

# **Coimisiún na Scrúduithe Stáit** State Examinations Commission

**Leaving Certificate 2019** 

**Marking Scheme** 

**Mathematics** 

V3

# **Ordinary Level**

#### Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

#### **Future Marking Schemes**

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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# Coimisiún na Scrúduithe Stáit State Examinations Commission

**Leaving Certificate 2019** 

# **Mathematics**

**Ordinary Level** 

Paper 1

Solutions and Marking Scheme

300 marks

## Marking Scheme – Paper 1, Section A and Section B

## Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	Α	В	С	D	Е
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5	
10 mark scales	0, 10	0, 5, 10	0, 3, 7, 10	0, 3, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

## B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

## C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

## D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

## E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

## Summary of mark allocations and scales to be applied

Section A		Section B	
Question 1	(25 marks)	Question 7	(55 marks)
(a)	10D	(a)	5B
(b)	5C	(b)(i)+(ii)	20D
(c)	10D	(c)(i)	5C
		(c)(ii)	5C
Question 2	(25 marks)	(d)(i)	5B
(a)(i)+(ii)	10D	(d)(ii)	10C
(a)(iii)	5D	(d)(iii)	5C
(b)	10C		
		Question 8	(55 marks)
Question 3	(25 marks)	(a)(i)	5C
(a)(i)	15D	(a)(ii)	10C
(a)(ii)	5B	(b)(i)	5C
(b)	5C	(b)(ii)	10C
		(b)(iii)	5C
Question 4	(25 marks)	(b)(iv)	5C
(a)	10C	(b)(v)	10C
(b)	15D	(b)(vi)	5C
Overstien F	(25 monto)	O	(40
Question 5	(25 marks)	Question 9	(40 marks)
(a)	15D	(a)	5B
(b)	10C	(b)	5C
Overstien C	/25 magniss)	(c)(i)	5B
Question 6	(25 marks)	(c)(ii)	10C
(a)	10D	(c)(iii)	10D
(b)	15C	(d)	5C

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Throughout the scheme indicate by use of # where an arithmetic error occurs.

## Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✓	Tick	Work of relevance	The work presented in the body of the script merits full credit
×	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
#	Hash	Rounding error Unit error Arithmetic error Misreading	
~~~	Horizontal wavy	Error	
<b>√</b> i	Tick L		The work presented in the body of the script merits low partial credit
✓m	Tick M		The work presented in the body of the script merits mid partial credit
<b>✓</b> h	Tick H		The work presented in the body of the script merits high partial credit
[	Left Bracket		Another version of this solution is presented elsewhere and is worth equal or higher credit
3	Vertical wavy	No work on this page (portion of the page)	
0	Oversimplify	The candidate has oversimplified the work	

**Note**: It may be necessary to use a combination of 2 symbols in the right margin to clearly show your judgement of the work in the body of the script:

# must be used to signify that Full Credit – 1 is merited by the work presented

Signifies that the work in the body of the script is worth mid partial credit but another effort at the work has been awarded this or higher credit

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

e.g. In a C scale where and and appear in the body of the work then should be placed in the right margin.

In the case of a **D scale** with the same level of annotation then should be placed in the right margin.

A in the body of the work may sometimes be used indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is them indicated in the right margin.

## **Model Solutions & Detailed Marking Notes**

**Note**: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$35300 \times 0.2 = €7060$ $4700 \times 0.4 = €1880$ $€7060 + 1880 = €8940$ $€8940 - 1650 = €7290 \text{ net tax}$	Scale10D (0, 3, 5, 8, 10) Low Partial Credit:  Use of 20%, 40% or similar Finds 4700  Mid Partial Credit: Both taxes formulated One tax found correctly  High Partial Credit: Both taxes found and added
(b)	40000 - (7290 + 1500) = €31210	Scale 5C (0, 2, 3, 5) Low Partial Credit:  Brings down answer from (a) Finds 8790  High Partial Credit: Income formulated correctly Correct answer without work
(c)	$35300 \times 0.2 + 2700 \times 0.4 = 8140$ $8140 - 1650 = 6490$ $38000 - 6490 = 31510$ $31510 - 31210 = €300$ $\Rightarrow €300 \text{ increase}$	Scale10D (0, 3, 5, 8, 10) Low Partial Credit:  Use of 7060 Finds 2700  Mid Partial Credit: Finds 8140 or equivalent  High Partial Credit: Finds 6490 and 31510 or equivalent

03	Model Solution 25 Marks	Marking Notes
Q2	Model Solution – 25 Marks	Marking Notes
(a) (i) + (ii)	$z_2 = 2(2+i) = 4+2i$ $\bar{z}_1 = 2-i$ $z_1 = 2-i$ $z_1 = 2-i$ $z_1 = 2-i$ $z_1 = 2z_1$ $z_1 = 2$	<ul> <li>Scale10D (0, 3, 5, 8, 10) Low Partial Credit: <ul> <li>Correct substitution into 2z<sub>1</sub></li> <li>Finds z<sub>2</sub> or z̄<sub>1</sub> correctly and stops</li> </ul> </li> <li>Mid Partial Credit: <ul> <li>Finds 4 + 2i and 2 - i</li> <li>z<sub>2</sub> or z̄<sub>1</sub> found correctly and plots, with or without label</li> </ul> </li> <li>High Partial Credit: <ul> <li>z<sub>2</sub> and z̄<sub>1</sub> found and at least one plotted correctly, without labels</li> <li>Mixes up axes but finds, plots and labels correctly</li> </ul> </li> <li>Note: Ignore extra incorrect plotting</li> </ul>
(a) (iii)	$ z_2  = \sqrt{16 + 4} = \sqrt{20}$ $ z_1 + \bar{z}_1  = 4 \neq \sqrt{20}$	Scale 5D (0, 2, 3, 4, 5)  Low Partial Credit:  Correct formula  Some correct substitution  Finds $z_1 + \bar{z}_1$ Mid Partial Credit:  Finds $ z_2 $ or $ z_1 + \bar{z}_1 $ High Partial Credit:  Finds $ z_2 $ and $ z_1 + \bar{z}_1 $ but no conclusion or incorrect conclusion

Q2	Model Solution – 25 Marks	Marking Notes
(b)	$(2+i)^{2} - 4(2+i) + 5 = 0$ $4+4i+i^{2} - 8 - 4i + 5 = 0$ $9-9+4i-4i = 0$	Scale 10C (0, 3, 7, 10)  Low Partial Credit:  • Some relevant substitution  High Partial Credit:  • Full substitution with correct multiplication of $(2+i)^2$ or $-4(2+i)$
	or	<b>Note</b> : failure to draw a relevant conclusion from incorrect work to be treated as an error
	$\frac{4 \pm \sqrt{16 - 4(1)5}}{2}$ $= \frac{4 \pm \sqrt{-4}}{2}$ $= 2 \pm i$	or  Scale 10C (0, 3, 7, 10)  Low Partial Credit:  Some relevant substitution into quadratic roots formula  Quadratic roots formula written  High Partial Credit:  Full substitution with some errors but with complex roots and relevant conclusion given  Gets ⁴±√-4/2  Note: failure to draw a relevant conclusion from incorrect work to be treated as an error

## 3 Model Solution - 25 Marks **Marking Notes** (a) (i) -1 0 1 2 3 4 $\boldsymbol{x}$ 2 2 -2 2 f(x)8 10 10 0 -1 Scale 15D (0, 4, 7, 11, 15) Low Partial Credit: • Relevant work towards finding a value for f(x)• One point found correctly • One point from candidates table plotted correctly Mid Partial Credit: • Table fully correct • All points from candidates incorrect table plotted and/or joined correctly **High Partial Credit:** • All correct points found and plotted correctly, but not joined or joined incorrectly • 4 or 5 correct points on table plotted correctly with joining

Q3	Model Solution – 25 Marks	Marking Notes
(a)(ii)	Roots $\approx -0.7, 0.7$	Scale 5B (0, 2, 5)  Mid Partial Credit:  One correct/relevant root given or indicated on graph
(b)	$f'(x) = -3x^2 + 8x + 1$ $f''(x) = -6x + 8 = 0$ $x = \frac{4}{3}$	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  • Any correct differentiation  • $f''(x)$ found from non-quadratic  • $f'(x)$ High Partial Credit:  • $f''(x)$ found

Q4	Model Solution – 25 Marks	Marking Notes
(a)	6x + 2 + 5x - 10 = 47 $x = 5$	Scale 10C (0, 3, 7, 10)  Low Partial Credit:  10 identified as common denominator Some relevant multiplication 2(3 $x$ + 1) + 5( $x$ - 2) without CD Tests a value/s for $x$ not equal to 5  High Partial Credit: 6 $x$ + 2 + 5 $x$ - 10 = 47 or equivalent  Note: Accept Trial and Improvement method for $x$ = 5 verified for full marks
(b)	$x = 5y - 13$ $(5y - 13)^{2} + y^{2} = 13$ $25y^{2} - 130y + 169 + y^{2} - 13 = 0$ $26y^{2} - 130y + 156 = 0$ $y^{2} - 5y + 6 = 0$ $(y - 2)(y - 3) = 0$ $y = 2 \text{ and } y = 3$ $\Rightarrow x = -3 \text{ and } x = 2$ $(-3, 2) \text{ and } (2, 3)$	<ul> <li>Scale 15D (0, 4, 7, 11, 15)  Low Partial Credit: <ul> <li>Isolates one variable or transposes correctly</li> </ul> </li> <li>Mid Partial Credit: <ul> <li>(5y - 13)² + y² = 13 or equivalent equation in one variable</li> </ul> </li> <li>High Partial Credit: <ul> <li>One relevant value (root) found from quadratic</li> </ul> </li> </ul>

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$A = \frac{3}{2} [0+0+2(15+18+15+12+15+18+15)]$ $A = 324 \text{ m}^2$	Scale 15D (0, 4, 7, 11, 15)  Low Partial Credit:  • Any correct dimension identified  • Correct answer without work  Mid Partial Credit  • Correct formula with some correct substitution  High Partial Credit:  • Correct formula fully substituted correctly
(b)	$1.6 \times 60 \times 60 \div 1000 = 5.76$	Scale 10C (0, 3, 7, 10)  Low Partial Credit:  • $1 \cdot 6 \times 60$ or equivalent  • Any work of merit  High Partial Credit:  • $1 \cdot 6 \times 60 \times 60 = 5760$ • $1 \cdot 6 \times 60 \times 60 \div 1000$ not finished

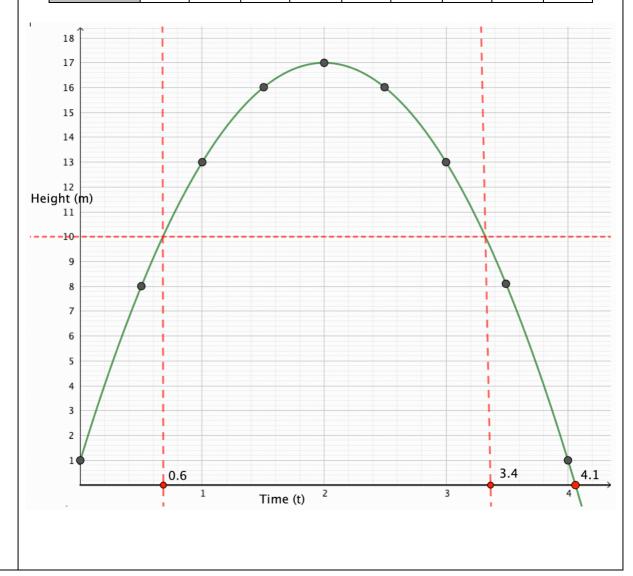
Q6	Model Solution – 25 Marks	Marking Notes
(a)	6 - 2x < 8 $2x > -2$ $x > -1$	<ul> <li>Scale 10D (0, 3, 5, 8, 10)  Low Partial Credit: <ul> <li>Any correct multiplication</li> </ul> </li> <li>Mid Partial Credit: <ul> <li>Correct transposing from correct work</li> </ul> </li> <li>High Partial Credit: <ul> <li>x &gt; -1 but no plot or incorrectly plotted</li> <li>x &lt; -1 and correctly plotted</li> </ul> </li> </ul>
(b)	$2^{2x-1} = 2^{6}$ $\Rightarrow 2x - 1 = 6$ $2x = 7$ $x = \frac{7}{2}$	Scale 15C (0, 5, 10, 15)  Low Partial Credit:  • $2^6$ • List of powers of 2  • Correct answer no work  High Partial Credit:  • Equation in $x$ , $2x - 1 = 6$ Zero Credit:  • $4x - 2 = 64$

## **Section B**

(ii)

Q7	Model Solution – 55 Marks					arking I	Notes				
(a)	f(0) = 1				M	Scale 5B (0, 2, 5)  Mid Partial Credit:  • $f(0)$ • $t = 0$					
(b) (i) &	(c)		Ta	able and	d Graph	(20 mai	rks)	2	2.5		

Time (t)	0	0.5	1	1.5	2	2.5	3	3.5	4
Height (m)	1	8	13	16	17	16	13	8	1



Q7	Model Solution – 55 Marks	Marking Notes
(b)		
(i)		Scale 20D (0,5,10,15,20)
&		Low Partial Credit:
(ii)		<ul> <li>Up to 6 correct entries</li> </ul>
		Mid Partial Credit:
		<ul> <li>Between 7 and 12 correct entries</li> </ul>
		High Partial Credit:
		• 13 to 17 correct entries, no joining
		or incorrect joined with a straight line
		<b>Note</b> : An entry is a calculation or a plot
		·
(c)	4·1	Carlo 50 (0. 3. 3. 5)
(i)	1 1	Scale 5C (0, 2, 3, 5) Low Partial Credit:
		• 4·1 or similar, but no extension shown
		on graph
		Extension to graph only
		High Partial Credit:
		Clear indication on graph, but no value
		given
(c)		
(ii)	Work to be shown on graph.	Scale 5C (0, 2, 3, 5)
	3.4 - 0.6 = 2.8	Low Partial Credit:
		• $f(3\cdot4)$ found or indicated on graph
		• $f(0.6)$ found or indicated on graph
		High Partial Credit:
		3·4 − 0·6 or equivalent and fails to finish

Q7	Model Solution – 55 Marks	Marking Notes
(d) (i)	f'(t) = -8t + 16	Scale 5B (0, 2, 5)  Mid Partial Credit:  • Any correct differentiation
(d) (ii)	-8(4.0) + 16 = -16 Speed = 16	<ul> <li>Scale 10C (0, 3, 7, 10)</li> <li>Low Partial Credit: <ul> <li>Some relevant substitution into candidate's f'(x)</li> <li>Brings down f'(x) to this part</li> </ul> </li> <li>High Partial Credit: <ul> <li>Candidate's f'(x) fully substituted, but not worked out</li> <li>Note: Accept -16</li> </ul> </li> </ul>
(d) (iii)	-8t + 16 = -8 $8t = 24$ $t = 3$	<ul> <li>Scale 5C (0, 2, 3, 5)</li> <li>Low Partial Credit:</li> <li>Candidate's f'(x) = z where z ≠ 8 or -8</li> <li>High Partial Credit:</li> <li>Candidate's f'(x) = 8 finished correctly where the solution is viable</li> <li>-8t + 16 = -8 and fails to finish</li> </ul>

Q8	Model Soluti	ion – 55 Ma	rks		Marking Notes				
(a) (i)		$P = \frac{480}{5}$ $= 228$ $= 228$	252		Scale 5C (0, 2, 3, 5)  Low Partial Credit:  Some relevant substitution into formula  High Partial Credit: Formula fully substituted				
(a) (ii)		$R = \frac{52}{}$	52 P T		Scale 10C (0, 3, 7, 10)  Low Partial Credit:  • Any relevant transposing  High Partial Credit:  • Transposing complete but with one error			one	
(b) (i)	Month 1 2 3				4 5 6				
	Profit (€)	-4000	-3750	-350	0	-3250	-3000	-2750	
	Scale 5C (0, 2, 3, 5)  • One additional correct entry  High Partial Credit:  • Two correct entries								
(b) (ii)	,	= -4000 + $= 250n - 4$	,	0	Lo		edit: $correct a a = a + (n - redit)$	- 1) <i>d</i>	

Q8	Model Solution – 55 Marks	Marking Notes
(b) (iii)	$T_{25} = 250(25) - 4250$ $= £2000$	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  Some relevant substitution into formula  Month by month method incomplete  High Partial Credit:  Formula fully substituted but fails to finish
(b) (iv)	$T_n = 0$ $250n - 4250 = 0$ $n = 17$	<ul> <li>Scale 5C (0, 2, 3, 5) Low Partial Credit: <ul> <li>Some relevant substitution into formula</li> <li>Month by month method incomplete</li> <li>n = 17 without work</li> <li>T<sub>n</sub> = 0 written</li> </ul> </li> <li>High Partial Credit: <ul> <li>250n - 4250 = 0</li> </ul> </li> </ul>
(b) (v)	$S_n = \frac{n}{2} [2(-4000) + (n-1)250]$ $S_n = \frac{n}{2} [-8000 + 250n - 250]$ $S_n = \frac{n}{2} [-8250 + 250n]$	Scale 10C (0, 3, 7, 10)  Low Partial Credit:  Identifies a and/or d  Correct formula for $S_n$ High Partial Credit:  Formula fully substituted correctly
(b) (vi)	$S_{37} = \frac{37}{2} [-8250 + 250(37)]$ $= 18500$	<ul> <li>Scale 5C (0, 2, 3, 5)</li> <li>Low Partial Credit:</li> <li>Some relevant substitution into either S<sub>n</sub> formula</li> <li>High Partial Credit:</li> <li>Correct formula fully substituted but work not complete</li> </ul>

Q9	Model Solution – 40 Marks	Marking Notes
(a)	$3000(1.8)^8 = 330598.817$ $\Rightarrow 330598 \text{ or } 330599$	Scale 5B (0, 2, 5)  Mid Partial Credit:  • Some relevant substitution into formula
(b)	$31493 \times 1.8 = 56687.4$ = 56687 or $3000(1.8)^4 = 31493$ $3000(1.8)^5 = 56687$	<ul> <li>Scale 5C (0, 2, 3, 5)</li> <li>Low Partial Credit: <ul> <li>Some relevant substitution into formula</li> <li>Incomplete trial and improvement to find m</li> </ul> </li> <li>High Partial Credit: <ul> <li>m = 4 found by trial and improvement but multiplied by 1·8 incorrectly</li> <li>3000(1·8)<sup>5</sup></li> </ul> </li> </ul>
(c) (i)	600000 × 0·0012 = €720	Scale 5B (0, 2, 5)  Mid Partial Credit:  Use of 600000 and/or 0·0012  • Decimal error
(c) (ii)	$\frac{1285 \cdot 37}{0 \cdot 0012} = 1071141 \cdot 667$ $= 1071142 \text{ or } 1071141$	Scale 10C (0, 3, 7, 10) Low Partial Credit:  • $1285 \cdot 3 \times 0.0012$ High Partial Credit:  • $1285 \cdot 37 \div 0.0012$ • $1285 \cdot 37 \times 0.0012$ or equivalent = $1.5$

Q9	Model Solution – 40 Marks	Marking Notes
(c) (iii)	Month 4: $3000(1\cdot8)^4 = 31492\cdot8$ $31492\cdot8 \times 0\cdot0012 = €37\cdot79$ $37\cdot79 - 80 = €42\cdot21 \text{ Loss}$ Month 12: $3000(1\cdot8)^{12} = 3470494\cdot144$ $3470494\cdot144 \times 0\cdot0012 = €4164\cdot59$ $4164\cdot59 - 80 = €4084\cdot59 \text{ Gain}$	Scale 10D (0, 3, 5, 8, 10)  Low Partial Credit:  Either month formulated correctly Finds number of users for either month  Mid Partial Credit  Income for either month evaluated correctly Finds number of users for both months  High Partial Credit: Incomes for both months evaluated correctly Profit/Loss for one month evaluated correctly
(d)	Exchange Rate $=\frac{55}{62.70}$ Exchange Rate $=0.8772$	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  • $\frac{62 \cdot 7}{55}$ High Partial Credit:  • $\frac{62 \cdot 7}{55} = 1 \cdot 14$ • $\frac{65}{55} = 1 \cdot 14$ • $\frac{55}{62 \cdot 7}$ or $\frac{50}{57}$ and stops

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# Coimisiún na Scrúduithe Stáit State Examinations Commission

Leaving Certificate 2019

# **Mathematics**

**Ordinary Level** 

Paper 2

Solutions and Marking Scheme

300 marks

## Marking Scheme - Paper 2, Section A and Section B

## **Structure of the marking scheme**

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	Α	В	С	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5	
10 mark scales	0, 10	0, 5, 10	0, 4, 6, 10	0, 3, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 9, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

## Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
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- response about half-right
- almost correct response
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E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

## Summary of mark allocations and scales to be applied

Section A		Section B	
Question 1	(25 marks)	Question 7	(50 marks)
(a)	10C	(a)	10C
(b)	5C	(b)	10C
(c)(i)	5C	(c)(i)	15C
(c)(ii)	5C	(c)(ii)	5A
		(c)(iii)	5C
Question 2	(25 marks)	(d)	5C
(a)	10C		
(b)	5C	Question 8	(45 marks)
(c)	5B	(a)(i)	10C
(d)	5D	(a)(ii)	5C
		(a)(iii)	10C
Question 3	(25 marks)	(b)(i)	10C
(a)	10B	(b)(ii)	5C
(b)	5C	(b)(iii)	5C
(c)	5C		
(d)	5C	Question 9	(55 marks)
		(a)	15C
Question 4	(25 marks)	(b)	10C
(a)	5C	(c)	10C
(b)	5C	(d)(i)	5D
(c)	5C	(d)(ii)	10C
(d)	10C	(d)(iii)	5D
Question 5	(25 marks)		
(a)	10D		
(b)	15D		
Question 6	(25 marks)		
(a)(i)	15D		
(a)(ii)	5C		
(b)	5D		

**Note:** In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale* 10C, 9 marks may be awarded.

Rounding and units penalty to be applied only once in each part (a), (b), (c) etc. Throughout the scheme indicate by use of # where an arithmetic error occurs.

## **Model Solutions & Detailed Marking Notes**

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

## **Section A**

Q1	Model Solution – 25 Marks								Marking Notes
(a)									
	1	8	8	9					Scale 10C (0, 4, 6, 10)
	2	0	1	1	2				Low Partial Credit:
	3	2	3	3	4	4	5	9	Attempt at ordering ages
	4	1	2	3	3	9			One correct entry
	5	4	5	7	7	8			With Burth Cond's
	6	3	3	4	5				High Partial Credit:
	KEY: 1 9 = 19 years of age						ge	1	• 15 correct entries
									Full Credit:
									Correct answer without work
(b)									
	14								Scale 5C (0, 2, 3, 5)
	28								Low Partial Credit:
	= 50%								Work of merit
					— .	30%			Mention of 14 or 28
									• Reference to × by 100
									High Partial Credit:
									<ul> <li>Answer given as <sup>14</sup>/<sub>28</sub> or equivalent</li> </ul>
									28 of equivalent
									Full Credit:
									<ul> <li>Correct answer without work</li> </ul>

Q1	Model Solution – 25 Marks	Marking Notes
(c)(i)	3 28	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  • Work of merit  High Partial Credit:  • $\frac{x}{28}$ , $(x \neq 3, x \leq 28)$ • $\frac{3}{x}$ , $(x \neq 28, x \geq 3)$ Full Credit:  • Correct answer without work  Zero Credit:  • Probability > 1, without work
(c) (ii)	$= \frac{3}{28} + \frac{4}{28}$ $= \frac{7}{28}$	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  Work of merit  High Partial Credit:  Both probabilities calculated correctly  Full Credit:  Correct answer without work  Zero Credit:  Probability > 1, without work

Q2	Model Solution – 25 Marks	Marking Notes
(a)	Model Solution – 25 Marks $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - 2}{8 - 4}$ $= \frac{3}{4}$	Scale 10C(0, 4, 6, 10)  Low Partial Credit:  Work of merit  Formula with some correct substitution  High Partial Credit:  Formula substituted correctly  Full Credit:  Correct answer without work  Misreading:  Finds slope of PR or QR
(b)	$y - y_1 = m(x - x_1)$ $y - 2 = \frac{3}{4}(x - 4)$ $3x - 4y - 4 = 0$	<ul> <li>Scale 5C (0, 2, 3, 5)</li> <li>Low Partial Credit:</li> <li>Formula with some correct or consistent substitution</li> <li>High Partial Credit:</li> <li>Formula substituted correctly or consistently</li> <li>One incorrect substitution followed by correct solution</li> <li>Answer not in the required format</li> <li>Note:</li> <li>Accept</li> <li>-3x + 4y + 4 = 0</li> </ul>
(c)	$-\frac{4}{3}$	Scale 5B (0, 2, 5)  Mid Partial Credit:  • Work of merit  Full Credit:  • Correct answer without work

Q2	Model Solution – 25 Marks	Marking Notes
(d)	$(4,2) (8,5) (2,11)$ $\downarrow \qquad \downarrow \qquad \downarrow$ $(0,0) (4,3) (-2,9)$ $= \frac{1}{2}  x_1 y_2 - x_2 y_1 $ $= \frac{1}{2}  (4)(9) - (-2)(3) $ $= \frac{1}{2}  36 + 6 $ $= 21 \text{ square units}$	<ul> <li>Scale 5D (0, 2, 3, 4, 5)</li> <li>Low Partial Credit:</li> <li>Translation of any relevant point to (0,0)</li> <li>Area of triangle formula with some substitution</li> <li>Mid Partial Credit:</li> <li>Area of triangle formula substituted fully with 2 from P,Q, and R</li> <li>High Partial Credit:</li> <li>Formula substituted correctly with translated points</li> <li>Area of triangle found from formula substituted fully with 2 from P,Q, R</li> <li>An incorrect translation applied correctly</li> </ul>

Q3	Model Solution – 25 Marks	Marking Notes
(a)		
	1 - 0.7 = 0.3	Scale 10B (0, 5, 10) Mid Partial Credit:  • Work of merit  Full Credit:  • Correct answer without work  Zero Credit:  • Probability > 1, without work
(b)	$0.7 \times 0.7 = 0.49$	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  Work of merit  High Partial Credit:  Correct answer indicated but not calculated  0·3 × 0·3 = 0·09  Full Credit:  Correct answer without work  Zero Credit:  Probability > 1, without work
(c)	$0.7 \times 0.7 \times 0.3 \times 3$ $= 0.147 \times 3$ $= 0.441$	Scale 5C (0, 2, 3, 5) Low Partial Credit:  Work of merit, e.g. List given  High Partial Credit:  Correct answer indicated but not calculated  Full Credit:  Correct answer without work  Zero Credit:  Probability > 1, without work

Q3	Model Solution – 25 Marks	Marking Notes
(d)	$(0.3)^{3} \times 0.7$ $= 0.0189$	Scale 5C (0, 2, 3, 5) Low Partial Credit:  • Work of merit, e.g. Correct list  High Partial Credit:  • Correct answer indicated but not calculated  Full Credit:  • Correct answer without work
		<ul><li>Zero Credit:</li><li>Probability &gt; 1, without work</li></ul>

Q4	Model Solution – 25 Marks	Marking Notes
(a)		
	$S(-4,11) \rightarrow P(2,3)$ $=> x \uparrow 6 \text{ and } y \downarrow 8$ $R(4,17) \rightarrow Q(10,9)$	<ul> <li>Scale 5C (0, 2, 3, 5)</li> <li>Low Partial Credit:</li> <li>Work of merit</li> <li>Translation identified or indicated on the diagram</li> </ul>
		<ul> <li>High Partial Credit:</li> <li>One co-ordinate correct</li> <li>An incorrect translation applied correctly</li> <li>Correct translation applied to an incorrect point</li> <li>Full Credit:</li> <li>Correct answer without work</li> </ul>
(b)	Mid-point of PR $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $= \left(\frac{2+4}{2}, \frac{3+17}{2}\right)$ $= (3, 10)$	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  • Formula with some relevant substitution  • Indicates the centre on the diagram  High Partial Credit:  • Formula substituted correctly  Full Credit:  • Correct answer without work
(c)	Radius = $\frac{1}{2}$  RS  or $\frac{1}{2}$  SP  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(4 + 4)^2 + (17 - 11)^2}$ $= \sqrt{64 + 36} = 10$ $r = \frac{1}{2}(10) = 5 \text{ units}$	Scale 5C (0, 2, 3, 5) Low Partial Credit:  • Distance formula with some relevant substitution  High Partial Credit:  • Formula substituted correctly  • Correct answer without work

Q4	Model Solution – 25	Marks	Marking Notes
(d)	Centre = $(3,10)$ $(x-3)^2 +$		Scale 10C (0, 4, 6, 10)  Low Partial Credit:  Work of merit  High Partial Credit:  Formula substituted with centre and radius i.e. $(x-3)^2 + (y-10)^2 = 5^2$ One error in substitution and finished correctly  Full Credit:  Correct or consistent answer without work  Note:  Accept work with candidates centre and radius

Q5	Model Solution – 25 Marks	Marking Notes
(a)		
	Area <sub>L</sub> = $\pi r^2$ = $\pi(3)^2$ = $9 \pi$ Area <sub>S</sub> = $\pi r^2$ = $\pi(2.5)^2$ = $6.25 \pi$ Area <sub>C</sub> = $9\pi - 6.25\pi$ = $8.64 \text{ cm}^2$ Perimeter <sub>L</sub> = $2\pi(3)$ = $6\pi$ Perimeter <sub>S</sub> = $2\pi(2.5)$ = $5\pi$	<ul> <li>Scale 10D (0, 3, 5, 8, 10)</li> <li>Low Partial Credit:</li> <li>Area formula with some substitution or Circumference formula with some substitution</li> <li>Mid Partial Credit:</li> <li>Area or perimeter of crescent worked correctly</li> <li>All four formulae substituted correctly</li> <li>High Partial Credit:</li> <li>Expression for Area and Perimeter substituted correctly with no conclusion</li> <li>Incorrect operator (- or +) in either part but finishes correctly</li> <li>Answers in terms of π</li> </ul>
	Perimeter <sub>C</sub> = $6\pi + 5\pi$ = $34.56$ cm	
(b)	Volume = $\frac{1}{3}\pi(7)^2(12)$ = $\frac{588}{3}\pi$ = $196\pi$ = $615.7521601$	Scale 15D (0, 4, 7, 11, 15)  Low Partial Credit:  Work of merit  Volume formula with some substitution  Mid Partial Credit:  Formula substituted correctly
	Time = $\frac{615.7521601}{0.5(1000)}$ = 1.23150432 minutes = 1.23150432 × 60 = 73.89025 = 74 seconds	High Partial Credit:  • Volume found correctly i.e. $\frac{588}{3}$ π or equivalent.

Q6	Model Solution – 25 Marks	Marking Notes
(a) (i)	5 9 R 5 5 65° P 9	Scale 15D (0, 4, 7, 11, 15)  Low Partial Credit:  • A labelled pilot diagram drawn  • One correct length or angle drawn  Mid Partial Credit:  • Any 2 components constructed correctly  High Partial Credit:  • Any three components constructed correctly
(a) (ii)	Area = $ab \sin C$ = $9(5) \sin 65^{\circ}$ = $45(0.906307787)$ = $40.78385042$ = $40.78 \text{ cm}^2$ or Area = $a \times h_{\perp}$ = $9[(5) \sin 65^{\circ}]$ = $9(4.531538935)$ = $40.78385042$ = $40.78 \text{ cm}^2$	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  • Formula with some substitution  • Perpendicular distance between sides measured and used correctly to find area  High Partial Credit:  • Formula substituted correctly  • Incorrect calculator mode (apply once only on paper)
(b)	$\alpha = 52^{\circ}$ $2\beta = 38^{\circ}$ $\beta = 19^{\circ}$	Scale 5D (0, 2, 3, 4, 5)  Low Partial Credit:  • Work of merit  • Pilot diagram  Mid Partial Credit:  •  ∠YOX  = 104° clearly stated or shown on the diagram  •  ∠WYX  = 90° clearly stated or shown on the diagram  High Partial Credit:  • One value correct  Full Credit:  • Correct answers without work

## **Section B**

Q7	Model Solution – 50 Marks			Marking Notes					
(a)	Table B (€)								
		Meat	Fisi		Milk, Cheese and Eggs		Fruits, Vegetables and Potatoes		
	Maximum 254		17	7 175		172			
	Minimum	54	59	)	65		48		
	Range	200	11	8	110		124		
(b)	1st 2nd 3rd 48 62 77 Median = 111		5th <b>7th</b> .04 <b>111</b>	_	Our corresponding to the second secon		11th 137	12th 150	13th 172
				Scale 10C (0, 4, 6, 10)  Low Partial Credit:					

Q7	Model Solution – 50 Marks	Marking Notes
(c) (i)	$132 + 117 + 106 + 63 + 56 + 111 + 157$ $+54 + 85 + 131 + 254 + 78 + 112$ $Sum = 1456$ $Mean = \frac{1456}{13}$ $Mean = €112$	Scale 15C (0, 5, 9, 15)  Low Partial Credit:  Work of merit  High Partial Credit:  Formula substituted correctly but not evaluated  Calculates the mean of an incorrect food category  Full Credit:  Correct answer without work
(c) (ii)	S. D. = € 51	Scale 5A (0, 5)
(c) (iii)	112 + 51 = 163 & $112 - 51 = 61$ [61,163]  Macedonia, Poland and Switzerland	Scale 5C (0, 2, 3, 5) Low Partial Credit:  • Work of merit  High Partial Credit:  • Range found

## **Model Solution – 50 Marks** Q7 **Marking Notes** (d) Table C (€) Milk, Fruits, Country Meat Fish Cheese Vegetables **Total Cost** and Eggs and Potatoes Ireland 36.04 31.32 24.32 31.28 122.96 Poland 18.36 18.56 12.35 14.26 63.53 Fish: Scale 5C (0, 2, 3, 5) Low Partial Credit: $\frac{31.32}{108} \times 64 = 18.56$ • Any work of merit **High Partial Credit:** Milk, Cheese, Eggs: • Two food categories filled correctly Correct answers without work $\frac{24.32}{128} \times 65 = 12.35$ Fruit, Veg, Potatoes: $\frac{31.28}{136} \times 62 = 14.26$

**Total Cost** 

= 63.53

= 18.36 + 18.56 + 12.35 + 14.26

Q8	Model Solution – 45 Marks	Marking Notes		
(a) (i)	$V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}\pi (3)^3$ $V = 36\pi \text{ cm}^3$	Scale 10C (0, 4, 6, 10)  Low Partial Credit:  • Work of merit  High Partial Credit:  • Formula substituted correctly  Full Credit:  • Correct answer without work		
(a) (ii)	$V = \pi r^2 h$ $\pi(5)^2 h = 36\pi$ $h = \frac{36\pi}{25\pi}$ $h = \frac{36}{25} \text{ cm or } 1.44 \text{ cm}$	Scale 5C (0, 2, 3, 5) Low Partial Credit:  • Work of merit  • Volume formula with some substitution  High Partial Credit:  • Formula substituted correctly and equation formed  • One error in substitution in the equation and finishes correctly		
(a) (iii)	C.S.A = $2\pi rh$ = $2\pi(5)(18)$ = $180\pi$ Material left over = $35 \times 20 - [180\pi]$ = $700 - 565.49$ = $134.51$ = $134.5 \text{ cm}^2$	Scale 10C (0, 4, 6, 10)  Low Partial Credit:  Work of merit  High Partial Credit:  2 area formulae substituted correctly and stops  One error in substitution in getting the area and finishes correctly		

Q8	Model Solution – 45 Marks	Marking Notes
(b) (i)	$\frac{1}{\sqrt{800}} = 3.5355$ $= 3.54\%$	Scale 10C (0, 4, 6, 10)  Low Partial Credit:  • Writes $\frac{1}{\sqrt{n}}$ .  • Sets up work as $\sqrt{800}$ High Partial Credit:  • Formula substituted correctly  • i.e.( $\frac{1}{\sqrt{800}}$ )  • Answer as $0.035355$ without work  Full Credit:  • Correct answer without work  Special Case  Without Work  Award $3.5355\%$ Full Credit -1
(b) (ii)	$\hat{p} = \frac{350}{800} = 43.75\%$ 95% confidence interval: $\left[\hat{p} - \frac{1}{\sqrt{n}}, \ \hat{p} + \frac{1}{\sqrt{n}}\right]$ $43.75 - 3.54  40.21\%$	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  • Work of merit  • $\hat{p} \pm \frac{1}{\sqrt{n}}$ .  High Partial Credit:  • One boundary formed:  • $(43.75 - 3.54 \text{ or } 43.75 + 3.54)$ • Correct answer without work  • Use of version of formula from HL  • Answer in decimal form
(b) (iii)	$H_0=$ the level of support is $50\%$ $H_1=$ the level of support is not $50\%$ $50\% \notin [40\cdot21\%,47\cdot29\%]$ $50\%$ is not within the $95\%$ CI so there is not enough evidence to support the claim. So, we reject $H_0$ and say that the level of support of the people in the area is not $50\%$	Scale 5C (0, 2, 3, 5)  Low Partial Credit:  • Null hypothesis stated  • C. I. written  High Partial Credit:  • Conclusion OK but reason incorrect or omitted  • 50% ∉ [40·21%, 47·29%]  with conclusion incorrect or omitted

Q9	Model Solution – 55 Marks	Marking Notes
(a)	$\sin 30^{\circ} = \frac{ QR }{120}$ $ QR  = 120 \sin 30^{\circ}$ $ QR  = 60 \text{ km}$	Scale 15C (0, 5, 9, 15)  Low Partial Credit:  Work of merit  Correct trigonometric ratio or relevant formula with some substitution  High Partial Credit:  Formula substituted correctly  One incorrect substitution and finished correctly  Incorrect calculator mode (apply once only on paper)
(b)	$ RS ^{2} =  RQ ^{2} +  QS ^{2}$ $100^{2} = 60^{2} +  QS ^{2}$ $ QS ^{2} = 10000 - 3600$ $ QS ^{2} = 6400$ $ QS  = \sqrt{6400}$ $ QS  = 80 \text{ km}$	<ul> <li>Scale 10C (0, 4, 6, 10)</li> <li>Low Partial Credit:</li> <li>Work of merit</li> <li>Pythagoras formulated with some substitution</li> <li>High Partial Credit:</li> <li>Pythagoras substituted correctly</li> <li>Answer as √6400 Correct answer without work</li> </ul>
(c)	$ PR ^{2} =  QR ^{2} +  PQ ^{2}$ $120^{2} = 60^{2} +  PQ ^{2}$ $ PQ ^{2} = 10800$ $ PQ  = 103.92304$ $ PS  =  PQ  -  QS $ $ PS  = 103.92 - 80$ $ PS  = 23.92$ $ PS  \approx 24 \text{ km}$	Scale 10C (0, 4, 6, 10)  Low Partial Credit:  • Work of merit  • Pythagoras formulated with some substitution  •  PS  =  PQ  - 80  High Partial Credit:  • Pythagoras substituted correctly and  PQ  evaluated correctly or consistently

Q9	Model Solution – 55 Marks	Marking Notes
(d) (i)	$ TS  = 2 QS $ $= 2 \times 80$ $= 160$ $a^{2} = b^{2} + c^{2} - 2bc \cos \theta$ $160^{2} = 100^{2} + 100^{2}$ $- 2(100)(100) \cos \theta$ $20000 \cos \theta = 10000 + 10000 - 25600$ $\cos \theta = -\frac{5600}{20000}$ $\cos \theta = -\frac{7}{25}$ $\theta = \cos^{-1} - \left(\frac{7}{25}\right)$ $\theta = 106.26$ $\theta = 106^{\circ}$	Scale 5D (0, 2, 3, 4, 5)  Low Partial Credit:  Work of merit  Cosine formula with some substitution  Mid Partial Credit:  Two variables correctly substituted into the Cosine formula  High Partial Credit:  Formula substituted correctly  One incorrect substitution followed by correct calculation  Incorrect calculator mode (apply once in paper)  No Credit:  Treats triangle as right angled
(d) (ii)	Arc TS = $2\pi r \times \frac{\theta}{360}$ = $2\pi (100) \times \frac{106}{360}$ = $\frac{530\pi}{9}$ = $185.0049$ = $185$ Difference = $185 - 160$ = $25 \text{ km}$	Scale 10C (0, 4, 6, 10)  Low Partial Credit:  • Work of merit  • Arc/circumference formula with some substitution  • Reference to $\frac{106}{360}$ High Partial Credit:  • Calculates arc length = 185

Q9	Model Solution – 55 Marks	Marking Notes
(d) (iii)	Area of sector RST = $\pi r^2 \times \frac{\theta}{360}$ = $\pi (100)^2 \times \frac{106}{360}$ = $\frac{26500\pi}{9}$ = $9250 \cdot 245$ Number of ships = $\frac{9250 \cdot 245}{25}$ = $370$	Scale 5D (0, 2, 3, 4, 5)  Low Partial Credit:  • Work of merit  • Formula with some substitution  Mid Partial Credit:  • Formula substituted correctly  High Partial Credit:  • Area of sector evaluated correctly  • One error in substitution and finishes correctly or consistently

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