2020.M30

2020L003A2EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2020

Mathematics

Paper 2

Higher Level

2 hours 30 minutes

300 marks

Examination Number

Day and Month of Birth

For example, 3rd February is entered as 0302

Centre Stamp

Do not write on this page

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your Examination Number into the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

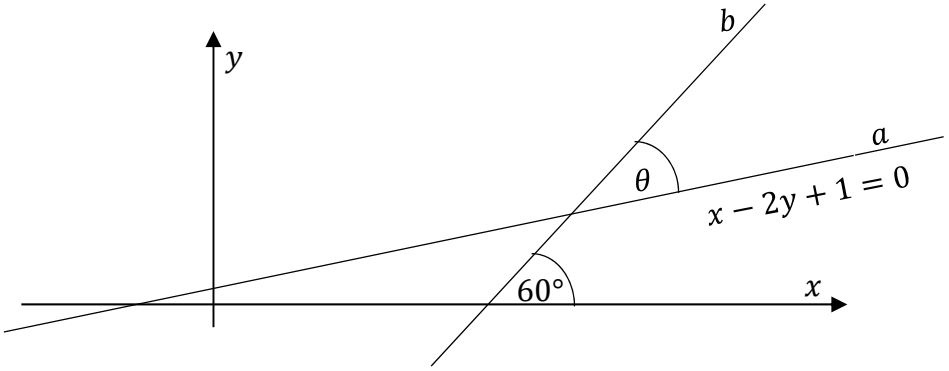
(a) The coordinates of three points are $A(2, -6), B(6, -12)$, and $C(-4, 3)$.
Find the perpendicular distance from A to BC.

Based on your answer, what can you conclude about the relationship between the points A, B, and C?

Distance:

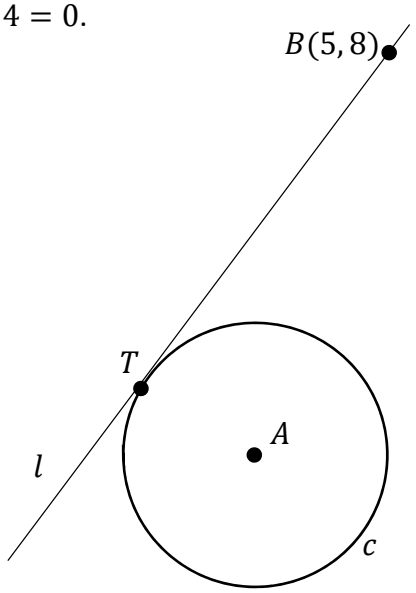
Conclusion:

(b) The diagram below shows two lines a and b. The equation of a is $x - 2y + 1 = 0$.
The acute angle between a and b is θ. Line b makes an angle of $60°$ with the positive sense of the x-axis, as shown in the diagram.
Find the value of θ, in degrees, correct to 3 decimal places.



Question 2 **(25 marks)**

(a) The circle c has equation $x^2 + y^2 - 4x + 2y - 4 = 0$.
The point A is the centre of the circle.
The line l is a tangent to c at the point T,
as shown in the diagram.
The point $B(5, 8)$ is on l.
Find $|BT|$.



(b) Two circles, c_1 and c_2, have their centres on the x-axis. Each circle has a radius of 5 units. The point $(1, 4)$ lies on each circle. Find the equation of c_1 and the equation of c_2.

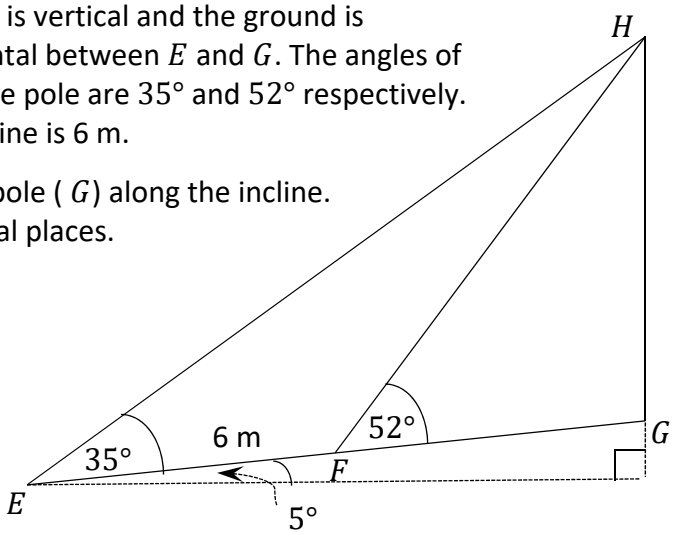
c_1:

c_2:

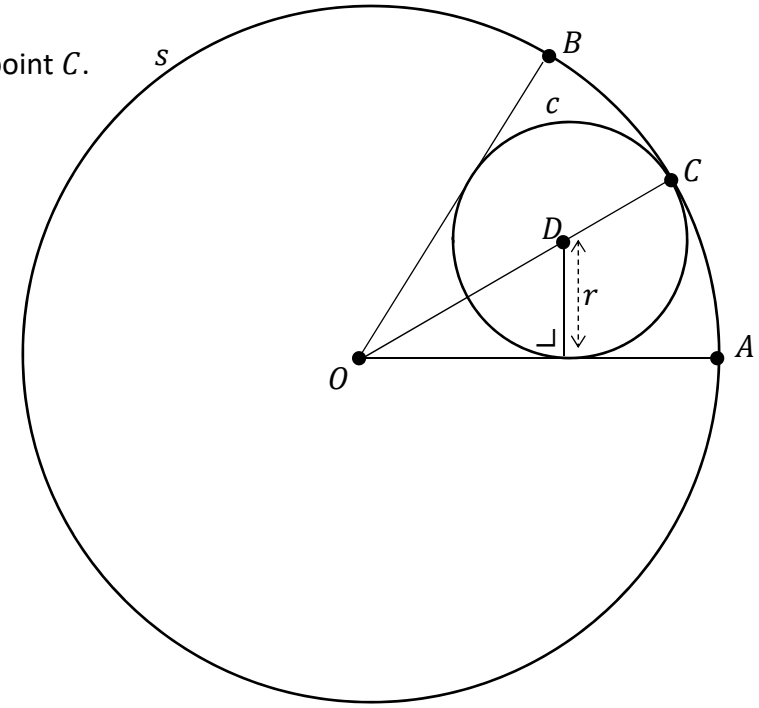
Question 3 **(25 marks)**

(a) A flagpole $[GH]$, shown in the diagram, is vertical and the ground is inclined at an angle of $5°$ to the horizontal between E and G. The angles of elevation from E and F to the top of the pole are $35°$ and $52°$ respectively. The distance from E to F along the incline is 6 m.

Find how far F is from the base of the pole (G) along the incline.
Give your answer correct to two decimal places.

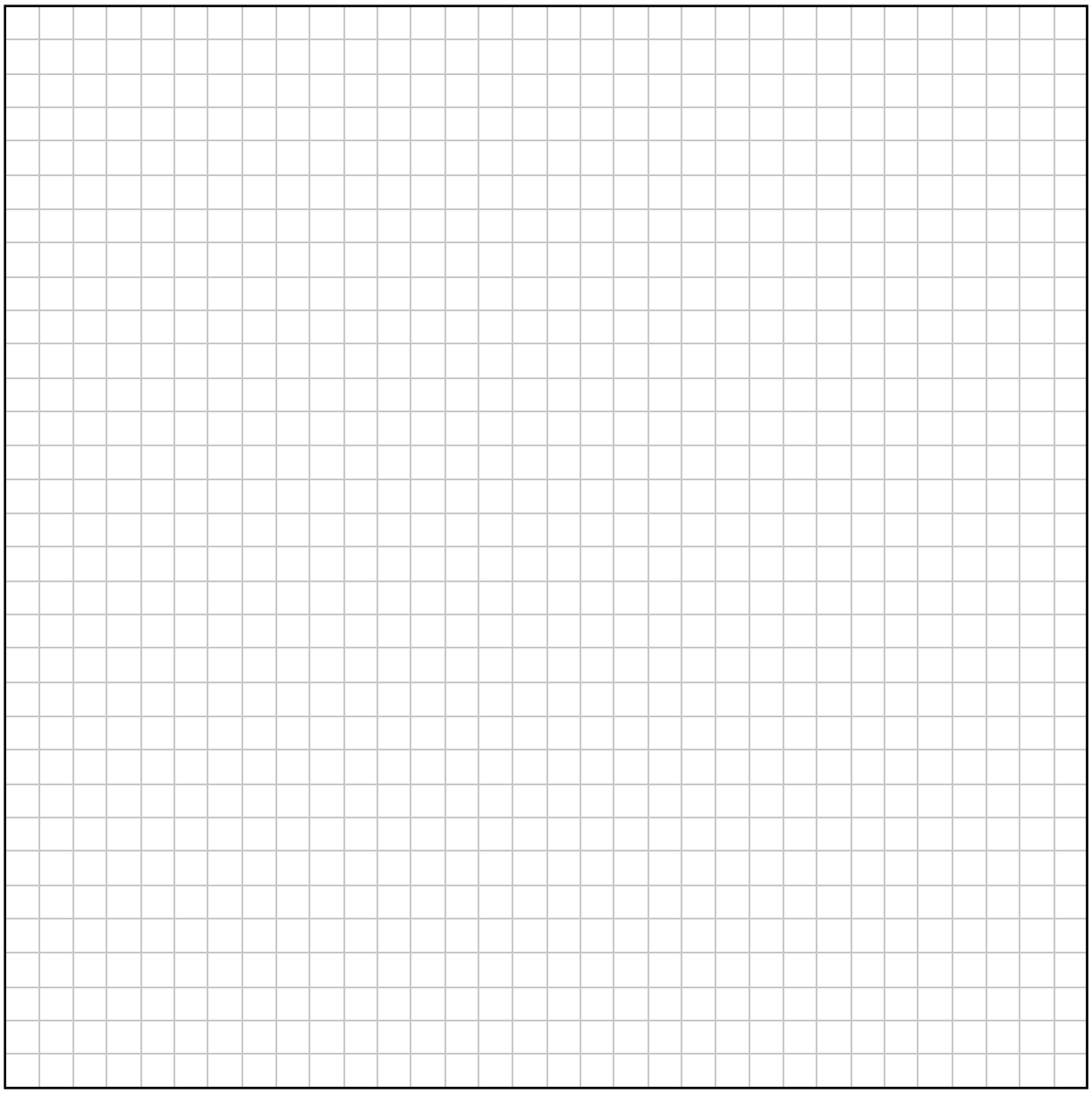


(b) In the diagram the large circle s has centre O and the small circle c has centre D.
The circle c touches the circle s at the point C.
OA and OB are tangents to c as shown.
The radius of c is r.
$|\angle BOA| = 60°$.
The ratio of the area of s to the area of c is $k : 1$.
Find the value of k.



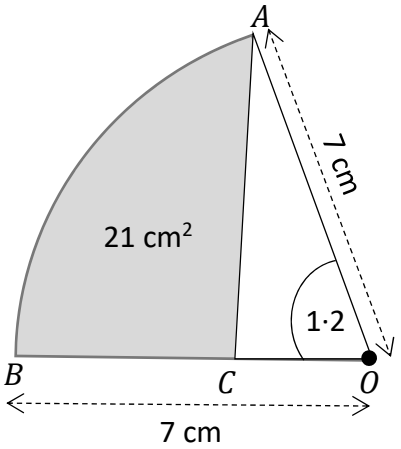
Question 4 **(25 marks)**

(a) Find the two values of θ for which $\tan\frac{\theta}{2} = -\frac{1}{\sqrt{3}}$, where $0 \leq \theta \leq 4\pi$.



(b) The diagram shows OAB, a sector of a circle of radius 7 cm with centre O.
In the sector, $|\angle BOA| = 1{\cdot}2$ radians.
The area of the shaded region is 21 cm^2.

Find $|BC|$.
Give your answer correct to 1 decimal place.



Question 5 **(25 marks)**

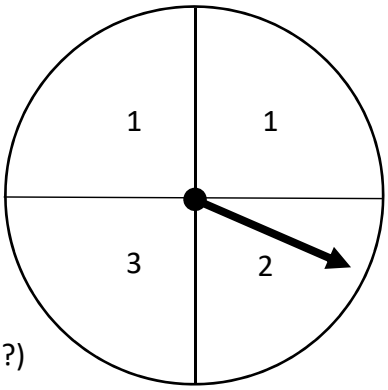
(a) Two events A and B are such that $P(A) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{2}$.

(i) Find $P(B|A)$. Give your answer as a fraction in its simplest form.

(ii) $P(A \cup B) = \frac{11}{12}$. Investigate if the events A and B are independent.

Conclusion:

(b) A spinner consists of 4 segments, as shown.
Each segment is equally likely to be landed on.
Liam, Sorcha and Lee play a game in which the spinner is spun twice and the numbers landed on are added together.
The result is divided by 3 and the remainder is recorded.



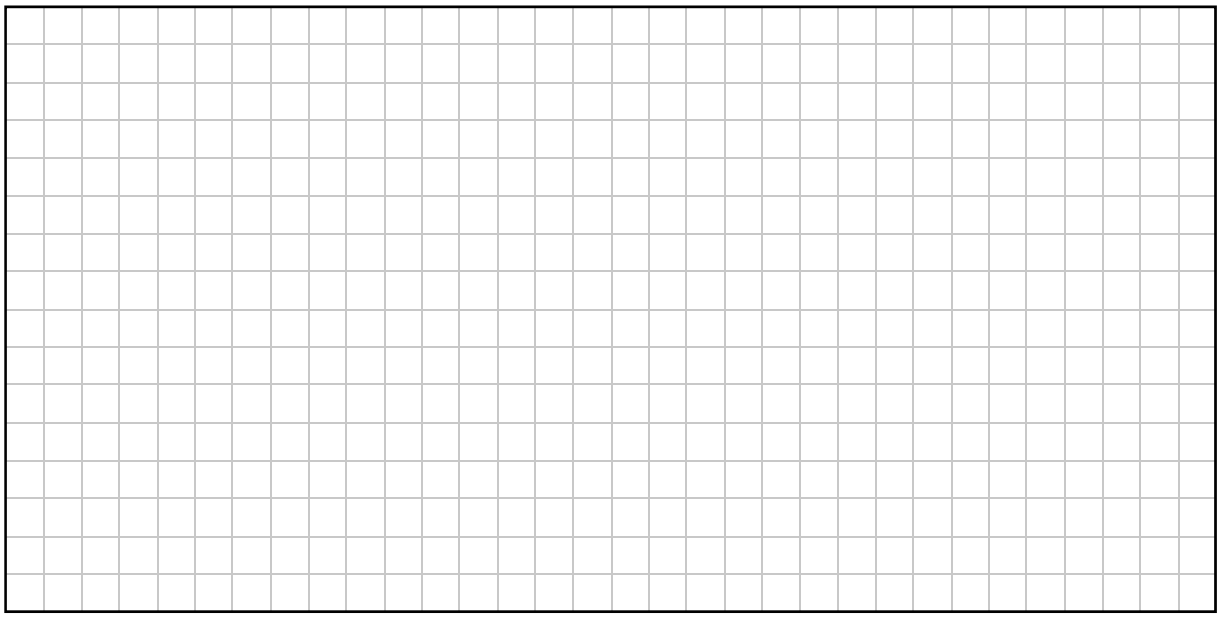
If the remainder is 0 then Liam wins the game.
If the remainder is 1 then Sorcha wins the game.
If the remainder is 2 then Lee wins the game.

Is this a fair game? (i.e. Are all 3 participants equally likely to win?)
Justify your answer by relevant calculations.

Fair Game (Yes / No):

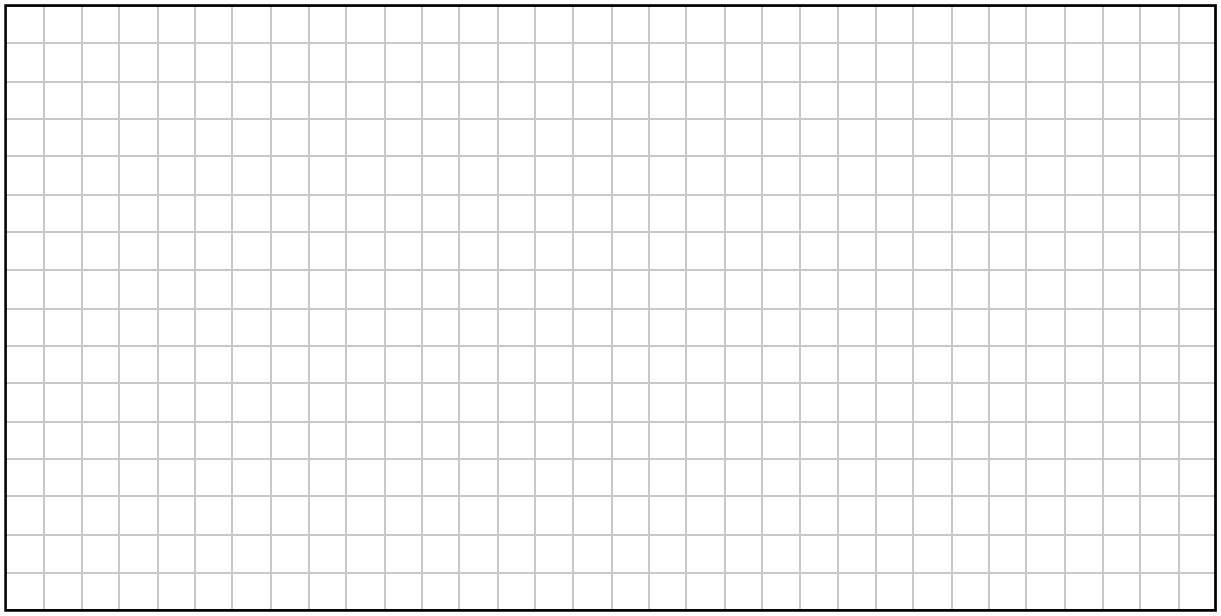
Question 6 **(25 marks)**

(a) A class group carried out a study of the makes and fuel types of cars in a large carpark.
It found that 30% of the cars ran on diesel and 70% of these diesel cars were Volkswagen.
It found that 60% of the cars ran on petrol and 25% of these petrol cars were Volkswagen.
It found that 10% of the cars were hybrid/electric and 9% of these cars were Volkswagen.
One car is selected at random from the car park.
Find the probability that it is a Volkswagen car.



(b) The Road Safety Authority has data on driving test pass rates at all its test centres.

(i) In a particular Driving Test Centre the probability that a person taking the test for the first time will pass is $\frac{1}{4}$. All of the test results are independent.
In this centre on a particular day Joe, along with 5 others, takes the test.
All six are taking the test for the first time.
Find the probability that Joe passes the test along with exactly 2 others.



(ii) The overall pass rate for all drivers at another centre is $\frac{1}{2}$ (Whether it is their first attempt or a subsequent attempt).
On a particular day, n people take the test in this centre.
The probability that two people or less than two people pass the test can be written in the form

$$\frac{an^2 + bn + c}{2^{n+1}}$$

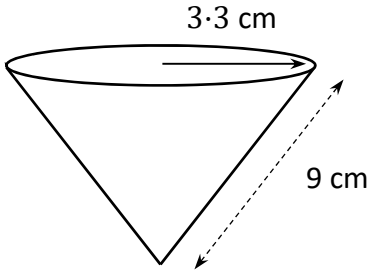
where $a, b, c \in \mathbb{N}$.
Find the value of a, the value of b, and the value of c.

$a =$ $b =$ $c =$

Section B — Contexts and applications — 150 marks

Question 7 **(55 marks)**

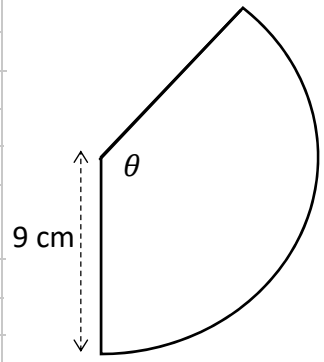
(a) A company makes biodegradable paper cups in the shape of a right circular cone. Each cup has a radius of $3{\cdot}3$ cm and a slant height of 9 cm, as shown.



(i) Show that the vertical height of the cup is 8·37 cm, correct to 2 decimal places.

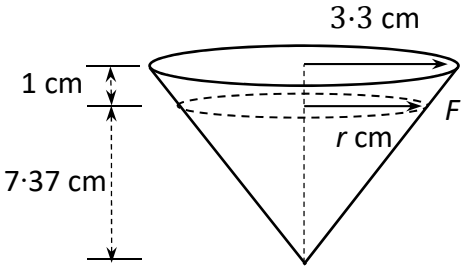
(ii) Find the curved surface area of the cup correct to 2 decimal places.

(iii) The diagram shows the net of the cup. Find, in degrees, the size of the angle θ.

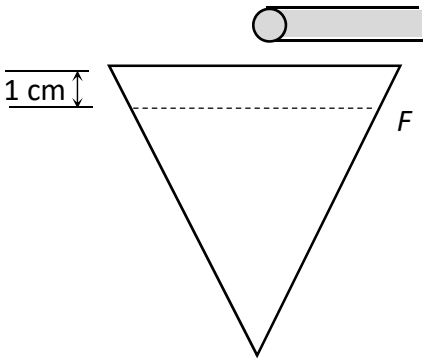


(b) In order to avoid spillages each cup is marked with a dotted line at *F* which is 1 cm vertically below the top of the cup, as shown.

Find the volume of water in the cup when it is filled as far as the dotted line.
Give your answer correct to 1 decimal place.

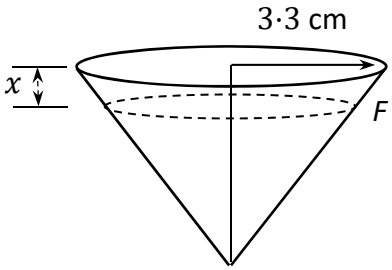


(c) Water flows into one of these cups through a cylindrical pipe of radius $0{\cdot}8$ cm at a flow rate of 2·5 cm/sec. Find, to the nearest second, how long it will take to fill the cup to the line at *F*.



This question continues on the next page.

(d) The company decides to change the position of the line *F* in order to limit the capacity of the cup to 60 cm^3.
How far, vertically below the rim of the cup, should the line *F* be drawn?
Give your answer, in cm, correct to 1 decimal place.



Question 8 **(70 marks)**

(a) An airline company *Trans-sky Airways* has designed an aptitude test for people applying for jobs as trainee pilots. The aptitude test is scored out of 500 marks. The results are normally distributed with a mean score of 280 and a standard deviation of 90.

(i) The top 25% of people taking the aptitude test are invited back for an interview.
Find the minimum mark needed on the test in order to be invited back for interview.

(ii) Anyone who scores above the 40^{th} percentile can re-sit the test later.
Eileen scored 260 marks in the test.
Find out whether or not Eileen is eligible to re-sit the test.

This question continues on the next page.

(b) **(i)** Explain the relevance of the z-scores $-1{\cdot}96$ and $1{\cdot}96$ in the standard normal distribution.

(ii) *Trans-sky Airways* surveyed 2500 of its passengers about a new service it proposed to introduce. The variable $\hat{p}$ is the proportion of respondents in the survey who said they would use the new service.
The radius of the 95% confidence interval of the survey was 0·01568.
Find the value of $\hat{p}$, where $0{\cdot}5 < \hat{p} \leq 1$.

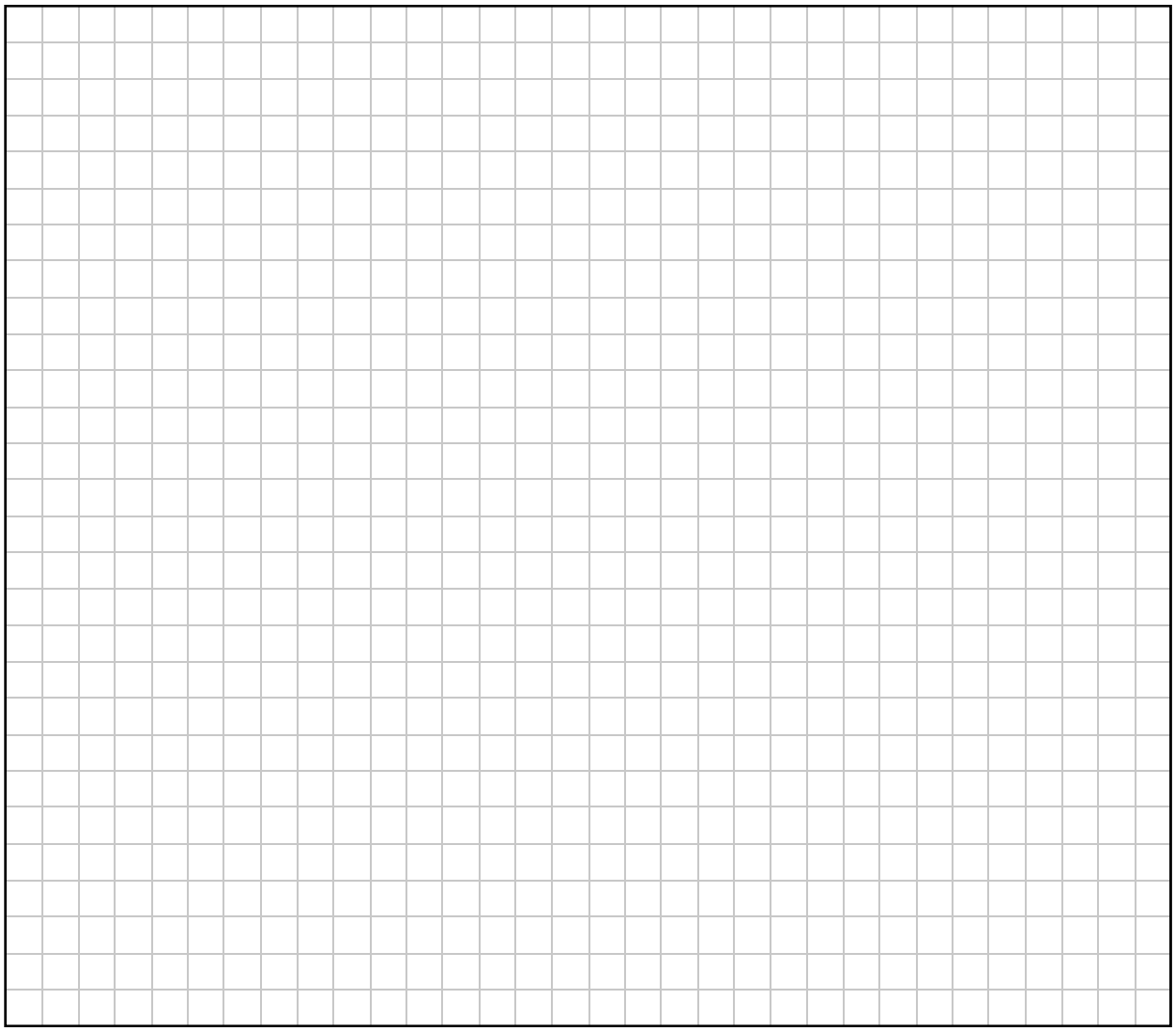
(c) The weight of the Airline passengers' carry-on luggage is normally distributed with a mean of 12 kg. The Airline has recently introduced a fee for non-carry-on luggage. After the fee was introduced, the Airline expected the mean weight of the carry-on luggage to change. They selected a random sample of 80 passengers and weighed their carry-on luggage. The sample mean was 13·1 kg and the sample standard deviation was 4·5 kg.

Test the hypothesis, at the 5% level of significance, that the mean weight of the carry-on luggage has changed. State the null hypothesis and the alternative hypothesis. Give your conclusion in the context of the question.

Conclusion:

This question continues on the next page.

(d) The company bus can carry passengers up to a total maximum weight allowance of 3000 kg. The weight of passengers is normally distributed with a mean of 73 kg and a standard deviation of 12 kg.
40 passengers board the bus.
Find the probability that the total passenger weight will be over the maximum weight allowance.
Give your answer as a percentage correct to 2 decimal places.



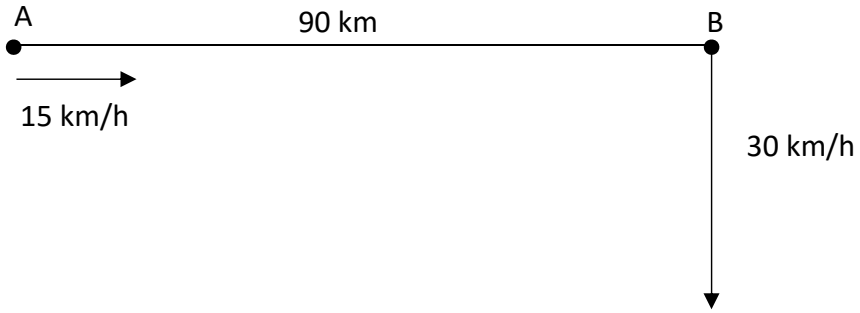
(e) A list consists of eight whole numbers. They are labelled from A to H as shown below.
The numbers are all greater than zero and are ordered from smallest to largest.
The difference between any two adjacent numbers is 2 or more.
The median of the list is 12·5.
The lower quartile (the median of the 4 lowest numbers) of the list is 7·5.
The interquartile range is 12.
The second largest number is 23, as shown.
The range of the list is 21.
The mean of the list is 13·5.

Find the numbers which satisfy all of the above conditions and write them into the boxes below.

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
						23	

Question 9 **(25 marks)**

Two ships set sail at the same time, Ship A from Port A and Ship B from Port B.
Port A is 90 km due west of Port B, as shown below.
Ship A is traveling due east at a speed of 15 km/h.
Ship B is travelling due south at a speed of 30 km/h.



(a) Find the distance between the two ships 30 minutes after they set sail.
Give your answer in km, correct to 2 decimal places.

(b) t is the time in hours after the ships set sail.
Show that the distance between the ships at time t can be given by the function

$$s(t) = (1125t^2 - 2700t + 8100)^{\frac{1}{2}},$$

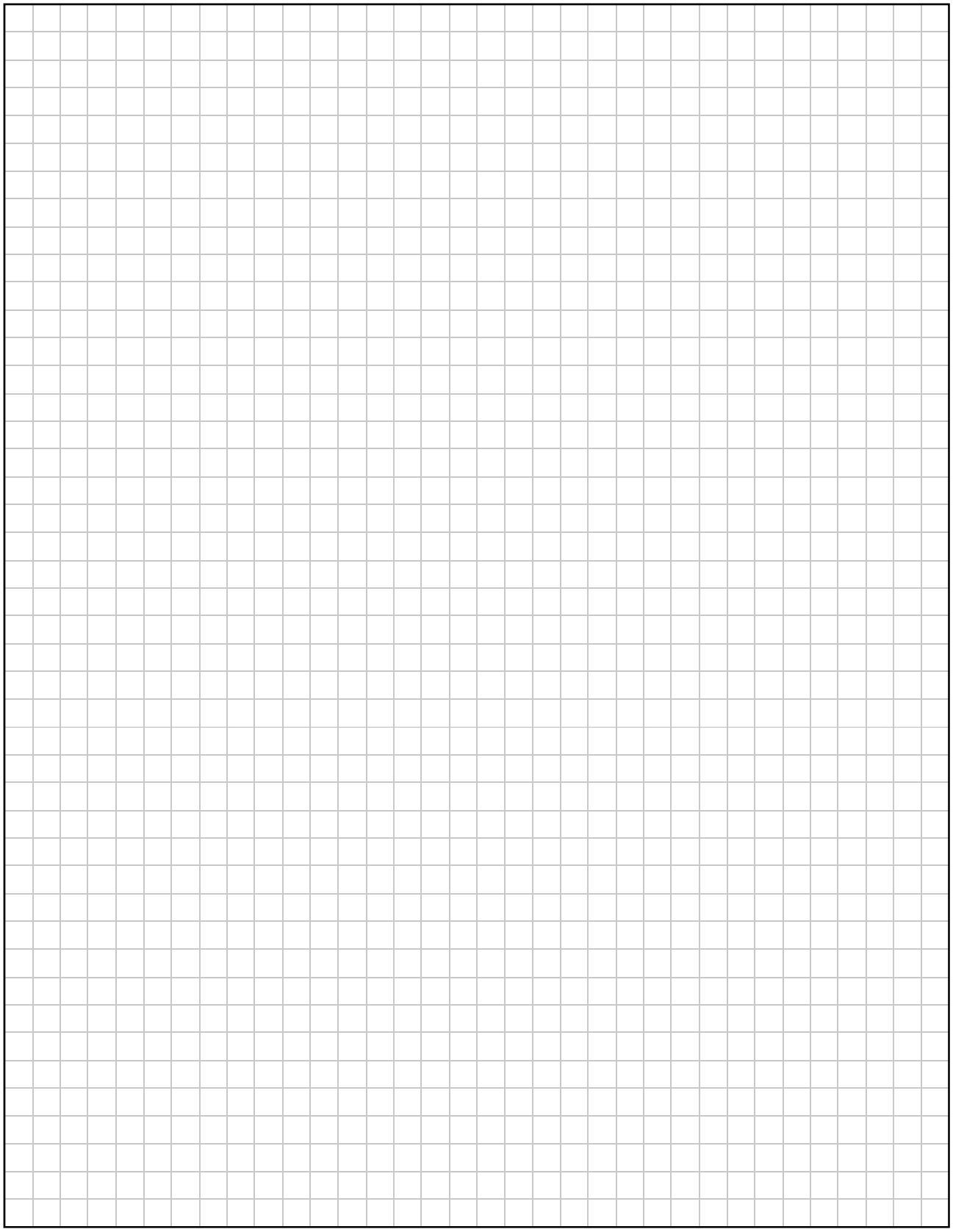
where $0 \leq t \leq 6$.

(c) Use calculus to find the value of t when the ships are closest to each other, **and** find the distance between the ships at your value of t.
Give the distance in km, correct to 1 decimal place.

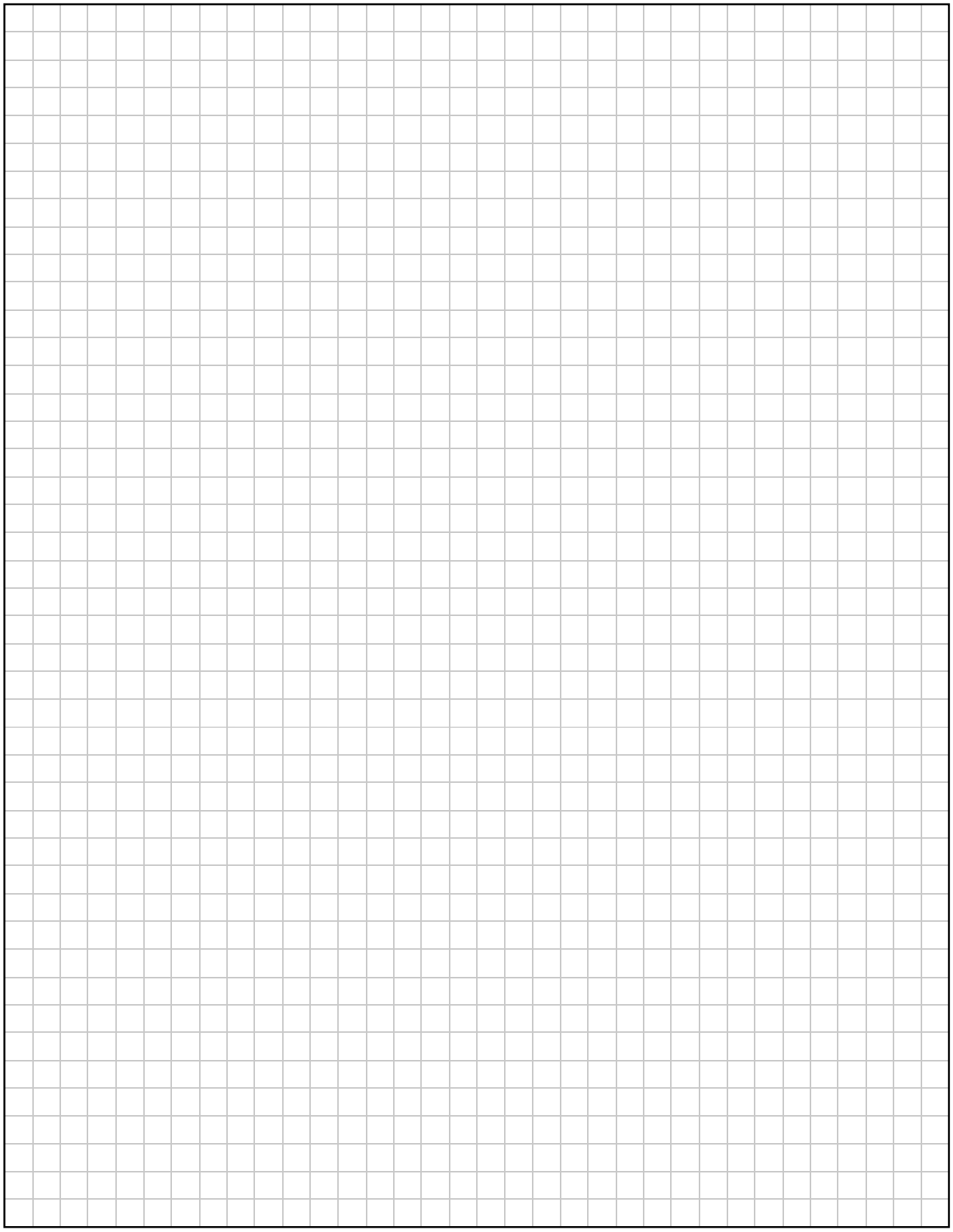
t:

Distance:

Page for extra work.
Label any extra work clearly with the question number and part.



Page for extra work.
Label any extra work clearly with the question number and part.



Do not write on this page

Leaving Certificate – Higher Level

Mathematics Paper 2

2 hours 30 minutes