Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2020

Marking Scheme

Mathematics

Higher Level

V4

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Marcanna breise as ucht freagairt trí Ghaeilge 44

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 3, 4, 5		
10 mark scales	0, 10	0, 5, 10	0, 4, 8, 10	0, 3, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

NOTE: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.
Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)(i)	10C
(a)(ii)	5C
(a)(iii)	5C
(b)	5D

Question 2

(a)	5D
(b) (i)	5C
(b) (ii)	15D

Question 3

(a)	10C
(b)(i)	5C
(b)(ii)	10D

Question 4

(a)	15D
(b)	5D
(c)	5C

Question 5

(a)	15C
(b)	10D

Question 6

(a)	15C
(b)(i)	5C
(b)(ii)	5D

Section B

Question 7 (50 Marks)

(a)(i)	10C
(a)(ii)	5C
(b)(i)	5C
(b)(ii)	5C
(b)(iii)	5C
(c)	5C
(d)	15D

Question 8 (45 Marks)

(a)(i)	5C
(a)(ii)	10C
(a)(iii)	15D
(a)(iv)	5C
(b)	10D

Question 9 (55 Marks)

(a)(i)	10C
(a)(ii)	10C
(b)	10D
(c)	10C
(d)	5C
(e)	10C

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✔	Tick	Work of relevance	The work presented in the body of the script merits full credit
✖	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding or Unit or Arithmetic error Misreading	
~~	Horizontal wavy	Error	
✔l	Tick L		The work presented in the body of the script merits low partial credit
✔m	Tick M		The work presented in the body of the script merits mid partial credit (or partial credit)
✔h	Tick H		The work presented in the body of the script merits high partial credit
F*	F star		The work presented in the body of the script merits Full Credit (– 1)
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
≀	Vertical wavy	No work on this page (portion of the page)	
o	Oversimplify	The candidate has oversimplified the work	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

e.g. In a **C scale** where * and ~~~ and ~~~ appear in the body of the work then ✔l should be placed in the right margin.

In the case of a **D scale** with the same level of annotation then ✔m should be placed in the right margin.

A ✔ in the body of the work may sometimes be used to indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is them indicated in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

<table>
<tr><th>Q1</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>$f(-3) = 0$
$f(-3) = -3^2 + 5(-3) + p = 0$
$9 - 15 + p = 0$
$p = 6$
Or
$x^2 + 5x + p = (x + 3)(x + a)$
$= x^2 + x(a + 3) + 3a$
$a + 3 = 5$
$a = 2$
$p = 3a$
$p = 6$
Or
$x + 2$
$x + 3 \,\big)\, x^2 + 5x + p$
$x^2 + 3x$
$2x + p$
$2x + 6$
$p - 6 = 0 \quad p - 6$
$p = 6$</td><td>Scale 10C (0, 4, 8, 10)
Low Partial Credit:
Demonstrates understanding of $x + 3$ as factor or -3 as root e.g. $(x + 3)$, $f(-3)$

High Partial Credit:
Relevant equation in p (with p as only unknown)</td></tr>
</table>

(a) (ii)	$x^2 + 5x + p = (x - \alpha)(x - \alpha - 3)$ $= x^2 + x(-\alpha - \alpha - 3) + \alpha^2 + 3\alpha$ $-2\alpha - 3 = 5$ $\alpha = -4$ $p = 16 - 12$ $p = 4$ **Or** $\alpha, \alpha + 3 =$ roots $\alpha + \alpha + 3 = -5$ $2\alpha = -8$ $\alpha = -4$ and $\alpha + 3 = -1$ $p = (-1)(-4) = 4$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Demonstrates understanding of 3 as difference of roots e.g. α with $\alpha \pm 3$ $x^2 - x(\text{sum}) + \text{product} = 0$ One correct value for p $x^2 + 5x + p > 0$ Sketch of U-shaped quadratic with turning point above the x-axis *High Partial Credit:* Relevant equation in α (with α as only unknown)
(a) (iii)	$b^2 - 4ac < 0$ $5^2 - 4(1)(p) < 0$ $25 - 4p < 0$ $4p > 25$ $p > 6{\cdot}25$ $p = 7$ and $p = 8$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $b^2 - 4ac$ One correct value for p $x^2 + 5x + p > 0$ *High Partial Credit:* Relevant inequality in p (with p as only unknown) *Full credit* (–1): $p > 6{\cdot}25$

<table>
<tr>
<td>(b)</td>
<td>

$$-1 \le 2x + 5 \le 1$$
$$-6 \le 2x \le -4$$
$$-3 \le x \le -2$$

$$2x + 5 \le 1$$
$$2x \le -4$$
$$x \le -2$$

$$-1 \le 2x + 5$$
$$-6 \le 2x$$
$$-3 \le x$$
$$-3 \le x \le -2$$

Or

$$(2x + 5)^2 \le 1$$
$$4x^2 + 20x + 25 \le 1$$
$$4x^2 + 20x + 24 \le 0$$
$$x^2 + 5x + 6 \le 0$$
$$(x + 2)(x + 3) \le 0$$
$$x = -2, x = -3$$
$$-3 \le x \le -2$$

</td>
<td>

Scale 5D (0, 2, 3, 4, 5)
Low Partial Credit:
$(2x + 5)^2 \le 1$
one linear inequality

Mid Partial Credit:
$-1 \le 2x + 5 \le 1$
Identifies both linear inequalities
Quadratic inequality involving 0

High Partial Credit:
Finding −3 and −2 in Methods 1 or 2
Roots of quadratic found
$-6 \le 2x \le -4$ or equivalent

Note: Accept $-3 < x < -2$

</td>
</tr>
</table>

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$iz_1 = -4 + 3i$ $i(iz_1) = i(-4 + 3i)$ $-z_1 = -4i + 3i^2$ $z_1 = 3 + 4i$ $3z_1 - z_2 = 3(3 + 4i) - z_2 = 11 + 17i$ $z_2 = 9 + 12i - 11 - 17i$ $z_2 = -2 - 5i$ **Or** $z_1 = \dfrac{(-4 + 3i)(-i)}{(i)(-i)}$ $z_1 = 3 + 4i$ and continues	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Either equation multiplied/divided by $\boldsymbol{i}$ *Mid Partial Credit:* z_1 found z_2 written in terms of z_1 with z_1 substituted z_1 eliminated *High Partial Credit* z_1 found and substituted into second equation z_2 found by elimination
(b) (i)	$r = \dfrac{T_2}{T_1} = \dfrac{5 - i}{3 + 2i} \times \dfrac{3 - 2i}{3 - 2i}$ $r = \dfrac{15 - 13i - 2}{9 + 4}$ $r = \dfrac{13 - 13i}{13}$ $r = 1 - i$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $\dfrac{T_2}{T_1}$ *High Partial Credit:* $\dfrac{5 - i}{3 + 2i} \times \dfrac{3 - 2i}{3 - 2i}$
(b) (ii)	$T_9 = ar^8$ $T_9 = (3 + 2i)(1 - i)^8$ $T_9 = (3 + 2i)\left(\sqrt{2}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)\right)^8$ $T_9 = (3 + 2i)(\sqrt{2})^8\left(\cos\dfrac{7\pi(8)}{4} + i\sin\dfrac{7\pi(8)}{4}\right)$ $T_9 = (3 + 2i)(16)(\cos 14\pi + i\sin 14\pi)$ $T_9 = (3 + 2i)(16)(1 + 0i)$ $T_9 = 48 + 32i$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* $T_9 = ar^8$ Any correct use of De Moivre Some use of De Moivre's Theorem on r *Mid Partial Credit:* Modulus and argument found for r *High Partial Credit:* Solution in polar form with some simplification **Note**: Accept candidates r from **(b)(i)**

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$fg(x) = f\left(\frac{x+5}{6}\right)$ $fg(x) = 6\left(\frac{x+5}{6}\right) - 5 = x$ $gf(x) = g(6x-5)$ $gf(x) = \frac{(6x-5)+5}{6} = \frac{6x}{x} = x$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $f\left(\frac{x+5}{6}\right)$ $g(6x-5)$ Particular case verification *High Partial Credit:* One correct composition simplified to x
(b) (i)	$\log_5 y = \log_5 5x^2$ $\log_5 y = \log_5 5 + \log_5 x^2$ $\log_5 y = 1 + 2\log_5 x$ $a = 1$ and $b = 2$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $\log_5 5x^2 = \log_5 y$ $\log_5 y = \log_5 5x^2$ *High Partial Credit:* $\log_5 y = \log_5 5 + \log_5 x^2$
(b) (ii)	$\log_5 y = \log_5 5x^2 = 2 + \log_5\left(\frac{126x}{25} - 1\right)$ $\log_5 5x^2 = \log_5\left(\frac{126x}{25} - 1\right) \times 25$ $5x^2 = 126x - 25$ $5x^2 - 126x + 25 = 0$ $(5x-1)(x-25) = 0$ $x = \frac{1}{5}$ **or** $x = 25$ $y = 5x^2 = 5\left(\frac{1}{5}\right)^2 2 = \frac{1}{5}$ **or** $y = 5(25)^2 = 3125$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Some relevant use of laws of logs *Mid Partial Credit:* Quadratic equation *High Partial Credit:* x values found **Note**: If 2 is incorrectly (non log) dealt with then award MPC at most **Note**: If incorrect work leads to a non-quadratic equation then award MPC at most

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$f'(x) = 3x^2 + 2kx + 15$ $3(3)^2 + 2k(3) + 15 = -12$ $27 + 6k + 15 = -12$ $6k = -54$ $k = -9$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Any relevant differentiation *Mid Partial Credit:* Expression fully differentiated *High Partial Credit:* Derivative fully substituted *No Credit:* No differentiation
(b)	$f'(x) = 3x^2 + 2(-9)x + 15$ $3x^2 - 18x + 15 = 0$ $x^2 - 6x + 5 = 0$ $x = 1 \quad x = 5$ $f(1) = 15 \qquad (1, 15)$ $f(5) = -17 \qquad (5, -17)$ $m_{g(x)} = -\frac{32}{4} = -8$ $y - 15 = -8(x - 1)$ $g(x)$: $\quad 8x + y - 23 = 0$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Any relevant differentiation *Mid Partial Credit:* Both x values found *High Partial Credit:* Turning points found
(c)	$f''(x) = 6x - 18 = 0$ $x = 3$ $f(3) = -1$ $(3, -1)$ is the point of inflection $8(3) + (-1) - 23 = 0$ $0 = 0$ $\Rightarrow (3, -1) \in g(x)$.	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $f''(x)$ *High Partial Credit:* x coordinate of point of inflection found Point of inflection found **Note**: Accept candidates $g(x)$ from **(b)** with relevant statement

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$$A = \frac{250000(0{\cdot}00287)(1{\cdot}00287)^{300}}{(1{\cdot}00287)^{300} - 1}$$ $A = €1244{\cdot}06$ **Or** $$\frac{A}{1{\cdot}00287^1} + \frac{A}{1{\cdot}00287^2} + \cdots \frac{A}{1{\cdot}00287^{300}}$$ $= 250000$ $$A\left[\frac{\frac{1}{1{\cdot}00287}\left(\left(\frac{1}{1{\cdot}00287}\right)^{300} - 1\right)}{\frac{1}{1{\cdot}00287} - 1}\right] = 250000$$ $200{\cdot}9544372 \times A = 250000$ $A = €1244{\cdot}06$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Formula with some correct substitution $(1{\cdot}00287)$ 300 *High Partial Credit:* Formula fully substituted
(b)	$$\frac{1771}{1{\cdot}003} + \frac{1771}{1{\cdot}003^2} + \cdots + \frac{1771}{1{\cdot}003^{167}} + \frac{1771}{1{\cdot}003^{168}}$$ $$S_{168} = \frac{\frac{1771}{1{\cdot}003}\left[\left(\frac{1}{1{\cdot}003}\right)^{168} - 1\right]}{\frac{1}{1{\cdot}003} - 1}$$ $= €233438{\cdot}25$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $\frac{1771}{1{\cdot}003}$ 168 *Mid Partial Credit:* S_{168} formula with some substitution *High Partial Credit:* Formula fully substituted

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$f(x) = (3x-5)(2x+4)$ $= 6x^2 + 2x - 20$ $f(x+h) = 6(x+h)^2 + 2(x+h) - 20$ $= 6x^2 + 12hx + 6h^2 + 2x + 2h - 20$ $f(x+h) - f(x) = 12hx + 6h^2 + 2h$ $\frac{f(x+h)-f(x)}{h} = 12x + 6h + 2$ $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = 12x + 2$ $f'(x) = 12x + 2$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Some substitution into $f(x+h)$ or $y + \Delta y$ *High Partial Credit:* $f(x+h) - f(x) = 12hx + 6h^2 + 2h$ *No Credit*: Not from first principles $(3x-5)(2x+4) = 6x^2 + 2x - 20$
(b) (i)	$h'(x) = \frac{1}{2}\left(\frac{1}{2x+3}\right)(2)$ $= \frac{1}{2x+3}$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Any relevant differentiation *High Partial Credit:* $\frac{1}{2}\left(\frac{1}{2x+3}\right)$
(b) (ii)	$\int_0^A \frac{1}{2x+3}\,dx = ln3$ $\frac{1}{2}\ln(2x+3) \;\; \vert_0^A = \ln 3$ $\frac{1}{2}(\ln(2A+3) - \ln 3) = \ln 3$ $\frac{1}{2}\ln\left(\frac{2A+3}{3}\right) = \ln 3$ $\ln\left(\frac{2A+3}{3}\right)^{\frac{1}{2}} = \ln 3$ $\left(\frac{2A+3}{3}\right)^{\frac{1}{2}} = 3$ $\frac{2A+3}{3} = 9$ $2A + 3 = 27$ $2A = 24$ $A = 12$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Integration indicated *Mid Partial Credit:* $\frac{1}{2}\ln(2x+3) \;\; \vert_0^A$ Substitutes limits into integral and stops Correct integration with some substitution *High Partial Credit:* Integral evaluated at $x = A$ only (i.e. omits $\ln 3$ on LHS and finishes) **Note**: Must have integration to gain any credit

<table>
<tr><th>Q7</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>
<table>
<tr><td>T.</td><td>T_1</td><td>T_2</td><td>T_3</td><td>T_4</td><td>T_5</td><td>T_6</td><td>T_7</td><td>T_8</td></tr>
<tr><td>No.</td><td>1</td><td>3</td><td>6</td><td>10</td><td>15</td><td>21</td><td>28</td><td>36</td></tr>
</table>
</td><td>Scale 10C (0, 4, 8, 10)
Low Partial Credit:
One correct new entry

High Partial Credit:
Three correct new entries</td></tr>
<tr><td>(a)
(ii)</td><td>$\frac{n}{2}(n+1) = 1275$
$n^2 + n - 2250 = 0$
$(n-50)(n+51) = 0$
$n = 50$
1275 is the 50th triangular number</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
$\frac{n}{2}(n+1) = 1275$

High Partial Credit:
$n = 50$
Note: accept T_{50} as valid reason</td></tr>
<tr><td>(b)
(i)</td><td>$T_{n+1} = T_n + (n+1)$
$= \frac{n}{2}(n+1) + (n+1)$
$= \frac{n(n+1) + 2(n+1)}{2}$
$= \frac{(n+1)(n+2)}{2}$</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
2 identified as C.D.
Correct numerator

High Partial Credit:
$= \frac{n(n+1) + 2(n+1)}{2}$</td></tr>
<tr><td>(b)
(ii)</td><td>$T_{n+1} + T_n$
$= \frac{(n+1)(n+2)}{2} + \frac{n}{2}(n+1)$
$= \frac{(n+1)(2n+2)}{2}$
$= \frac{2(n+1)(n+1)}{2}$
$= (n+1)^2$</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
$T_{n+1} + T_n$ with some substitution
Particular case verification

High Partial Credit:
$\frac{(n+1)(n+2)}{2} + \frac{n}{2}(n+1)$</td></tr>
</table>

(b) (iii)	$(n+1)^2 = 12544$ $n+1 = \sqrt{12544} = 112$ $n = 111$ $n = 111$ T_{111} is the smaller term $T_{111} = \frac{111(112)}{2}$ $T_{111} = 6216$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $(n+1)^2$ *High Partial Credit:* $n = 111$, or $n = 112$
(c)	$N_3 = \left(\frac{\left(3+2\sqrt{2}\right)^3 - \left(3-2\sqrt{2}\right)^3}{4\sqrt{2}}\right)^2$ $= 1225$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Formula with some substitution *High Partial Credit:* Formula fully substituted *Full Credit:* Correct answer with no work shown

(d)		
	$1^2+2^2+3^2+\cdots+n^2=\dfrac{n(n+1)(2n+1)}{6}$ $\boldsymbol{P(1)}$: $1=\frac{1(2)(3)}{6}$ $\boldsymbol{P(k)}$: $1+4+9+\cdots+k^2=\dfrac{k(k+1)(2k+1)}{6}$ $\boldsymbol{P(k+1)}$: $1+4+9+\cdots+k^2+(k+1)^2=\dfrac{(k+1)(k+2)(2k+3)}{6}$ $LHS=\dfrac{k(k+1)(2k+1)}{6}+(k+1)^2$ $LHS=\dfrac{k(k+1)(2k+1)+6(k+1)^2}{6}$ $LHS=\dfrac{(k+1)[k(2k+1)+6(k+1)]}{6}$ $LHS=\dfrac{(k+1)[2k^2+7k+6]}{6}$ $\dfrac{(k+1)(k+2)(2k+3)}{6}=RHS$ Thus the proposition is true for $n=k+1$ provided it is true for $n=k$ but it is true for $n=1$ and therefore true for all positive integers.	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Step $P(1)$ *Mid Partial Credit:* Step $P(k+1)$ *High Partial Credit:* Uses *Step* $P(k)$ to prove *Step* $P(k+1)$ *Full Credit(−1):* Concluding statement missing **Note:** Accept *Step* $P(1)$, *Step* $P(k)$, *Step* $P(k+1)$ in any order

Q8	Model Solution – 45 Marks	Marking Notes
(a) (i)	$\cos\theta = \frac{x}{5}$ $\quad$ $\sin\theta = \frac{y}{5}$ $5\cos\theta = x$ $\quad$ $5\sin\theta = y$ $(x, y) = (5\cos\theta\,,\ 5\sin\theta)$ $\therefore a = 5,\ \ b = 5$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $\cos\theta = \frac{x}{5}$ or equivalent *High Partial Credit:* a or b found Correct answer without work
(a) (ii)	$A(\theta) = (10\cos\theta) \times (10\sin\theta)$ $A(\theta) = 100\cos\theta\sin\theta$ $= 50 \times 2\cos\theta\sin\theta$ $= 50(\sin 2\theta)$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* xy $(10\cos\theta) \times (10\sin\theta)$ *High Partial Credit:* $100\cos\theta\sin\theta$
(a) (iii)	$A(\theta) = 50\sin 2\theta$ $A'(\theta) = 50\cos 2\theta \times 2$ $A'(\theta) = 100\cos 2\theta = 0$ $\cos 2\theta = 0$ $2\theta = \frac{\pi}{2}$ $\theta = \frac{\pi}{4}$ $2x = 2\left(5\cos\left(\frac{\pi}{4}\right)\right) = 5\sqrt{2}$ $2y = 2\left(5\sin\left(\frac{\pi}{4}\right)\right) = 5\sqrt{2}$ $\Longrightarrow$ Square	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* $a'(\theta)$ States $\frac{dy}{dx} = 0$ *Mid Partial Credit:* Correct differentiation *High Partial Credit:* Value of θ at maximum found Value of x or y at maximum fully substituted *No Credit:* No differentiation
(a) (iv)	Max area $= 5\sqrt{2} \times 5\sqrt{2}$ $= 50$ Square units **Or** Max area $= 50(\sin 2\theta)$ $50(\sin\frac{\pi}{2})$ $= 50$ Square units	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* xy length $\times$ width $50(\sin 2\theta)$ *High Partial Credit:* Area formula fully substituted

(b)		
	$$\frac{dx}{dt}=\frac{dx}{dl}\cdot\frac{dl}{dt}$$ $$\frac{2}{5}=\frac{x}{l+x}$$ $$2l+2x=5x$$ $$x=\frac{2}{3}l$$ $$\frac{dx}{dl}=\frac{2}{3}$$ $$\left\|\frac{dx}{dt}\right\|=\frac{2}{3}\times\frac{3}{2}$$ $$\left\|\frac{dx}{dt}\right\|=1\text{ m/sec}$$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $\frac{dx}{dt}$ or $\frac{dx}{dl}$ or $\frac{dl}{dt}$ given Reference to similar triangles $\frac{2}{5}$ or $\frac{5}{2}$ *Mid Partial Credit:* $\frac{dx}{dt}=\frac{dx}{dl}\cdot\frac{dl}{dt}$ or equivalent with one relevant substitution $x=\frac{2}{3}l$ *High Partial Credit:* $\frac{dx}{dl}$ and $\frac{dl}{dt}$ found

Q9	Model Solution – 55 Marks	Marking Notes
(a) (i)	$N(t) = 450e^{0{\cdot}065t}$ $N(4{\cdot}5) = 450e^{0{\cdot}065(4{\cdot}5)}$ $= 602{\cdot}89$ $= 603$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Some substitution into function Correct answer without work *High Partial Credit:* $450e^{0{\cdot}065(4{\cdot}5)}$
(a) (ii)	$N(t) = 450e^{0{\cdot}065t}$ $\frac{N(t)}{450} = e^{0{\cdot}065t}$ Convert to log equation $\ln(\frac{N(t)}{450}) = 0{\cdot}065t$ $\ln\big(N(t)\big) - \ln 450 = 0{\cdot}065t$ $\frac{\ln\big(N(t)\big) - \ln 450}{0{\cdot}065} = t$ $t = \frac{\ln(790) - \ln 450}{0{\cdot}065}$ $t = 8{\cdot}7$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Some substitution into function Full substitution and stops *High Partial Credit:* Equation in t (i.e. logs handled correctly)

(b)	$\frac{1}{9}\int_{3}^{12} 450e^{0\cdot065t}dt$ $= \frac{450}{9(0\cdot065)}\left[e^{12(\cdot065)} - e^{3(0\cdot065)}\right]$ $= 743\cdot2$ Average no. $= 743$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Integration indicated *Mid Partial Credit:* Integration correct *High Partial Credit:* Substitutes limits into integral and stops **Note**: Must have integration to gain any credit
(c)	$N'(t) = 450e^{0\cdot065t} \times 0\cdot065$ $N'(t) = 29\cdot25e^{0.065t}$ $N'(12) = 29\cdot25e^{0\cdot065(12)}$ $= 63\cdot8$ At hour 12 the population is growing at a rate of 64 bacteria per hour **or** At hour 12 the population is growing at a rate of 63·8 bacteria per hour	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $N'(t)$ stated or indicated *High Partial Credit:* Derivative fully substituted $N'(12) = 63\cdot8$ and stops

(d)	$N'(t) = 29{\cdot}25e^{0{\cdot}065k} > 90$ $e^{0{\cdot}065k} > 90/\ 29{\cdot}25$ $k > \dfrac{\ln\frac{90}{29{\cdot}25}}{0{\cdot}065}$ $k > 17{\cdot}29$ $k = 18$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $29{\cdot}25e^{0{\cdot}065k} > 90$ *High Partial Credit:* Equation in k (i.e. taking logs handled correctly) *No Credit:* No differentiation **Note:** if $k > 17{\cdot}29 \Rightarrow k = 17$ Award Full credit (–1)
(e)	$450e^{0{\cdot}065t} = 220e^{0{\cdot}17t}$ $\dfrac{450}{220} = \dfrac{e^{0{\cdot}17t}}{e^{0{\cdot}065t}}$ $\dfrac{450}{220} = e^{0{\cdot}105t}$ $\ln\left(\dfrac{450}{220}\right) = 0{\cdot}105t$ $\dfrac{\ln\left(\frac{450}{220}\right)}{0{\cdot}105} = t$ $t = 6{\cdot}82$ $t = 7$ hours	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $450e^{0{\cdot}065t} = 220e^{0{\cdot}17t}$ *High Partial Credit:* Equation in t (i.e. taking logs handled correctly)

Leaving Certificate 2020

Marking Scheme

Mathematics

Higher Level

Paper 2

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Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales		0, 2, 5	0, 3, 4, 5	0, 2, 3, 4, 5	
10 mark scales			0, 4, 8, 10	0, 3, 5, 8, 10	
15 mark scales			0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales					
25 mark scales					

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

NOTE: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.
Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)	15D
(b)	10D

Question 2

(a)	10D
(b)	15D

Question 3

(a)	15C
(b)	10D

Question 4

(a)	10C
(b)	15D

Question 5

(a)(i)	15C
(a)(ii)	5C
(b)	5C

Question 6

(a)	10D
(b)(i)	10C
(b)(ii)	5C

Section B

Question 7

(a)(i)	10C
(a)(ii)	10C
(a)(iii)	10C
(b)	10D
(c)	5D
(d)	10D

Question 8

(a)(i)	15D
(a)(ii)	10D
(b)(i)	5B
(b)(ii)	10D
(c)	10C
(d)	10D
(e)	10D

Question 9

(a)	15C
(b)	5C
(c)	5D

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	Slope of *BC* $m = \frac{3+12}{-4-6} = -\frac{3}{2}$ Equation *BC* $3x + 2y + 6 = 0.$ Perp. Distance from *A* to line *BC* $\frac{3(2)+2(-6)+6}{\sqrt{3^2+2^2}} = \frac{6-12+6}{\sqrt{13}} = \frac{0}{\sqrt{13}} = 0.$ Therefore *A*, *B* and *C* are collinear.	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Slope formula with some substitution Equation of line formula with some substitution Effort at finding are of triangle *ABC* *Mid Partial Credit:* Equation of *BC* *High Partial Credit:* Perp. Distance formula with some substitution from relevant line Area of triangle *ABC* $= 0$ but perp. distance not explicit *Full credit* (*–1*) Distance $= 0$ but conclusion omitted Area of triangle *ABC* $= 0$ **and** perp. dist. $= 0$ but conclusion omitted

<table>
<tr>
<td>(b)</td>
<td>
Slope of $a = \frac{1}{2}$

Slope of $b = \tan 60° = \sqrt{3}$

$\tan\theta = \pm\dfrac{\sqrt{3} - \frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} = \pm\dfrac{2\sqrt{3} - 1}{2 + \sqrt{3}}$

$= \pm\dfrac{(2\sqrt{3} - 1)(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}$

$= \pm(-8 + 5\sqrt{3})$

$\theta = \tan^{-1}(-8 + 5\sqrt{3})$

$\theta = 33{\cdot}435°$

Or

$\theta + \tan^{-1}\frac{1}{2} + 120° = 180°$`

$\theta + 26{\cdot}565° + 120° = 180°$

$\theta = 33{\cdot}435°$
</td>
<td>
Scale 10D (0, 3, 5, 8, 10)
Low Partial Credit:
Slope of $a = \frac{1}{2}$
Slope of $b = \tan 60°$

Mid Partial Credit:
Tan formula with some relevant substitution

High Partial Credit:
Tan formula fully substituted

Full credit (–1)
$\theta = +\tan^{-1}(-8 + 5\sqrt{3})$

Scale 10D (0, 3, 5, 8, 10)
Low Partial Credit:
Slope of $a = \frac{1}{2}$
120°

Mid Partial Credit:
$\tan^{-1}\dfrac{1}{2} + 120°$

High Partial Credit:
$\theta + 26{\cdot}565° + 120° = 180°$
and fails to finish
</td>
</tr>
</table>

<table>
<tr><th>Q2</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>Centre: $(2,-1)$
Radius: $\sqrt{2^2+(-1)^2+4}=3$
Distance from centre to B: $\sqrt{90}$
Pythagoras: $|BT|^2=90-3^2=81$
$\Rightarrow |BT|=9$</td><td>Scale 10D (0, 3, 5, 8, 10)
Low Partial Credit:
Centre or radius

Mid Partial Credit:
$\sqrt{90}$

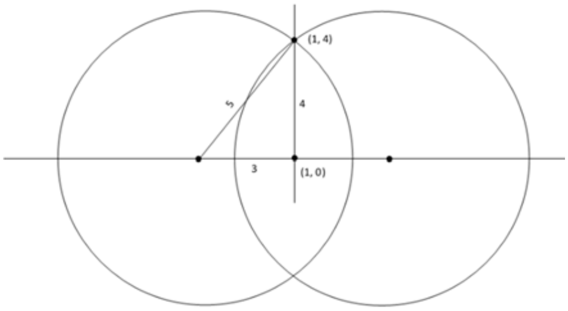
High Partial Credit:
Pythagoras fully substituted (: $|BT|^2$)</td></tr>
<tr><td>(b)</td><td>Centre $(-g,0)$.
Radius = $\sqrt{g^2+(0)^2-c}=5$
$\Rightarrow\ g^2-c=25$ Equation (i)

Equation is $x^2+y^2+2gx+c=0$
Sub (1, 4):
$1^2+4^2+2g(1)+c=0$
$\Rightarrow 17+2g+c=0$ Equation (ii)
Solve (i) and (ii)
$17+2g+(g^2-25)=0$
$\Rightarrow\ g^2+2g-8=0$

Solve for g:
$g=2$ and $g=-4$
Centres are $(-2,0)$ and $(4,0)$
Equations:
$(x+2)^2+y^2=25$,
$(x-4)^2+y^2=25$
Or</td><td>Scale 15D (0, 4, 7, 11, 15)
Low Partial Credit:
Centre $(-g,0)$ or equivalent
Some substitution of (1, 4) into general equation of circle

Mid Partial Credit:
2 relevant equations in g and c

High Partial Credit:
Quadratic in g ($g^2+2g-8=0$ or equivalent)</td></tr>
</table>

	Centre: $(-g, 0)$ $$\sqrt{(1+g)^2+(4-0)^2}=5$$ $$(1+g)^2=9$$ $$1+g=\pm 3$$ $g=-4$ or $g=2$ Equations: $(x+2)^2+y^2=25,$ $(x-4)^2+y^2=25$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Centre $(-g, 0)$ or equivalent Some substitution into distance formula *Mid Partial Credit:* Distance formula fully substituted *High Partial Credit:* Quadratic in g
	or  Centres $(-2, 0)$ and $(4, 0)$; radius =5 Equations: $(x+2)^2+y^2=25,$ $(x-4)^2+y^2=25$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Diagram with (1, 0) identified *Mid Partial Credit:* -2 or 4 identified *High Partial Credit:* $g=-4$ and $g=2$

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$\dfrac{6}{\sin 17°} = \dfrac{\lvert HF\rvert}{\sin 35°}$ $\lvert HF\rvert = \dfrac{6\sin 35°}{\sin 17°} = 11{\cdot}771$ $\dfrac{11{\cdot}771}{\sin 95°} = \dfrac{x}{\sin 33°}$ $x = \dfrac{11{\cdot}771(\sin 33°)}{\sin 95°}$ $x = 6{\cdot}44$ m (Or $x = 6{\cdot}43$ m from rounded $\lvert HF\rvert$)	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* $\lvert\angle FHE\rvert = 17°$ $\lvert\angle GHF\rvert = 33°$ Some relevant substitution into relevant formula *High Partial Credit:* $\lvert HF\rvert$ found and stops $\lvert HE\rvert = 16{\cdot}17$ found and stops Incorrect value of $\lvert HF\rvert$ (or $\lvert HE\rvert$) used correctly to find x
(b)	$\lvert\angle BOA\rvert = 60°$ => $\lvert\angle COA\rvert = 30°$ $\sin\angle COA = \dfrac{r}{DO} = \dfrac{1}{2}$ $\Rightarrow\ \lvert DO\rvert = 2r$ $\Rightarrow\ \lvert OC\rvert = 3r$ Area $c = \pi r^2$ Area $s = \pi(3r)^2 = 9\pi r^2$ Area s : Area $c = 9 : 1 \Rightarrow k = 9$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* 30° Area $c = \pi r^2$ *Mid Partial Credit:* $\lvert DO\rvert = 2r$ *High Partial Credit:* $\lvert OC\rvert = 3r$

Q4	Model Solution – 25 Marks	Marking Notes
(a)	Reference angle: $\frac{\pi}{6}$ 2[nd] Quadrant: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\frac{\theta}{2} = \frac{5\pi}{6} + 2n\pi$ $\theta = \frac{5\pi}{3} + 4n\pi$ $n = 0 \implies \theta = \frac{5\pi}{3} = 300°$ 4[nd] Quadrant: $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ $\frac{\theta}{2} = \frac{11\pi}{6} + 2n\pi$ $\theta = \frac{11\pi}{3} + 4n\pi$ $n = 0 \implies \theta = \frac{11\pi}{3} = 660°$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* 30° or −30° Mention of 2[nd] or 4[th] quadrants *Mid Partial Credit* 150° or 330° or equivalent *High Partial Credit:* 150° and 330° or equivalent
(b)	Area of $\Delta COA =$ Area of Sector $- 21$ $= \frac{1}{2}r^2\theta - 21 = 8{\cdot}4$ Area of ΔCOA: $\frac{1}{2}\|CO\|\|7\| \sin 1{\cdot}2 = 8{\cdot}4$ $\|CO\| = \frac{8{\cdot}4}{3{\cdot}5 \sin 1{\cdot}2} = 2{\cdot}57$ $\|BC\| = 7 - 2{\cdot}6 = 4{\cdot}4$ cm	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Area of ΔCOA Area of Sector COA *Mid Partial Credit:* Area of $\Delta COA =$ Area of Sector $- 21$ *High Partial Credit:* $\frac{1}{2}\|CO\|\|7\| \sin 1{\cdot}2 = 8{\cdot}4$ *Full credit (–1)* Distance $\|CO\|$ found and stops

Q5	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	$$P(B\|A) = \frac{P(A \cap B)}{P(A)}$$ $$P(B\|A) = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Formula for $P(B\|A)$ *High Partial Credit:* Formula fully substituted
(a) **(ii)**	$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$ $$\frac{11}{12} = \frac{3}{4} + P(B) - \frac{1}{2}$$ $$\frac{11}{12} - \frac{1}{4} = P(B) = \frac{2}{3}$$ Check if: $P(A) \times P(B) = P(A \cap B)$ $$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2} = P(A \cap B)$$ $\Rightarrow$ Independent **or** $$P(B\|A) = P(B)$$ $$\frac{2}{3} = \frac{2}{3}$$ $\Rightarrow$ Independent	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Condition for independent events *High Partial Credit:* $P(B) = \frac{2}{3}$ $P(A) \times P(B) = P(A \cap B)$ fully checked for any relevant value (< 1) of $P(B)$ with a valid conclusion

<table>
<tr>
<td>(b)</td>
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Add	**1**	**1**	**2**	**3**
1	2	2	3	4
1	2	2	3	4
2	3	3	4	5
3	4	4	5	6

Rem.	**1**	**1**	**2**	**3**
1	2	2	0	1
1	2	2	0	1
2	0	0	1	2
3	1	1	2	0

Lee has 6 chances to win.
The others only have 5 chances
$\Longrightarrow$ It is not a fair game
</td>
<td>

Scale 5C (0, 3, 4, 5)
Low Partial Credit:
Any relevant listing of remainders/sums

High Partial Credit:
All remainders listed but no conclusion or incorrect conclusion or unsound conclusion
</td>
</tr>
</table>

<table>
<tr><th>Q6</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

	D	P	H/E
	$0{\cdot}3$	$0{\cdot}6$	$0{\cdot}1$
	$\times 0{\cdot}7$	$\times 0{\cdot}25$	$\times 0{\cdot}09$
VW	$0{\cdot}21$	$0{\cdot}15$	$0{\cdot}009$

$$P(VW) = 0{\cdot}21 + 0{\cdot}15 + 0{\cdot}009$$
$$= 0{\cdot}369$$

</td><td>

Scale 10D (0, 3, 5, 8, 10)

Low Partial Credit:
Any relevant probability from line 1 written

Mid Partial Credit:
Any 1 relevant probability from line 3 formulated or written

High Partial Credit:
All 3 relevant probability from line 3 formulated or written

</td></tr>
<tr><td>(b)
(i)</td><td>

$$\binom{5}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3\frac{1}{4} = \frac{135}{2048}$$

</td><td>

Scale 10C (0, 4, 8, 10)

Low Partial Credit:
$\binom{5}{2}$ or $\frac{3}{4}$ or $\left(\frac{1}{4}\right)^2$ or $\left(\frac{1}{4}\right)^3$

High Partial Credit:
$\binom{5}{2}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^y$ where $x, y \neq 1$

</td></tr>
<tr><td>(b)
(ii)</td><td>

P(2 or less) = P(0 pass + 1 pass + 2 pass)

P(0 pass) = $\left(\frac{1}{2}\right)^n$

P(1 pass) = $\left[\binom{n}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1}\right]$

P(2 pass) = $\left[\binom{n}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2}\right]$

P(≤ 2) = $\frac{1}{2^n} + \left[\frac{n}{2^n}\right] + \left[\frac{n(n-1)}{2^{n+1}}\right]$

$$= \frac{2 + 2n + n^2 - n}{2^{n+1}} = \frac{n^2 + n + 2}{2^{n+1}}$$

$$\Rightarrow a = 1, \; b = 1, \; c = 2.$$

</td><td>

Scale 5C (0, 3, 4, 5)

Low Partial Credit:
P(0 pass + 1 pass + 2 pass)

High Partial Credit:
Any two of $\left(\frac{1}{2}\right)^n$ or $\left[\binom{n}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1}\right]$ or $\left[\binom{n}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2}\right]$

</td></tr>
</table>

Q7	Model Solution – 55 Marks	Marking Notes
(a) **(i)**	$9^2 = 3{\cdot}3^2 + h^2$ $h^2 = 81 - 10{\cdot}89$ $h = 8{\cdot}37$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Pythagoras formulated *High Partial Credit:* $\sqrt{9^2 - 3{\cdot}3^2}$ or equivalent
(a) **(ii)**	$CSA = \pi rl = \pi 3{\cdot}3(9) = 93{\cdot}31$ cm^2	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Formula for **CSA** with some substitution *High Partial Credit:* Formula fully substituted
(a) **(iii)**	Circumference of cup = $2\pi r = 2\pi(3{\cdot}3)$ Arc length of sector $= \frac{2\pi \times 9\theta}{360°}$ $2\pi(3{\cdot}3) = \frac{2\pi \times 9\theta}{360°}$ $\theta = \frac{3{\cdot}3(360)}{9} = 132°$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Formula for circumference **or** arc length with some substitution *High Partial Credit:* Both formulas fully substituted
(b)	$\frac{3{\cdot}3}{8{\cdot}37} = \frac{r}{7{\cdot}37}$ $r = 2{\cdot}905$ cm $v = \frac{1}{3}\pi(2{\cdot}905)^2 7{\cdot}37$ $65{\cdot}16$ cm^3 $65{\cdot}2$ cm^3	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Any relevant effort to find r using similar triangles *Mid Partial Credit:* $\boldsymbol{r}$ found *High Partial Credit:* Volume formula fully substituted Note: If $r = 3{\cdot}3$ used then award MPC at most

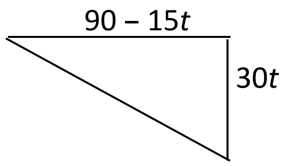
(c)	Volume of water in one second $\pi 0{\cdot}8^2(2{\cdot}5)$ $= 5{\cdot}0265$ cm^3 Time taken is $\frac{65{\cdot}2}{\pi 0{\cdot}8^2(2{\cdot}5)} = 13$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Any relevant effort to find volume of water *Mid Partial Credit:* $\pi 0{\cdot}8^2(2{\cdot}5)$ *High Partial Credit:* Time formula fully substituted **Note**: Accept work using candidates volume from part (b)
(d)	$\frac{3{\cdot}3}{8{\cdot}37} = \frac{r}{h}$ $r = \frac{3{\cdot}3h}{8{\cdot}37}$ $v = \frac{1}{3}\pi\left(\frac{3{\cdot}3h}{8{\cdot}37}\right)^2 h = 60$ $h^3 = \frac{60 \times 8{\cdot}37^2 \times 3}{\pi 3{\cdot}3^2}$ $h = \sqrt[3]{\frac{60 \times 8{\cdot}37^2 \times 3}{\pi 3{\cdot}3^2}} = 7{\cdot}169$ $x = 8{\cdot}37 - 7{\cdot}169 = 1{\cdot}2$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Effort to link r and h *Mid Partial Credit* r and h linked *High Partial Credit:* $h^3 = \frac{60 \times 8{\cdot}37^2 \times 3}{\pi 3{\cdot}3^2}$ or equivalent

Q8	Model Solution – 70 Marks	Marking Notes
(a) (i)	$$z = \frac{x - \bar{x}}{\sigma}$$ $$\frac{x - 280}{90} = 0{\cdot}68$$ $$\Rightarrow x = 341{\cdot}2$$ $$x = 342$$	**Scale 15D(0, 4, 7, 11, 15)** *Low Partial Credit:* μ or σ identified *Mid Partial Credit:* $0{\cdot}68$ *High Partial Credit:* Equation in x fully substituted and stops or continues incorrectly
(a) (ii)	Eileen's z-score $= \frac{260-280}{90} = -0{\cdot}222 = z$ 40% z-score $= -0{\cdot}25$ i.e. z score for 60% $-0{\cdot}222 > -0{\cdot}25$ Eileen is eligible to re-sit the test. or $P(0{\cdot}222) = 0{\cdot}5871$ $1 - 0{\cdot}5871 = 0{\cdot}4129$ 41·29%	**Scale 10D(0, 3, 5, 8, 10)** *Low Partial Credit:* μ or σ identified *Mid Partial Credit:* $\frac{260-280}{90}$ or $-0{\cdot}222$ or $-0{\cdot}25$ *High Partial Credit:* $-0{\cdot}222$ and $-0{\cdot}25$ Note: Allow $-0{\cdot}26$
(b) (i)	95% of the of the data lies in the interval $-1{\cdot}96 \leq z \leq 1{\cdot}96$	**Scale 5B (0, 2, 5)** *Partial Credit:* 95% without context

(b) (ii)	$$1{\cdot}96\sqrt{\frac{\hat{p}(1-\hat{p})}{2500}} = 0{\cdot}01568$$ $$=> \hat{p}(1-\hat{p}) = 2500\left(\frac{0{\cdot}01568^2}{1{\cdot}96^2}\right)$$ $$\Rightarrow \hat{p}^2 - \hat{p} + \frac{4}{25} = 0$$ $$\hat{p} = \frac{1 \pm \sqrt{1-4\left(\frac{4}{25}\right)}}{2} = \frac{1 \pm \frac{3}{5}}{2}$$ $$\hat{p} = \frac{4}{5} \text{ or } \frac{1}{5}$$ $\frac{1}{5}$ outside the range $$\Rightarrow \hat{p} = \frac{4}{5}$$	**Scale 10D(0, 3, 5, 8, 10)** *Low Partial Credit:* $\sqrt{\frac{\hat{p}(1-\hat{p})}{2500}}$ or equivalent written *Mid Partial Credit:* Formula fully substituted *High Partial Credit:* Quadratic in form $a\hat{p}^2 + b\hat{p} + c = 0$
(c)	H_0: Mean weight of bags has not changed H_1: Mean weight of bags has changed $$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{13{\cdot}1 - 12}{\frac{4{\cdot}5}{\sqrt{80}}} = 2{\cdot}186$$ $$2{\cdot}186 > 1{\cdot}96$$ Mean weight of the bags has changed	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* CI formulated with some correct substitution $1{\cdot}96$ H_0 or H_1 *High Partial Credit:* z score fully substituted

(d)	$P(\text{weight} > 3000)$ $= P(\text{Average of those on bus} > \frac{3000}{40})$ $P(\bar{x} > 75) = 1 - P(\bar{x} < 75)$ $z = \frac{75-73}{\frac{12}{\sqrt{40}}}$ $= 1{\cdot}054$ This gives a proportion of $0{\cdot}8531$. $1 - 0{\cdot}8531 = 0{\cdot}1469$ $= 14{\cdot}69\%$ This is the probability that the bus with 40 passengers will be above the maximum weight allowance.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $\frac{3000}{40}$ μ or σ identified *Mid Partial Credit:* z formula fully substituted *High Partial Credit:* $1{\cdot}054$

(e)	Median is 12·5 $\Longrightarrow$ D + E = 25 LQ is 7·5 $\Longrightarrow$ B + C = 15 IQR is 12 and 12 + 7·5=19·5 $\Longrightarrow$The upper quartile = 19·5 F + G = 39 G = 23 so F = 39 – 23 = 16 Now B + C + D + E + F+G = 79 The total is 8 x 13·5 = 108 So A + H = 108 – 79 = 29 H – A = 21 (range) A = 4 and H = 25 D + E = 25 so D = 11, E = 14 (cannot be 12 and 13 also cannot be 10 and 15) B + C = 15 so B = 6, C = 9 (cannot be 7 and 8 also cannot be 5 and 10) The list is: *A* *B* *C* *D* *E* *F* *G* *H* 4 6 9 11 14 16 23 25	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* One unknown number given One relevant equation written *Mid Partial Credit* Three unknown numbers given Three relevant equations written *High Partial Credit:* Five unknown numbers given Five relevant equations written

Q9	Model Solution – 25 Marks	Marking Notes
(a)	$d = \sqrt{\left(90 - \frac{15}{2}\right)^2 + \left(\frac{30}{2}\right)^2}$ $d = \sqrt{(82{\cdot}5)^2 + (15)^2}$ $d = 83{\cdot}85$ km	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* $\frac{15}{2}$ or $\frac{30}{2}$ Indication of Pythagoras *High Partial Credit:* Pythagoras fully substituted
(b)	 $s^2 = (90 - 15t)^2 + (30t)^2$ $s^2 = 8100 - 2700t + 225t^2 + 900t^2$ $s^2 = 1125t^2 - 2700t + 8100$ $s = (1125t^2 - 2700t + 8100)^{\frac{1}{2}}$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* 90 – 15t or 30t *High Partial Credit:* Pythagoras fully substituted
(c)	$s = (1125t^2 - 2700t + 8100)^{\frac{1}{2}}$ $\frac{ds}{dt} = \frac{(2250t - 2700)}{2\sqrt{1125t^2 - 2700t + 8100}}$ $\Rightarrow 2250t - 2700 = 0$ $t = \frac{2700}{2250} = 1{\cdot}2$ hours $s = (1125t^2 - 2700t + 8100)^{\frac{1}{2}}$ $s = (1125(1{\cdot}2)^2 - 2700(1{\cdot}2) + 8100)^{\frac{1}{2}}$ $s = 80{\cdot}4984 \approx 80{\cdot}5$ km	**Scale 5D(0, 2, 3, 4, 5)** *Low Partial Credit:* Any correct differntiation *Mid Partial Credit:* Value of t found *High Partial Credit:* Formula for s fully substituted Incorrect value of t (found through calculus) substituted and worked correctly. **Note**: No calculus ⇒ 0 credit

Marcanna breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc × 5% = 9·9 ⇒ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] × 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0