
(b) A generic drug used to treat a particular condition has a success rate of 51%. A company is developing two new drugs, *A* and *B*, to treat the condition. They carried out clinical trials on two groups of 500 patients suffering from the condition. The results showed that Drug *A* was successful in the case of 296 patients. The company claims that Drug *A* is more successful in treating the condition than the generic drug.

(i) Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to justify the company's claim. State the null hypothesis and state your conclusion clearly.

(ii) The null hypothesis was accepted for Drug *B*. Estimate the greatest number of patients in that trial who could have been successfully treated with Drug *B*.

Question 9 **(60 marks)**

(a) The diagram shows a circular clock face, with the hands not shown. The square part of the clock face is glass so that the mechanism is visible. Two circular cogs, h and k, which touch externally are shown.



The point C is the centre of the clock face. The point D is the centre of the larger cog, h, and the point E is the centre of the smaller cog, k.

(i) In suitable co-ordinates, the equation of the circle h is

$$x^2 + y^2 + 4x + 6y - 19 = 0.$$

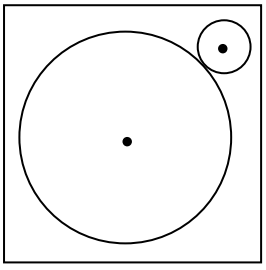
Find the radius of h, and the co-ordinates of its centre, D.

(ii) The point E has co-ordinates $(3, 2)$. Find the radius of the circle k.

(iii) Show that the distance from $C(-2, 2)$ to the line DE is half the length of $[DE]$.

(iv) The translation which maps the midpoint of $[DE]$ to the point C maps the circle k to the circle j. Find the equation of the circle j.

(v) The glass square is of side length l. Find the smallest whole number l such that the two cogs, h and k, are fully visible through the glass.



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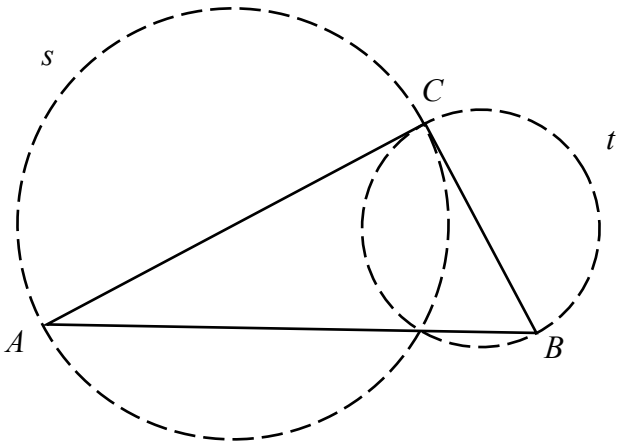
(b) The triangle ABC is right-angled at C.

The circle s has diameter $[AC]$ and the circle t has diameter $[CB]$.



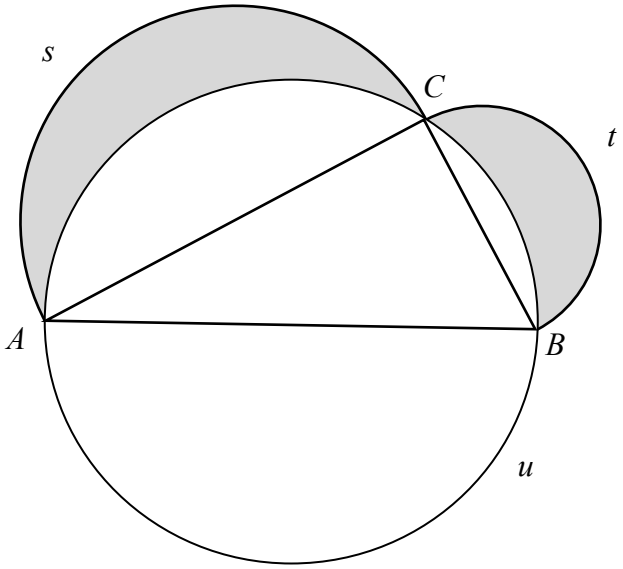
(i) Draw the circle u which has diameter $[AB]$.

(ii) Prove that in any right-angles triangle ABC, the area of the circle u equals the sum of the areas of the circles s and t.

(iii) The diagram shows the right-angled triangle ABC and arcs of the circles s, t and u.

Each of the shaded areas in the diagram is called a lune, a crescent-shaped area bounded by arcs of the circles.

Prove that the sum of the areas of the two shaded lunes is equal to the area of the triangle ABC.



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