Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2019

Marking Scheme

Mathematics

Higher Level

V2

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Paper 1

Paper 2

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5		
10 mark scales	0, 10	0, 5, 10	0, 4, 7, 10	0, 4, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Throughout the scheme indicate by use of # where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1	
(a)	10D
(b)	15D
Question 2	
(a)(i)	5C
(a) (ii)	5C
(b)	15D
Question 3	
(a)	5B
(b)	10D
(c)	10D
Question 4	
(a)	5C
(b)(i)	10D
(b)(ii)	10D
Question 5	
(a)	10C
(b)(i)	5C
(b)(ii)	10C
Question 6	
(a)(i)	10C
(a)(ii)	5C
(b)	10D

Section B

Question 7	(45 Marks)
(a)(i)	10C
(a)(ii)	5C
(a)(iii)	5C
(b)(i)	10C
(b)(ii)	5B
(b)(iii)	10C
Question 8	(50 Marks)
(a)	10C
(b)	10D
(c)	5C
(d)	10C
(e)	15D
Question 9	(55 Marks)
(a)(i)	5C
(a)(ii)	5C
(b)(i)+(ii)	15D
(b)(iii)	5C
(c)(i)	5C
(c)(ii)	5B
(c)(iii)	15D

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✔	Tick	Work of relevance	The work presented in the body of the script merits full credit
✘	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
#	Hash	Rounding error Unit error Arithmetic error Misreading	
	Horizontal wavy	Error	
✔l	Tick L		The work presented in the body of the script merits low partial credit
✔m	Tick M		The work presented in the body of the script merits mid partial credit
✔h	Tick H		The work presented in the body of the script merits high partial credit
[	Left Bracket		Another version of this solution is presented elsewhere and is worth equal or higher credit
	Vertical wavy	No work on this page (portion of the page)	
$\boxed{O}$	Oversimplify	The candidate has oversimplified the work	

Note: It may be necessary to use a combination of 2 symbols in the right margin to clearly show your judgement of the work in the body of the script:

✔ # must be used to signify that Full Credit – 1 is merited by the work presented

[✔m Signifies that the work in the body of the script is worth mid partial credit but another effort at the work has been awarded this or higher credit

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

e.g. In a **C scale** where # and and appear in the body of the work then should be placed in the right margin.

In the case of a **D scale** with the same level of annotation then should be placed in the right margin.

A in the body of the work may sometimes be used indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme.

The level of credit is them indicated in the right margin.

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$(2x+1)(x^2+px+4)$ $2x^3+2px^2+8x+x^2+px+4$ $8+p=2(2p+1)$ $8+p=4p+2$ $3p=6$ $p=2$ Or Coefficient of x is $8+p$ Coefficient of x^2 is $2p+1$ $8+p=2(2p+1)$ $8+p=4p+2$ $3p=6$ $p=2$	**Scale10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant multiplication *Mid Partial credit:* - Multiplication completed without error(**s**) - Multiplication completed with errors and correctly identifies (in terms of p) the coefficient of **either** x^2 or x - Correctly identifies the coefficient of **either** x or x^2 *High Partial credit:* - Multiplication completed with error(s) but finishes correctly without further errors - Relevant coefficients equated (equation in p) - Multiplication completed and coefficients of x^2 and x identified but solves incorrect equation in p

<table>
<tr>
<td>(b)</td>
<td>

$$\frac{3}{2x+1}+\frac{2}{5}=\frac{2}{3x-1}$$

CD: $5(2x+1)(3x-1)$

$$15(3x-1)+(4x+2)(3x-1)$$
$$=10(2x+1)$$
$$12x^2+27x-27=0$$
$$4x^2+9x-9=0$$
$$(x+3)(4x-3)=0$$
$$x=-3 \text{ or } x=\frac{3}{4}$$

Or

$$\frac{3}{2x+1}+\frac{2}{5}=\frac{2}{3x-1}$$
$$\frac{15+2(2x+1)}{5(2x+1)}=\frac{2}{3x-1}$$
$$\frac{4x+17}{10x+5}=\frac{2}{3x-1}$$
$$(4x+17)(3x-1)=2(10x+5)$$
$$12x^2+47x-17=20x+10$$
$$12x^2+27x-27=0$$
$$4x^2+9x-9=0$$
$$(x+3)(4x-3)=0$$
$$x=-3 \text{ or } x=\frac{3}{4}$$

</td>
<td>

Scale 15D (0, 4, 7, 11,15)

Low Partial Credit:

- CD or partial CD identified
- Cross multiply on LHS
- Multiplies one term correctly by one of the denominators
- $x=-3$ **or** $x=\frac{3}{4}$ substituted and justified as a solution

Mid Partial Credit:

- Equation without fractions

High Partial Credit:

- Relevant quadratic in the form: $ax^2+bx+c=0$

<u>*Note*</u>: No quadratic $\Rightarrow$ low partial credit at most, except in the case where the candidate has reached the mid partial stage

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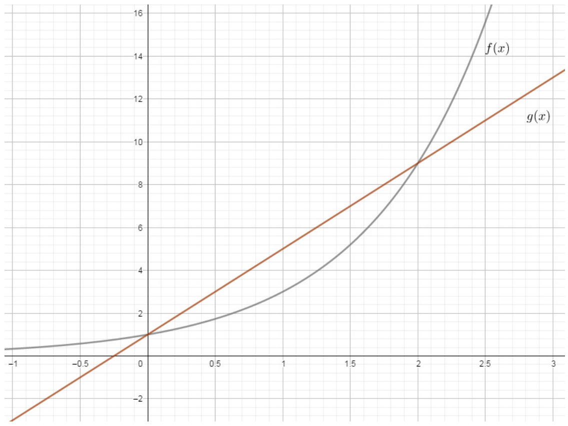
Q2	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	(0, 1) (2, 9) 	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - 1 point on line found *High Partial Credit:* - 2 points on line found - 1 point found and plotted (apart from (0, 1) and (2, 9)) *Full Credit −1*: - Freehand graph drawn
(a) **(ii)**	$g(1{\cdot}9) = 4(1{\cdot}9) + 1 = 8{\cdot}6$ $f(1{\cdot}9) = 3^{1{\cdot}9} = 8{\cdot}06$ $f(x) < g(x)$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - $g(1{\cdot}9)$ written or found - $f(1{\cdot}9)$ written or found *High Partial Credit:* - $g(1{\cdot}9)$ and $f(1{\cdot}9)$ found

<table>
<tr>
<td>(b)</td>
<td>To Prove: $3^n \geq 4n + 1$ for $n \geq 2$
$P(2): 3^2 \geq 4(2) + 1$
$9 \geq 9$, True
Assume $P(n)$ is true for $n = k$,
Now prove $P(n)$ is true for $n = k + 1$
$P(k)$: $3^k \geq 4k + 1$ for $k \geq 2$
$P(k+1): 3^{k+1} \geq 4(k+1) + 1$
$3^{k+1} \geq 4k + 5$

Proof: $P(k) \times 3$: $3^{k+1} \geq 3(4k+1)$
$= 12k + 3$
$\Rightarrow$ $3^{k+1} \geq 4k + 5$
since $4k + 5 < 12k + 3$ for $k \geq 2$

True for
$n = k + 1$ provided true for $n = k$
but true for $n = 2$
$\therefore$ True for all $n \geq 2, n \in \mathbb{N}$.</td>
<td>Scale 15D (0, 4, 7, 11, 15)
Low Partial Credit:
- Step $P(2)$
- $P(k)$ or $P(k+1)$ with incorrect inequality sign

Mid Partial Credit:
- Any two of $P(2)$, $P(k)$ or $P(k+1)$

High Partial Credit:
- Uses Step $P(k)$ to prove Step $P(k+1)$

Full Credit −1:
- Omits conclusion but otherwise correct

<u>Note</u>: Accept Step $P(2)$, Step $P(k)$, Step $P(k+1)$ in any order

<u>Note</u>: Accept $f(k) \geq g(k)$, $k \geq 2$ for Step $P(k)$</td>
</tr>
</table>

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$(3x+4)(y-3)$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* - Any relevant factorisation
(b)	$3xlnx - 9x + 4lnx - 12 =$ $3x(lnx-3) + 4(lnx-3) =$ $(3x+4)(lnx-3)$ $3x+4=0 \Rightarrow x = -\frac{4}{3}$ $lnx - 3 = 0$ $lnx = 3$ $x = e^3$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant factorisation of $g(x)$ - Trial and improvement with at least two values tested - Substitutes $20 \leq x \leq 20{\cdot}1$ - $y = lnx$ *Mid Partial Credit* - Expression fully factorised *High Partial Credit:* - $lnx = 3$ *Full Credit :* - Both solutions presented <u>Note</u>: Accept $x = 20{\cdot}1$ for $x = e^3$ in the last line of the solution <u>Note</u>: If no reference is made to $3x+4$ in the solution, then award high partial credit **at most**

(c)	$g'(x) = 3x\left(\frac{1}{x}\right) + (3)lnx - 9 + 4\left(\frac{1}{x}\right)$ $g'(e) = 3(e)\left(\frac{1}{e}\right) + (3)\,ln(e) - 9 + 4\left(\frac{1}{e}\right)$ $g'(e) = 3 + 3 - 9 + \frac{4}{e} = -1{\cdot}53$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant differentiation - $g(e)$ evaluated correctly to at least 2 decimal places *Mid Partial Credit* - Expression fully differentiated - Product rule not applied but finishes correctly *High Partial Credit:* - Derivative fully substituted

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$\frac{4x^4}{4} - \frac{6x^2}{2} + 10x + C$ $x^4 - 3x^2 + 10x + C$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Any relevant integration *High Partial Credit:* - 3 correct terms
(b) (i)	$\int (6x^2 - 54x + 109)\, dx$ $= 2x^3 - 27x^2 + 109x + C = f(x)$ $(2, 0) \in f(x)$ $2(2)^3 - 27(2)^2 + 109(2) + C = 0$ $2(8) - 27(4) + 218 + C = 0$ $16 - 108 + 218 + C = 0$ $16 + 110 + C = 0$ $126 + C = 0$ $C = -126$ $\therefore f(x) = 2x^3 - 27x^2 + 109x - 126$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant integration *Mid Partial Credit* - 3 correct terms *High Partial Credit:* - Relevant equation in C *Note:* Must integrate or indicate integration to gain any credit

(b) (ii)	2 is a root $\Rightarrow (x-2)$ is a factor $2x^3 - 27x^2 + 109x - 126 = 0$ $2x^2(x-2) - 23x(x-2) + 63(x-2)$ $2x^2 - 23x + 63 = 0$ $(2x - 9)(x-7) = 0$ $x = 4{\cdot}5$ or $x = 7$ $\therefore B(4{\cdot}5, 0)$ and $C(7, 0)$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - 2 identified as root - 0 given as the y co-ordinate - Sets up equation - Any integer fully substituted in $f(x)$ fully worked - $(x-2)$ is a factor - Sets up the correct equation *Mid Partial Credit* - Division completed with no remainder - 7 identified as a root - One coordinate pair found *High Partial Credit:* - x values found from factors

<table>
<tr><th>Q5</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$3-2i =$ other root
$-p=(3+2i)+(3-2i)=6$
$p=-6$
$q=(3+2i)(3-2i)=13$

Or

$(3+2i)^2+p(3+2i)+q=0$
$5+12i+3p+2pi+q=0$
$2p=-12 \Rightarrow p=-6$
$5+3p+q=0 \Rightarrow q=13$

Or

$\dfrac{-p\pm\sqrt{p^2-4q}}{2}=3\pm 2i$
$-p\pm\sqrt{p^2-4q}=6\pm 4i$

$-p=6$
$\therefore p=-6$

$\sqrt{4q-p^2}=4$
$4q-p^2=16$
$4q-(-6)^2=16$

$4q=52$
$\therefore q=13$</td><td>Scale 10C (0, 4, 7, 10)
Low Partial Credit:
- Second root identified
High Partial Credit:
- Sum and product of roots formulated into equations for p and q
- p or q found correctly

Low Partial Credit:
- Root substituted into equation
- Any correct substitution

High Partial Credit:
- Real and imaginary terms formulated into equations for p and for q

Low Partial Credit:
- Some substitution into quadratic formula

High Partial Credit:
- Finds p
- Full substitution into quadratic formula and equated to either root.</td></tr>
</table>

(b) (i)	$\lvert v\rvert = \sqrt{4+12} = 4$ $\theta = 300°$ $v = 4(\cos 300° + i\sin 300°)$ Or $\lvert v\rvert = \sqrt{4+12} = 4$ $\theta = \frac{5\pi}{3}$ $v = 4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Correct plot on the Argand diagram - Some use of Pythagoras to find modulus - Some use of trigonometry to find argument *High Partial Credit:* Modulus or argument found <u>Note</u>: Accept $4\left(\cos -\frac{\pi}{3} + i\sin -\frac{\pi}{3}\right)$ and $4(\cos -60° + i\sin -60°)$
(b) (ii)	$w = \pm v^{\frac{1}{2}}$ $w = \pm 2(\cos 300 + i\sin 300)^{\frac{1}{2}}$ $w = \pm 2(\cos 150 + i\sin 150)$ $w = \pm(-\sqrt{3} + i)$ $w = -\sqrt{3} + i$ or $\sqrt{3} - i$ Or $w = [4(\cos(300 + 360n) + i\sin(300 + 360n)]^{\left(\frac{1}{2}\right)}$ $w = 4^{\frac{1}{2}}[\cos(150 + 180n) + i\sin(150 + 180n)]$ $\underline{n = 0}$ $w = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\sqrt{3} + i$ $\underline{n = 1}$ $w = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt{3} - i$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - w written in polar form with index - Some use of De Moivre's Theorem - $w = v^{\frac{1}{2}}$ *High Partial Credit:* - De Moivre's theorem applied to w - One solution found - Solutions in polar form <u>Note</u>: Accept candidates answer from (b)(i)

Q6	Model Solution – 25 Marks	Marking Notes
(a) (i)	$x + 5x = \sqrt{128} + \sqrt{32}$ $6x = 8\sqrt{2} + 4\sqrt{2}$ $6x = 12\sqrt{2}$ $x = 2\sqrt{2}$ Or $x - \sqrt{32} = \sqrt{128} - 5x$ $\left(x - \sqrt{32}\right)^2 = \left(\sqrt{128} - 5x\right)^2$ $\left(x - 4\sqrt{2}\right)^2 = \left(8\sqrt{2} - 5x\right)^2$ $x^2 - 8\sqrt{2}x + 32 = 128 - 80\sqrt{2}x + 25x^2$ $x^2 - 3\sqrt{2}x + 4 = 0$ $\left(x - \sqrt{2}\right)\left(x - 2\sqrt{2}\right) = 0$ $x = \sqrt{2}$ or $x = 2\sqrt{2}$ Check solutions: $x = \sqrt{2}$ $\sqrt{2} - \sqrt{32} = \sqrt{128} - 5\sqrt{2}$ $-3\sqrt{2} = 3\sqrt{2}$ (False) Solution: $x = 2\sqrt{2}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Any relevant transposing - $\sqrt{32}$ or $\sqrt{128}$ in the form a$\sqrt{2}$ *High Partial Credit* - x term isolated in equation *Low Partial Credit:* - $\sqrt{32}$ or $\sqrt{128}$ in the form $a\sqrt{2}$ - Any relevant multiplication *High Partial Credit:* - LHS and RHS squared correctly - Solution not in the form $a\sqrt{2}$ *Full Credit –1:* - Both solutions presented Note: If $\sqrt{128}$ and $\sqrt{32}$ are converted to decimals, then award low partial credit at most
(a) (ii)	$\sqrt{32k^2}, \sqrt{128k^2}, \sqrt{98k^2}, \sqrt{50k^2}$ $4\sqrt{2}\,k,\quad 8\sqrt{2}\,k,\quad 7\sqrt{2}\,k,\quad 5\sqrt{2}\,k$ $4\sqrt{2}\,k,\quad 5\sqrt{2}\,k,\quad 7\sqrt{2}\,k,\quad 8\sqrt{2}\,k$ Mean $= \dfrac{24\sqrt{2}\,k}{4} = 6\sqrt{2}\,k$ Median $= 6\sqrt{2}\,k$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - List in ascending or descending order - Any term written in the form $a\sqrt{2}k$ or in the form $a\sqrt{2k^2}$ *High Partial Credit:* - Mean or median found - Verified for a particular value of k Note: If decimals are used then award low partial credit at most

(b)		
	Assume $\sqrt{2}$ is rational i.e. $\sqrt{2} = \frac{p}{q}$ where p and q have no common factors (simplest form) $\Rightarrow 2 = \frac{p^2}{q^2}$ $\Rightarrow 2q^2 = p^2$ $\Rightarrow p^2$ is even $\Rightarrow$ p is even $\Rightarrow p = 2k$ for some $k \in \mathbb{Z}$ $2q^2 = p^2$ becomes $2q^2 = 4k^2$ $\Rightarrow q^2 = 2k^2$ $\Rightarrow q^2$ is even $\Rightarrow$ q is even $\Rightarrow q = 2m$ for some $m \in \mathbb{Z}$ $\therefore \sqrt{2} = \frac{p}{q} = \frac{2k}{2m}$ $\Rightarrow$ common factor of 2 (contradiction) $\therefore \sqrt{2}$ cannot be rational.	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - $\sqrt{2} = \frac{p}{q}$ or similar *Mid Partial Credit* - deduces that p is even or equivalent - $p = 2k$ or equivalent deduced - $p^2 = 2q^2$ *High Partial Credit:* - $q = 2m$ or equivalent deduced

Section B

Q7	Model Solution – 45 Marks	Marking Notes
(a) (i)	<table><tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr><tr><td>Fraction</td><td>$\frac{1}{3}$</td><td>$\frac{2}{9}$</td><td>$\frac{4}{27}$</td><td>$\frac{8}{81}$</td><td>$\frac{16}{243}$</td></tr></table>	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - 1 correct fraction given in table - 1 correct denominator - 1 correct numerator *High Partial Credit:* - 2 correct fractions given in table - All numerators correct - All denominators correct
(a) (ii)	$a = \frac{1}{3} \;\; r = \frac{2}{3}$ $S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{\frac{1}{3}\left(1-\left(\frac{2}{3}\right)^n\right)}{1-\frac{2}{3}}$ $S_n = 1 - \left(\frac{2}{3}\right)^n$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - S_n formula with some substitution - Correct a or correct r identified *High Partial Credit:* - S_n formula fully substituted
(a) (iii)	Infinite Geometric Series $a = \frac{1}{3} \;\; r = \frac{2}{3}$ $S_\infty = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{2}{3}} = 1$ Or $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(1 - \left(\frac{2}{3}\right)^n\right) = 1$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - S_∞ indicated - Correct a or correct r identified *High Partial Credit:* - S_∞ fully substituted *<u>Note</u>*: If $\|r\| > 1$, then award low partial credit at most

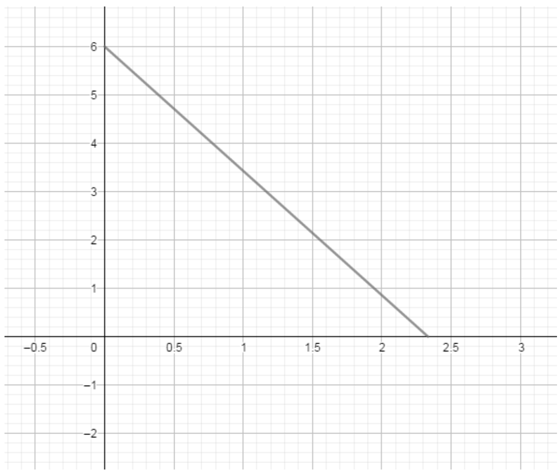
(b) (i)	| Label | A | B | C | D | E | F | | End-point | $\frac{2}{3}$ | $\frac{2}{9}$ | $\frac{7}{9}$ | $\frac{8}{9}$ | $\frac{7}{27}$ | $\frac{25}{27}$ |	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - 1 correct fraction given in table - All denominators correct *High Partial Credit:* - 4 correct fractions given in table
(b) (ii)	It is the end point (start point) of a segment Or $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}=\frac{20}{81}$ $\frac{6}{27}$ ∘ ∘ $\boldsymbol{E}\,\frac{7}{27}$ $\frac{18}{81}$ $\frac{19}{81}$ $\frac{20}{81}$ $\frac{21}{81}$ Or $\frac{7}{27}-\frac{1}{81}=\frac{20}{81}$ is a point in the Cantor Set	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* - Relevant but incomplete reason given - Sum of fractions $=\frac{20}{81}$
(b) (iii)	$S_\infty=\frac{a}{1-r}=\frac{\frac{1}{3}}{1-\left(-\frac{1}{3}\right)}=\frac{1}{4}$ Or $\frac{1}{3}+\frac{1}{27}+\frac{1}{243}+\cdots=\frac{\frac{1}{3}}{1-\frac{1}{9}}$ $=\frac{3}{8}$ $-\left(\frac{1}{9}+\frac{1}{81}+\frac{1}{729}+\cdots\right)=-\left(\frac{\frac{1}{9}}{1-\frac{1}{9}}\right)$ $=-\frac{1}{8}$ $S_\infty=\frac{3}{8}-\frac{1}{8}$ $=\frac{1}{4}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - S_∞ indicated - S_∞formula with some substitution - Correct a or correct r *High Partial Credit:* - S_∞ formula fully substituted

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$r(20) = 22500\cos\left(\frac{\pi}{26}(20)\right) + 37500$ $= 22500\cos\left(\frac{20\pi}{26}\right) + 37500$ $= €20658{\cdot}51$ $\approx €20659$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Any relevant substitution - $r(20)$ or $t = 20$ *High Partial Credit:* - Correct substitution *Full Credit –1:* - Uses degrees as unit of measurement, giving an answer of €59980
(b)	$22500\cos\left(\frac{\pi}{26}t\right) + 37500 = 26250$ $22500\cos\left(\frac{\pi}{26}t\right) = -11250$ $\cos\left(\frac{\pi}{26}t\right) = -\frac{1}{2}$ $\frac{\pi}{26}t = \frac{2\pi}{3}$ and $\frac{\pi}{26}t = \frac{4\pi}{3}$ $t = \frac{52}{3}$ and $t = \frac{104}{3}$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Equation formed - Trial and improvement with at least two values tested *Mid Partial Credit:* - Equation simplified to: $\cos\left(\frac{\pi}{26}t\right) = -\frac{1}{2}$ - Equation simplified to: $\cos\left(\frac{\pi}{26}t\right) = -\frac{11250}{22500}$ *High Partial Credit:* - 1 correct solution to equation found

(c)	$r'(t) = 22500\left[-\sin\left(\frac{\pi}{26}t\right)\right]\left(\frac{\pi}{26}\right)$ $= -\frac{11250}{13}\pi\left[\sin(\frac{\pi}{26}t)\right]$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Some relevant differentiation *High Partial Credit:* - Chain rule applied
(d)	$r'(30) = -\frac{11250}{13}\pi\left[\sin(\frac{\pi}{26}(30))\right]$ $= 402{\cdot}164\pi$ $= 1263{\cdot}44$ > 0 ⇒ Increasing	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Some relevant substitution into answer from (c) - $r'(t) > 0$ - $\frac{dy}{dx} > 0$ *High Partial Credit:* - $r'(30)$ found but no conclusion or incorrect conclusion <u>*Note*</u>: If calculus is not used then award no credit for the solution

(e)	$-\frac{11250}{13}\pi\left[\sin\left(\frac{\pi}{26}t\right)\right]=0$ $\sin\left(\frac{\pi}{26}t\right)=0$ $\frac{\pi}{26}t=0$ and $\frac{\pi}{26}t=\pi$ $t=0$ and $t=26$ $r''(t)=-\frac{11250}{13}\pi\left[\cos\left(\frac{\pi}{26}t\right)\right]\left(\frac{\pi}{26}\right)$ $t=0$: $r''(t)<0 \Rightarrow$ Max $t=26$: $r''(t)>0 \Rightarrow$ Min Or Range: $[37500-22500, 37500+22500]$ $=[15{,}000, 60{,}000]$ $22500\cos\left(\frac{\pi}{26}t\right)+37500=15000$ $22500\cos\left(\frac{\pi}{26}t\right)=15000-37500$ $22500\cos\left(\frac{\pi}{26}t\right)=-22500$ $\cos\left(\frac{\pi}{26}t\right)=-1$ $\frac{\pi}{26}t=\pi$ $\therefore t=26$ $r''(26)=-\frac{11250}{13}\pi\left[\cos\left(\frac{\pi}{26}26)\right)\right]\left(\frac{\pi}{26}\right)$ >0 $\Rightarrow$ Min	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* - $r'(t)=0$ - $\frac{dy}{dx}=0$ - States $r''(t)>0$ at minimum value - $t=26$ and no further work *Mid Partial Credit* - $t=0$ or $t=26$ found with supporting work - $r''(t)$ found *High Partial Credit:* - $t=26$ found with supporting work and $r''(t)$ found (including use of chain rule)

Q9	Model Solution – 55 Marks	Marking Notes
(a) **(i)**	$= 2(x) + 2(y) + \frac{1}{2}(2\pi)(x)$ $= 2x + 2y + \pi x$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Some relevant substitution into perimeter formula - Circumference of circle of radius x found i.e. $2\pi x$ *High Partial Credit:* - Two of the three terms found
(a) **(ii)**	$2x + 2y + \pi x = 12$ $2y = 12 - 2x - \pi x$ $y = \dfrac{12 - 2x - \pi x}{2}$ $y = \dfrac{12 - (2 + \pi)x}{2}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Some relevant substitution into equation *High Partial Credit:* - y term isolated correctly in equation <u>Note</u>: Accept candidates answer from (a)(i) provided it doesn't oversimplify the work. <u>Note</u>: Must draw a relevant conclusion from incorrect work

(b) **(i)** **+** **(ii)**	**Table and Graph** <table><tr><td>x</td><td>0</td><td>$\frac{12}{2+\pi}$</td></tr><tr><td>$y = \frac{12-(2+\pi)x}{2}$</td><td>6</td><td>0</td></tr></table> 	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* - One correct table entry - One correct plot of incorrect point *Mid Partial Credit:* - 2 table entries correct - 2 incorrect points plotted and joined *High Partial Credit:* - 2 correct points plotted but not joined with correct table entries *Full Credit −1:* - Two correct points plotted and joined but the function is not graphed in the stated domain <u>*Note*</u>: Accept $2{\cdot}25 \le x \le 2{\cdot}5$ for x−intercept

(b) (iii)	$y = \frac{12 - (2+\pi)x}{2}$ $y = 6 - \left(\frac{2+\pi}{2}\right)x$ $m = -\left(\frac{2+\pi}{2}\right)$ $m = -2{\cdot}57$ Or $m = \frac{0-6}{\frac{12}{2+\pi} - 0}$ $m = -\left(\frac{2+\pi}{2}\right)$ $m = -2{\cdot}57$ Intepretation: For each 1m rise in the radius of the semi-circle, the height of the rectangle falls by approximately 2·57 m	. **Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Some substitution into slope formula - Slope isolated in the equation of the line formula - $\frac{dy}{dx}$ - $\frac{rise}{run}$ with some relevant substitution - Some effort at differentiation *High Partial Credit:* - Slope found *Note*: Accept $-2{\cdot}7 \leq$ slope $\leq -2{\cdot}5$ from relevant work
(c) (i)	$a = 2xy + \frac{\pi x^2}{2}$ $= \frac{2x[(12-(2+\pi)x]}{2} + \frac{\pi x^2}{2}$ $= \frac{24x - 4x^2 - 2\pi x^2}{2} + \frac{\pi x^2}{2}$ $= \frac{24x - (\pi+4)x^2}{2}$	0. **Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - area of rectangle correct - area of semi-circle correct *High Partial Credit:* - Both areas correct in terms of x and added

(c) (ii)	$a(x) = \frac{1}{2}(24x - (\pi + 4)x^2)$ $a'(x) = \frac{1}{2}(24 - 2(\pi + 4)x)$ $= 12 - (\pi + 4)x$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* - Some correct differentiation
(c) (iii)	$a'(x) = 0$ $12 - (\pi + 4)x = 0$ $(\pi + 4)x = 12$ $x = \frac{12}{\pi+4}$ (1·68) $y = \frac{12-(2+\pi)x}{2}$ $(= \frac{12-(5\cdot14)\,1\cdot68}{2} \approx 1\cdot68)$ $= \frac{12-(2+\pi)(\frac{12}{\pi+4})}{2}$ $= \frac{12(\pi+4)-(2+\pi)(12)}{2(\pi+4)}$ $= \frac{12\pi+48-24-12\pi}{2(\pi+4)}$ $= \frac{24}{2(\pi+4)}$ $= \frac{12}{\pi+4}$ $= x$ Area Max when height equals the radius	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* - $a'(x)$ used - States $\frac{dy}{dx} = 0$ *Mid Partial Credit* - Value of x at maximum found *High Partial Credit:* - Value of y at maximum fully substituted

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2019

Marking Scheme

Mathematics

Higher Level

Paper 2

Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5		
10 mark scales			0, 4, 7, 10	0, 4, 5, 8, 10	
15 mark scales			0, 5, 10, 15	0, 5, 7, 11, 15	
20 mark scales					
25 mark scales					

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

Note: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.

Throughout the scheme indicate by use of # where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)(i)	10C
(a)(ii)	10D
(b)	5C

Question 2

(a)	10C
(b)(i)	5B
(b)(ii)	10D

Question 3

(a)	10C
(b)	15D

Question 4

(a)	10C
(b)(i)	15D

Question 5

(a)	15D
(b)(i)	10D

Question 6

(a)(i)	10C
(b)	15D

Section B

Question 7 (50)

(a)(i)	10C
(a)(ii)	5C
(a)(iii)	10D
(a)(iv)	5C
(b)(i)	15D
(b)(ii)	5C

Question 8 (45)

(a)(i)	10C
(a)(ii)	15D
(a)(iii)	10C
(b)(i)	5B
(b)(ii)	5C

Question 9 (55)

(a)	10C
(b)	10C
(c)	15C
(d)(i)	5B
(d)(ii)	5C
(d)(iii)	5C
(e)	5C

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	$\frac{12}{20}\times\frac{8}{19}+\frac{8}{20}\times\frac{12}{19}$ $=\frac{96}{380}+\frac{96}{380}$ or $2\left(\frac{96}{380}\right)$ $\frac{12}{20}\times\frac{8}{19}+\frac{8}{20}\times\frac{12}{19}=\frac{192}{380}$ or $\frac{48}{95}$ Or $\frac{\binom{12}{1}\binom{8}{1}}{\binom{20}{2}}=\frac{96}{190}$ or $\frac{48}{95}$ Or $1-\left[\frac{12}{20}\times\frac{11}{19}+\frac{8}{20}\times\frac{7}{19}\right]=1-\frac{188}{380}$ $=\frac{192}{380}$ or $\frac{48}{95}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* 1 probability given e.g. $\frac{12}{20}$ or equivalent 1 combination indicated e.g. $\binom{12}{1}$ or $\binom{8}{1}$ or $\binom{20}{2}$ $\frac{12}{20}\times\frac{8}{19}$ or $\frac{8}{20}\times\frac{12}{19}$ or equivalent and stops $\frac{\binom{12}{1}}{\binom{20}{2}}$ or $\frac{\binom{8}{1}}{\binom{20}{2}}$ and stops $1-\frac{12}{20}\times\frac{11}{19}$ or $1-\frac{8}{20}\times\frac{7}{19}$ and stops *High Partial Credit:* $\frac{12}{20}\times\frac{8}{19}$ or $\frac{8}{20}\times\frac{12}{19}$ or equivalent and continues $\frac{\binom{12}{1}}{\binom{20}{2}}$ or $\frac{\binom{8}{1}}{\binom{20}{2}}$ and continues $1-\frac{12}{20}\times\frac{11}{19}$ or $1-\frac{8}{20}\times\frac{7}{19}$ and continues
(a) **(ii)**	$\frac{12}{20}\times\frac{11}{19}\times\frac{10}{18}\times\frac{8}{17}=\frac{10560}{116280}$ or $\frac{88}{969}$ **Or** $\frac{\binom{12}{1}}{\binom{20}{1}}\times\frac{\binom{11}{1}}{\binom{19}{1}}\times\frac{\binom{10}{1}}{\binom{18}{1}}\times\frac{\binom{8}{1}}{\binom{17}{1}}$ $=\frac{10560}{116280}$ or $\frac{88}{969}$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* 1 probability given 1 combination indicated *Mid Partial Credit* 3 or 4 correct probabilities indicated *High Partial Credit:* 3 correct probabilities with multiplication completed 4 probabilities with correct operator

(b)	$\binom{6}{3} \times \binom{8}{4} = 1400$ or $\binom{1}{1} \times \binom{6}{3} \times \binom{8}{4} = 1400$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* $\binom{6}{3}$ or $\binom{8}{4}$ or $\binom{1}{1}$ *High Partial Credit:* $\binom{6}{3} \times \binom{8}{4}$ and stops

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$m = \frac{b-0}{0-a} = \frac{-b}{a}$ $y - 0 = \frac{-b}{a}(x - a)$ $ay = -bx + ab$ $bx + ay = ab$ Now divide across by ab $\frac{x}{a} + \frac{y}{b} = 1$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* Slope formula with some substitution *High Partial Credit:* Equation of line formula fully substituted
	Or $m = \frac{b-0}{0-a} = \frac{-b}{a}$ $y = mx + c \;\; => \;\; y = \frac{-b}{a}x + c$. But (o, b) is on this line, thus $b = \;\frac{-b}{a}(o) + c$ $\therefore \;\; b = c$ Equation $y = \frac{-b}{a}x + b$ $ay = -bx + ab$ $bx + ay = ab$ Now divide across by ab $\frac{x}{a} + \frac{y}{b} = 1$	*Low Partial Credit:* Slope formula with some substitution *High Partial Credit:* m expressed in terms of *a* and *b*, ***and*** c in terms of *b*
	Or $(a, 0) \; \in \; y = mx + c => 0 = ma + c$ $=> -ma = c$ $(0, b) \; \in \; y = mx + c => b = c$ $\therefore -ma = b => m = \frac{-b}{a}$ Equation $y = \frac{-b}{a}x + b$ $ay = -bx + ab$ $bx + ay = ab$ Now divide across by ab $\frac{x}{a} + \frac{y}{b} = 1$	
	Or $\frac{x}{a} + \frac{y}{b} = 1$ LHS: $\frac{x}{a} + \frac{y}{b}$ $(a, 0)$: $\frac{a}{a} + \frac{0}{b}$ = 1=1 or RHS $(0, b)$: $\frac{0}{a} + \frac{b}{b}$ = 1=1 or RHS	*Low Partial Credit:* $(a, 0)$ or ($(0, b)$ correctly substituted e.g. $\frac{a}{a} + \frac{0}{b}$ *High Partial Credit:* $(a, 0)$ **and** $(0, b)$ correctly substituted

<table>
<tr>
<td>(b)
(i)</td>
<td>$y - 0 = m(x - 6)$ <u>or</u> $y = m(x - 6)$
Or
$y = mx - 6m$
Or
$y = mx + c$
$\therefore 0 = 6m + c \Rightarrow c = -6m$</td>
<td>Scale 5B (0, 2, 5)
Mid Partial Credit:
Equation of line formula with some relevant substitution</td>
</tr>
<tr>
<td>(b)
(ii)</td>
<td>$y = m(x - 6)$
$4x + 3y = 25$
=> $4x + 3m(x - 6) = 25$

=>$x = \frac{25+18m}{3m+4}$

Substitute this into $y = m(x - 6)$
$y = m\left(\frac{25 + 18m}{3m + 4}\right) - 6m$
$= \frac{25m + 18m^2 - 18m^2 - 24m}{3m + 4}$
$= \frac{m}{3m + 4}$

Or

$4x + 3y = 25 \cap mx - y = 6m$

$4x + 3y = 25$
$\underline{3mx - 3y = 18m}$
$4x + 3mx = 18m + 25$

$x = \frac{25+18m}{3m+4}$

$4mx + 3my = 25m$
$\underline{4mx - 4y = 24m}$
$(3m + 4)y = m$

$\therefore y = \frac{m}{3m + 4}$</td>
<td>Scale 10D (0, 4, 5, 8, 10)
Low Partial Credit:
Indication of use of simultaneous equations

Mid Partial Credit
One relevant substitution

High Partial Credit:
x or y value found

Low Partial Credit:
Indication of use of simultaneous equations

Mid Partial Credit
One successful elimination in equations

High Partial Credit:
x or y value found</td>
</tr>
</table>

<table>
<tr><th>Q3</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

$$(-2-2)^2 + (k-3)^2 = 65$$
$$16 + (k-3)^2 = 65$$
$$(k-3)^2 = 49$$
$$k - 3 = \pm\sqrt{49} = \pm 7$$
$$k = 10 \text{ and } k = -4$$

Or

$$k^2 - 6k + 9 = 49$$
$$k^2 - 6k - 40 = 0$$
$$(k-10)(k+4) = 0$$
$$k = 10 \text{ and } k = -4$$

Or

$$x^2 - 4x + 4 + y^2 - 6y + 9 = 65$$
$$x^2 + y^2 - 4x - 6y = 52$$
$$4 + k^2 + 8 - 6k = 52$$
$$k^2 - 6k - 40 = 0$$

$(k-10)(k+4) = 0, \therefore k = 10,\ k = -4$

Or

Centre (2, 3) , radius $\sqrt{65}$

$$\sqrt{(2+2)^2 + (3-k)^2} = \sqrt{65}$$

and proceed as above

</td><td>

Scale 10C (0, 4, 7, 10)
Low Partial Credit:
Some relevant substitution
Centre or radius

High Partial Credit:
Equation in k^2

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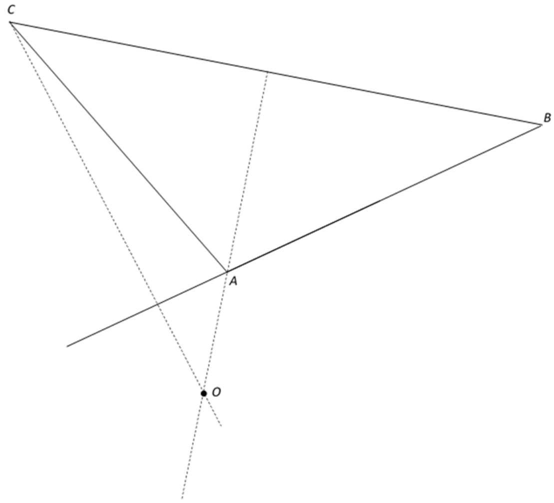
</table>

(b)		
	Both axes are tangents to the circle. centre (-g,-g) and radius is g Perpendicular distance (-g,-g) to $3x - 4y + 6 = 0$ is equal to the radius $$\frac{-3g+4g+6}{5} = -g$$ $$g + 6 = \pm 5g$$ $g + 6 = -5g, \therefore\ -g = 1$ Centre (1, 1) and radius 1 Equation: $(x-1)^2 + (y-1)^2 = 1$ or $x^2 + y^2 - 2x - 2y + 1 = 0$ Or s is a tangent to both axes therefore $c = g^2 = f^2$ So equation is in the form $x^2 + y^2 + 2gx + 2gy + g^2 = 0$ $3x - 4y + 6 = 0$ => $y = \frac{3x+6}{4}$ Substitute into circle: $x^2 + \left(\frac{3x+6}{4}\right)^2 + 2gx + \frac{2g(3x+6)}{4} + g^2 = 0$ => $25x^2 + x(36 + 56g) + 36 + 48g + 16g^2 = 0$ Tangent therefore $b^2 = 4ac$ $(36 + 56g)^2 = 4(25)(36 + 48g + 16g^2)$ $2g^2 - g - 3 = 0$ $g = -1$ and $g = \frac{3}{2}$ But can't have positive g as the co-ordinate $-g$ is in first quadrant. => $g = -1$. Therefore equation is $x^2 + y^2 - 2x - 2y + 1 = 0$ or $(x-1)^2 + (y-1)^2 = 1$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* centre *(-g,-g)* or equivalent *Mid Partial Credit:* Substitution into perpendicular distance formula completed Perpendicular distance of centre to tangent equals radius with some substitution *High Partial Credit:* equation in g or equivalent *Low Partial Credit:* $c = g^2$ or f^2 Effort to express $\boldsymbol{x}$ in terms of $\boldsymbol{y}$ or equivalent *Mid Partial Credit:* Substitution into circle equation completed *High Partial Credit:* Quadratic equation in g or f

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\cos 2A=\cos^2 A-\sin^2 A$ $\cos 2A=(1-\sin^2 A)-\sin^2 A$ $\cos 2A=1-2\sin^2 A$ Or Taking RHS $1-2\sin^2 A=1-2(1-\cos^2 A)$ $=-1+2\cos^2 A$ $=-(\cos^2 A+\sin^2 A)+2\cos^2 A$ $=\cos^2 A-\sin^2 A$ $=\cos A\cos A-\sin A\sin A=\cos 2A$ Or $(\cos A+i\sin A)^2=\cos 2A+i\sin 2A$ $(\cos A+i\sin A)^2$ $=\cos^2 A+2i\sin A\cos A+(i\sin A)^2$ $\cos 2A=\cos^2 A-\sin^2 A$ $\cos 2A=(1-\sin^2 A)-\sin^2 A$ $\cos 2A=1-2\sin^2 A$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* $\cos(A+B)$ formula with some substitution $\cos^2 A+\sin^2 A=1$ indicated or clearly implied *High Partial Credit:* $\cos 2A=\cos^2 A-\sin^2 A$ *Low Partial Credit:* $\cos^2 A+\sin^2 A=1$ indicated or clearly implied $(\cos A+i\sin A)^2$ expanded *High Partial Credit:* $\cos 2A=\cos^2 A-\sin^2 A$

(b)		
	Let length of side be x Diagonal of any face $= \sqrt{x^2 + x^2} = \sqrt{2}x$ Internal diagonal $= x^2 + \left(\sqrt{2}x\right)^2 = \sqrt{3}x$ By cosine rule: $x^2 = \left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - 2\frac{\sqrt{3}x}{2}\frac{\sqrt{3}x}{2}\cos A$ $\cos A = \dfrac{\left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - x^2}{2\left(\frac{\sqrt{3}x}{2}\right)\left(\frac{\sqrt{3}x}{2}\right)}$ $\cos A = \dfrac{1}{3}$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* Length of any diagonal formulated *Mid Partial Credit* Internal diagonal found *High Partial Credit:* Fully substituted cosine rule **Note:** Accept and mark work where a consistent numerical value is assigned to one side of the cube.
	Or Drop perpendicular from intersecting diagonals to side of cube, thereby creating angle $A/2$ at vertex in a right-angled triangle. $\sin\dfrac{A}{2} = \dfrac{\frac{x}{2}}{\frac{\sqrt{3}x}{2}} = \dfrac{1}{\sqrt{3}}$ $\therefore \cos\dfrac{A}{2} = \dfrac{\sqrt{2}}{\sqrt{3}}$ $\cos A = 2\cos^2\dfrac{A}{2} - 1 = 2\left(\dfrac{2}{3}\right) - 1 = \dfrac{1}{3}$ Also: $\sin\frac{A}{2} = \frac{1}{\sqrt{3}} \rightarrow \frac{A}{2} = 35{\cdot}2643896^{\circ}$ $A = 70{\cdot}5287792^{\circ}$ $\cos A = 0{\cdot}33236$	*Low Partial Credit:* Length of any diagonal formulated *Mid Partial Credit* Internal diagonal found *High Partial Credit:* $\sin\frac{A}{2}$ fully substituted

Q5	Model Solution – 25 Marks	Marking Notes
(a)	Standard Orthocentre Construction 	**Scale 15D (0, 5, 7, 11,15)** *Low Partial Credit:* Some correct element of construction Some evidence of understanding of term orthocentre *Mid Partial Credit* One correct altitude *High Partial Credit:* Two correct altitudes but not intersecting.
(b)	$\|DC\| = \|OB\|$ Given $\Rightarrow \|DC\| =$ Radius $\Rightarrow \Delta ODC$ is equilateral $\Rightarrow \angle ODC = 60$ $\Rightarrow \angle AOD = 60$ Alternate ΔAOD is isosceles as $\|OA\| = \|OD\|$ $\angle OAD = \angle ODA = \frac{120}{2} = 60$ $\|\angle ABE\| = 90°$ as BE tangent $\|\angle BEA\| = 180 - 90 - 60 = 30°$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* 1 relevant step listed or shown on diagram *Mid Partial Credit* 3 relevant steps listed or shown on diagram *High Partial Credit:* All valid steps included but with no justification

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$P(F \cap S) = P(F) \times P(S)$ since the events are independent. $$\frac{1}{5} = \frac{9}{20} \times P(S)$$ => $P(S) = \frac{4}{9}$ So $P(S \setminus F) = \frac{4}{9} - \frac{1}{5} = \frac{11}{45} = x$ Or $$P(S) = \frac{1}{5} + x$$ $$\frac{1}{5} = \frac{9}{20}\left(\frac{1}{5} + x\right) => \frac{11}{45} = x$$ Or $$P(F\|S) = \frac{P(F \cap S)}{P(S)} = P(F)$$ $$\frac{\frac{1}{5}}{\frac{1}{5} + x} = \frac{9}{20} \quad => x = \frac{11}{45}$$ Or $$P(S\|F) = \frac{P(S \cap F)}{P(F)} = P(S)$$ $$\frac{\frac{1}{5}}{\frac{9}{20}} = \frac{1}{5} + x => x = \frac{11}{45}$$ $$y = 1 - \frac{11}{45} - \frac{1}{5} - \frac{1}{4} = \frac{11}{36}$$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* $P(F \cap S) = P(F) \times P(S)$ or equivalent $P(F) = \frac{1}{4} + \frac{1}{5}$ $P(S) = x + \frac{1}{5}$ $\frac{1}{4} + \frac{1}{5} + x + y = 1$ *High Partial Credit* x found

(b)	If n Germans then $2n$ Irish and $3n+10$ children in total $\frac{n}{3n+10} \times \frac{2n+10}{3n+9} = \frac{1}{6}$ $\frac{2n^2+10n}{9n^2+57n+90} = \frac{1}{6}$ $3n^2 + 3n - 90 = 0$ $n^2 + n - 30 = 0$ $(n+6)(n-5) = 0$ $n = 5$ German children. There are 10 Irish (and 10 Spanish) so 25 children in the club.	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* *2n* *3n+10* One correct probability e.g. $\frac{n}{3n+10}$ *Mid Partial Credit:* $\frac{n}{3n+10}$ and $\left(\frac{2n+10}{\blacksquare} \text{ or } \frac{\blacksquare}{3n+9}\right)$ *High Partial Credit:* $\frac{n}{3n+10} \times \frac{2n+10}{3n+9} = \frac{1}{6}$
	Or 25 by trial and improvement method: 5 German, 10 Irish, 10 Spanish and verified to indicate $\frac{5}{25} \times \frac{20}{24} = \frac{1}{6}$	*Low Partial Credit:* Some correct element in approach *Mid Partial Credit* Tests more than one value *High Partial Credit:* Correct number of each nationality but not verified that probability is $\frac{1}{6}$ Correct answer (25) with no supporting work

Section B

Q7	Model Solution – 50 Marks	Marking Notes
(a) (i)	$\vert AD\vert^2 = 90^2 - 60^2$ $90^2 = 60^2 + \vert AD\vert^2$ $\vert AD\vert = \sqrt{8100-3600} = \sqrt{4500} = 30\sqrt{5}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* $\vert OD\vert = 60$ Pythagoras formulated Effort to find angle other than $\angle ODA$ *High Partial Credit:* $\sqrt{8100-3600}$ or equivalent
(a) (ii)	$\cos(\angle DOA) = \dfrac{60}{90}$ $\cos^{-1}\left(\dfrac{6}{9}\right) = 0{\cdot}84$ Or $\sin(\angle DOA) = \dfrac{30\sqrt{5}}{90} = \dfrac{\sqrt{5}}{3} = 0{\cdot}745356$ $\vert\angle DOA\vert = 48{\cdot}189^{\circ}$ $\vert\angle DOA\vert = 0{\cdot}84139 = 0{\cdot}84$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Relevant trigonometric ratio formulated *High Partial Credit:* Relevant trigonometric ratio fully substituted
(a) (iii)	Area of sector: $\dfrac{1}{2}r^2\theta$ $\frac{1}{2}(0{\cdot}9)^2 \times 2(0{\cdot}84) = 0{\cdot}6804\text{ m}^2$ Area Δ ACO : $\frac{1}{2}\vert AC\vert\vert OD\vert = \frac{1}{2}(60\sqrt{5})60\text{ cm}^2$ $\frac{1}{2}(1{\cdot}34164)(0{\cdot}6) = 0{\cdot}40\text{ m}^2$ Or Area Δ ACO : $\frac{1}{2}\vert AO\vert\vert OC\vert\sin(\angle AOC) =$ $\dfrac{1}{2}(90)(90)\sin 2(48{\cdot}189^{\circ})$ $= 4024{\cdot}9174\text{ cm}^2 = 0{\cdot}40\text{ m}^2$ Area of segment = 0·6804 – 0·40 = 0·28	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* Formula for area of sector with some substitution Formula for area of Δ ACO with some substitution *Mid Partial Credit:* One relevant area fully substituted *High Partial Credit:* Both relevant areas fully substituted Mishandling conversion of units
(a) (iv)	Volume = 0·28 × 2·5 = 0·7	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Formula for **volume** of trough with some substitution Indicates some relevant use of 2·5 *High Partial Credit:* Formula fully substituted

(b) (i)	Volume = $\pi\left[\left(\left(\frac{2}{3}\right)1{\cdot}25^3\right)\right]$ $+\pi[(1{\cdot}25^2 \times 3{\cdot}5)]$ $+\pi\left[\left(\left(\frac{1}{3}\right)1{\cdot}25^2 \times 1{\cdot}5\right)\right]$ $=$ 4·0906 + 17·1805 + 2·4544 $= 23{\cdot}73$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* 1 volume formula with some substitution *Mid Partial Credit* 2 volumes fully substituted *High Partial Credit:* 3 volumes fully substituted
(b) (ii)	$23{\cdot}73 \times 0\ 02 = 0{\cdot}4746$ cm^3 $\frac{r}{h} = \frac{1{\cdot}25}{1{\cdot}5} = \frac{5}{6}$ $r = \frac{5h}{6}$ Volume in cone $= \frac{1}{3}\pi\left(\frac{5h}{6}\right)^2 \times h = 0{\cdot}4746$ $h^3 = \frac{0{\cdot}4746.3.6}{25\pi} = 0{\cdot}65262$ $h = \sqrt[3]{0{\cdot}65262} = 0{\cdot}8674$ $h = 0{\cdot}87$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* volume $\times$ 0·98 or equivalent volume multiplied by 2% effort at *r* : *h* *High Partial Credit:* Volume formula expressed in one variable

<table>
<tr><th>Q8</th><th>Model Solution – 45 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>Confidence interval
$$0{\cdot}2175 \pm 1{\cdot}96\sqrt{\frac{\left(\frac{174}{800}\right)\left(\frac{626}{800}\right)}{800}}$$
$$0{\cdot}2175 - 1{\cdot}96\sqrt{\frac{(0{\cdot}2175)(0{\cdot}7825)}{800}} < p$$
$$< 0{\cdot}2175 + 1{\cdot}96\sqrt{\frac{(0{\cdot}2175)(0{\cdot}7825)}{800}}$$
$$0{\cdot}2175 - 1{\cdot}96\sqrt{0{\cdot}00021274} < p$$
$$< 0{\cdot}2175 + 1{\cdot}96\sqrt{0{\cdot}00021274}$$
$$0{\cdot}188913 < p < 0{\cdot}246087$$
$$0{\cdot}1889 < p < 0{\cdot}2461$$
or
$$18{\cdot}89\% < p < 24{\cdot}61\%$$</td><td>Scale 10C(0, 4, 7,10)

Low Partial Credit:
$0{\cdot}2175$ or $\frac{174}{800}$
CI formulated with some substitution

High Partial Credit:
CI fully substituted</td></tr>
<tr><td>(a)
(ii)</td><td>$$\frac{x - \bar{x}}{\sigma}$$
$\frac{95-87{\cdot}3}{12} = 0{\cdot}64167$ (z score)

=> $p(Z \leq 0{\cdot}64167) = 0{\cdot}7389$
$$P(z \geq 0{\cdot}64) = 1 - 0{\cdot}7389$$
$$= 0{\cdot}2611 \text{ or } 26{\cdot}11\%$$</td><td>Scale 15D(0, 5, 7, 11, 15)
Low Partial Credit:
μ or σ identified

Mid Partial Credit:
z found

High Partial Credit:
$P(z < 0{\cdot}64)$ and stops or continues incorrectly</td></tr>
<tr><td>(a)
(iii)</td><td>$$z = -0{\cdot}52 = \frac{x - 87{\cdot}3}{12}$$
$$=> x = 81{\cdot}06 \text{ km/h}$$
$$x = 81 \text{ km/h}$$</td><td>Scale 10C (0, 4, 7, 10)
Low Partial Credit:
$\frac{x - 87{\cdot}3}{12}$
$z \in [0{\cdot}52, 0{\cdot}53]$ or $z \in [-0{\cdot}52, -0{\cdot}53]$ and stops

High Partial Credit:
Formula for x fully substituted</td></tr>
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(b) (i)	Average speed has changed p-value $< 0{\cdot}05$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* Answer or reason correct
(b) (ii)	$$0{\cdot}024 = 2\big(1 - P(z \leq \lvert T \rvert)\big)$$ => $P(z \leq \lvert T \rvert) = 0{\cdot}988$ Therefore $z = 2{\cdot}26$ or $- 2{\cdot}26$. Because the mean has reduced $z = -2{\cdot}26$ $$-2{\cdot}26 = \frac{x - 87{\cdot}3}{\frac{12}{\sqrt{100}}}$$ $=>$ $x = 84{\cdot}588$ km/h $=> x = 84{\cdot}6$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* $0{\cdot}024 = 2(0{\cdot}012)$ Value(s) of *z* found *High Partial Credit:* Formula for x fully substituted

Q9	Model Solution – 55 Marks	Marking Notes
(a)	$\lvert SG\rvert^2 = 30^2 + 58^2 - 2(30)(58)(\cos 68)$ $= 2960{\cdot}369$ $\lvert SG\rvert = 54{\cdot}409$ m $\lvert SG\rvert = 54{\cdot}4$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* Some relevant substitution into correct cosine formula *High Partial Credit:* Formula fully substituted
(b)	$\dfrac{54{\cdot}4}{\sin 68} = \dfrac{30}{\sin \angle HSG}$ $\sin \angle HSG = 0{\cdot}51131$ $\lvert\angle HSG\rvert = 30{\cdot}75$ Or $\cos \angle HSG = \dfrac{54{\cdot}4^2 + 58^2 - 30^2}{2(54{\cdot}4)(58)}$ $= 0 \cdot 859432$ $\lvert\angle HSG\rvert = 30{\cdot}747^\circ$=30·75	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* Some relevant substitution into relevant formula *High Partial Credit:* Formula fully substituted Note: Finds $\lvert\angle HGS\rvert$ => ✔#
(c)	Area $\Delta GSH = \frac{1}{2}(30)(58)\sin 68 = 806{\cdot}65$ Also Area ΔGSH: $\dfrac{1}{2}(54{\cdot}4)(58)\sin 30{\cdot}75$ and $\frac{1}{2}(54{\cdot}4)(30)\sin 81{\cdot}25$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Some substitution into area formula *High Partial Credit:* Formula fully substituted
(d) (i)	$\dfrac{1}{2}(58)(r)$ or $29r$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* Right angle indicated Relevant triangle indicated on diagram Area of triangle formula with some substitution

(d) (ii)	Area ΔGHS $= \frac{1}{2}(30)(r) + \frac{1}{2}(54{\cdot}4)(r) + \frac{1}{2}(58)(r)$ $= 15r + 27{\cdot}2r + 29r = 71{\cdot}2r$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Relevant use of previous answer in this part Indication of 3 relevant triangle areas to be added Area of 1 additional triangle (in terms of r) *High Partial Credit:* Addition of 2 areas (each written in terms of r)
(d) (iii)	$71{\cdot}2r = 806{\cdot}62$ $r = \frac{806{\cdot}62}{71{\cdot}2}$ $= 11{\cdot}3289 = 11{\cdot}3$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Both relevant answers presented *High Partial Credit:* Areas equated
(e) (ii)	$\tan 14 = \frac{\lvert TS\rvert}{\lvert PS\rvert}$ $\sin 15{\cdot}375 = \frac{11{\cdot}3}{\lvert PS\rvert} = 42{\cdot}51$ $=> \lvert PS\rvert = 42{\cdot}619$ $\tan 14 = \frac{\lvert TS\rvert}{42{\cdot}619}$ $\lvert TS\rvert = 10{\cdot}626 = 10{\cdot}6$ Or $\lvert\angle HPS\rvert = 180 - 15{\cdot}375 - 34$ $= 130{\cdot}625^{\circ}$ $\frac{\sin 130{\cdot}625}{58} = \frac{\sin 34}{\lvert PS\rvert}$ $\lvert PS\rvert = 42{\cdot}73$ $\tan 14 = \frac{\lvert TS\rvert}{42{\cdot}73}$ $\lvert TS\rvert = 10{\cdot}653 = 10{\cdot}7$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* Some relevant substitution *High Partial Credit:* Formula fully substituted