2020.M27

2020L003G1EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2020

Mathematics

Paper 1

Ordinary Level

2 hours 30 minutes

300 marks

Examination Number

Day and Month of Birth

For example, 3rd February is entered as 0302

Centre Stamp

Do not write on this page

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) John works as part of a sales team. He earns a basic rate of €12·60 per hour.
In addition to his hourly pay, he earns a commission of 22% on any sales he makes above €200 each week.

(i) During a particular week, John worked 45 hours at the basic rate and made sales amounting to €350. Find John's gross pay for this week.

(ii) During the following week John worked 51 hours. This included 3 hours on Sunday.
If John works on a Sunday he receives 1·5 times the basic rate for those hours.
His gross pay for that week was €713·20.
Find the amount of sales John made in that week.

(b) John pays tax at the Standard Rate of 20% and at the Higher Rate of 40%.
He has a weekly Tax Credit of €26.
The weekly Standard Rate Cut-off Point is €678.
Find John's net income for the week where his salary was €713·20.

Question 2 **(25 marks)**

(a) Solve the equation:

$$\frac{9x-6}{2}=\frac{3x-14}{3}+\frac{9x}{4}.$$

(b) Solve the simultaneous equations:

$$3x - y = 4$$
$$4x^2 - 3xy = 4.$$

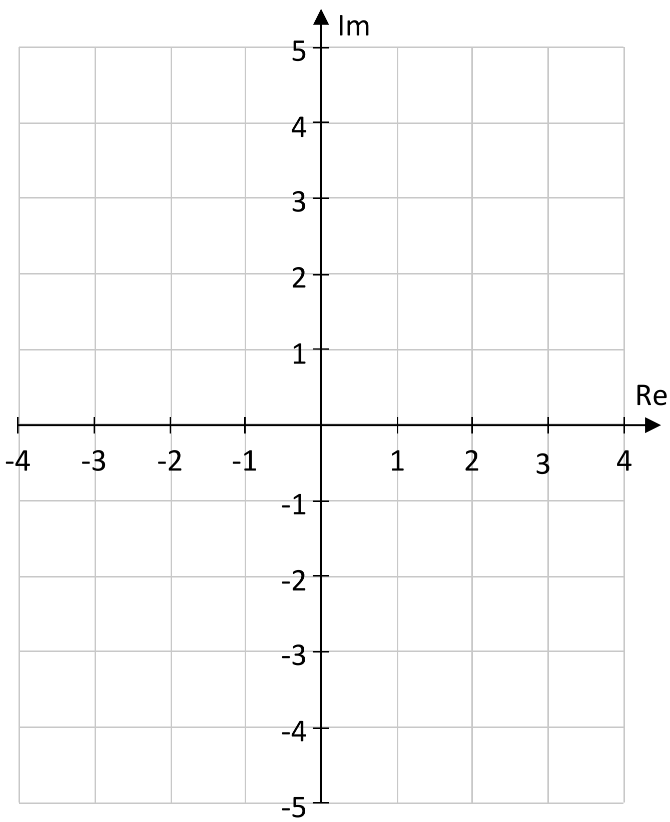
Question 3 **(25 marks)**

$z_1 = 3 - 4i$, $z_2 = -2 + i$ and $z_3 = 2iz_2$, where $i^2 = -1$.

(a) **(i)** Write z_3 in the form $a + bi$, where $a, b \in \mathbb{Z}$.

$z_3 = 2iz_2 =$

(ii) Plot z_1, z_2 and z_3 on the given Argand Diagram.
Label each point clearly.

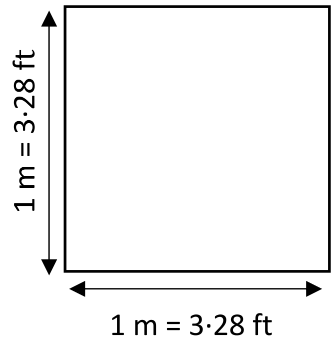


(iii) Find $|z_1|$.

(b) If $z_1 \times z_4 = 29 + 3i$, write z_4 in the form $a + bi$, where $a, b \in \mathbb{R}$.

Question 4 **(25 marks)**

(a) The Golden Gate Bridge in San Francisco is constantly being repainted. It is estimated that the surface area of exposed steel that needs to be painted is approximately 10 million square feet.



(i) Given that 1 metre is equal to 3·28 feet, convert the surface area of the bridge into square metres, giving your answer in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{N}$. Give the value of a correct to two significant figures.

(ii) A litre of the paint used on the Golden Gate Bridge will cover approximately 5 square metres. This paint comes in 25 litre tins. Find the minimum number of tins of paint that will be needed to paint the entire bridge.

(b) Solve the equation $2^{9x-1} = 8^{2x}$.

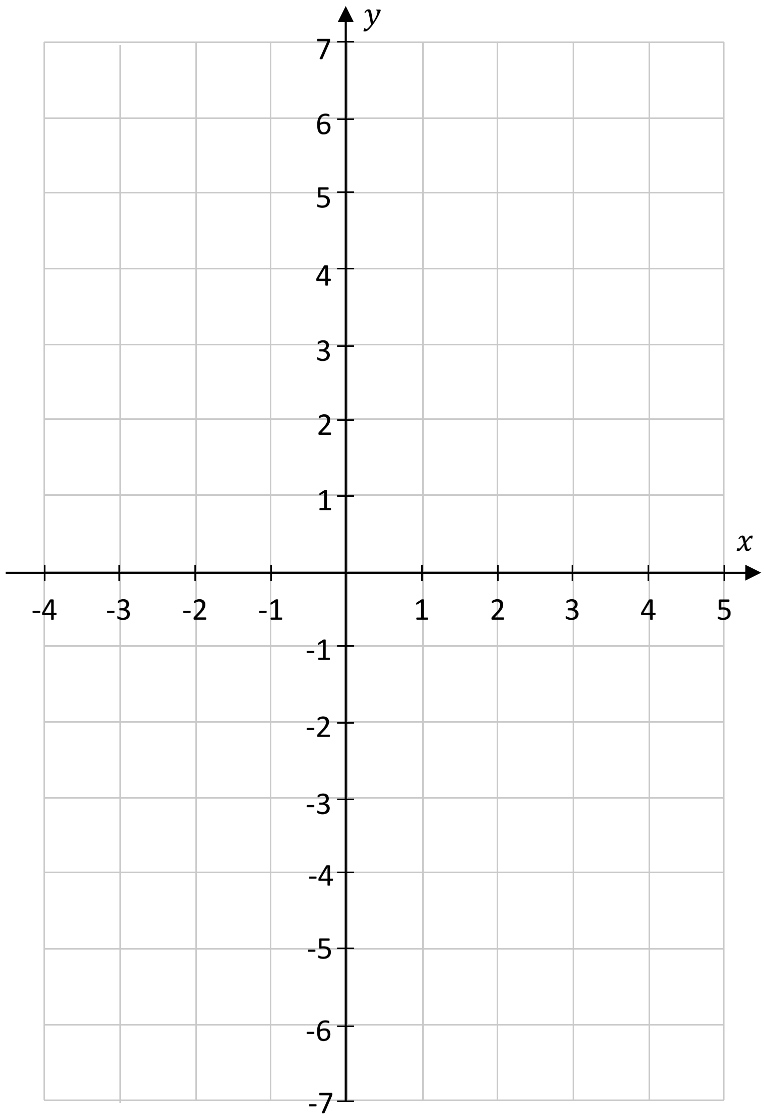
Question 5 **(25 marks)**

(a) Solve the equation $x^2 - 3x - 4 = 0$.

(b) **(i)** Complete the table below to show the value of the function $f(x) = -x^2 - x + 6$ for each of the given values of x.

x	-4	-3	-2	-1	0	1	2
$f(x)$			4				

(ii) Hence draw the graph of f in the domain $-4 \leq x \leq 2$, where $x \in \mathbb{R}$, on the diagram below.



(iii) On the same diagram as the function f, draw the graph of the function $g(x) = f(x - 2)$, in the domain $-2 \leq x \leq 4$, where $x \in \mathbb{R}$.

Label the graphs of $f(x)$ and $g(x)$ clearly.

Question 6 **(25 marks)**

(a) **(i)** Differentiate the function $f(x) = 4x^3 - 3x^2 + x - 7$, where $x \in \mathbb{R}$, with respect to x.

(ii) Find the slope of the tangent to the graph of $f(x) = 4x^3 - 3x^2 + x - 7$ at the point $(1, -5)$.

(iii) Hence find the equation of the tangent to the graph at this point.

(b) The function $g(x) = 2x^2 + px + q$, where $p, q \in \mathbb{Z}$, and $x \in \mathbb{R}$.
Given that $g(2) = 6$ and $g'(3) = 9$, find the value of p and the value of q.

Note: $g'(3)$ is the value of the derivative of $g(x)$ at $x = 3$.

$p =$ $q =$

Section B Contexts and applications 150 marks

Answer **all three** questions from this section.

Question 7 **(35 marks)**

(a) Pat buys a new car for €32 000.
He trades in his old car and is given an allowance of €20 000 by the garage.
He borrows the balance of the money from the credit union.
His fixed monthly repayment over three years is €443·66 per month.

(i) How much money does Pat pay in total to the credit union for the loan?

(ii) Show the amount that Pat repays as a percentage of the amount that he borrows from the credit union is 133·1%, correct to one decimal place.

(b) A sum of money is invested at r% per annum compound interest for 3 years.
At the end of the 3 years the value of the investment has increased by 33·1%.
Find the value of r.

(c) (i) It is estimated that the value of cars depreciates at a compound rate of 20% per year. Use this percentage to find the value of Pat's car after three years (original price €32 000).

(ii) Pat's friend Caitlín bought a new car three years ago.
Its value also depreciated by 20% per year.
It is now worth €17 920.
Find the original value of the car.

Question 8 **(55 marks)**

A swimmer is on a starting block at the beginning of a race. When she dives off the block until she resurfaces, the level of the swimmer relative to the level of the water is given by the function:

$$h(x) = \frac{1}{60}x^2 - \frac{1}{4}x + \frac{3}{5}.$$

In the function, x is the horizontal distance in metres of the swimmer from the block, $0 \leq x \leq 12$, where $x \in \mathbb{R}$ and $h(x)$ is measured in metres.

(a) Find the height of the block above the water.

(b) **(i)** Show that the swimmer is on the surface of the water (i.e. $h(x) = 0$) when she is 12 metres from the starting block.

(ii) Find the horizontal distance, in metres, from the starting block to the point where the swimmer enters the water.

(c) **(i)** Find $h'(x)$, the derivative of $h(x) = \frac{1}{60}x^2 - \frac{1}{4}x + \frac{3}{5}$.

(ii) Use your answer to **Part (c)(i)** to find the horizontal distance (x), in metres, from the starting block to the point at which the swimmer reaches her greatest depth.

(iii) **Hence** find this greatest depth.

This question continues on the next page.

(d) In the 2016 Summer Olympics, Michael Phelps, won the 200 m Butterfly final in a time of 1 minute and $53{\cdot}36$ seconds ($1{:}53{\cdot}36$). The time it took to swim each of the four 50 m sections of the race (split time) is given in the table below. The percentage increase in the time taken to swim one section is also given. Thus, it took the swimmer $14{\cdot}7\%$ longer to swim the second 50 metres compared to the first 50 metres.

Race Section	**Split Time (secs)**	**Percentage Increase**
0 – 50 m	24·85	
50 – 100 m	28·5	$14{\cdot}7\%$
100 – 150 m	29·33	
150 – 200 m	30·68	
Total	**1:53·36**	

(i) Complete the table by finding the percentage increase in split time by comparing each split time with the previous one.
Give your answers correct to one decimal place.

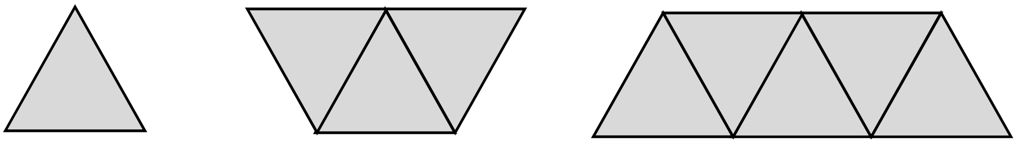
(ii) Another swimmer in the race completed the first 50 m in a time of 25·01 seconds. His subsequent 50 m split times increased at the same rates as each of Michael Phelps' times. Using the table below, or otherwise, find the difference between his finishing time and that of Michael Phelps.
Give your answers in seconds correct to 2 decimal places.

Race Section	**Split Time (secs)**	**Percentage Increase**
0 – 50 m	25·01	
50 – 100 m		14·7%
100 – 150 m		
150 – 200 m		
Total		

Difference:

Question 9 **(60 marks)**

The following sequence of patterns is created using matchsticks to form equilateral triangles.



Pattern 1 Pattern 2 Pattern 3

(a) Complete the table below to show the number of matchsticks required to make each of the first six patterns of the above sequence.

Pattern Number	1	2	3	4	5	6
Number of Matchsticks	3	7				

(b) **(i)** How many matchsticks are required to make pattern 10 of the sequence?

(ii) Find a formula for T_n, the number of matchsticks required to make pattern n of the sequence.

(iii) Pattern k has 147 matchsticks, where $k \in \mathbb{N}$. Find the value of k.

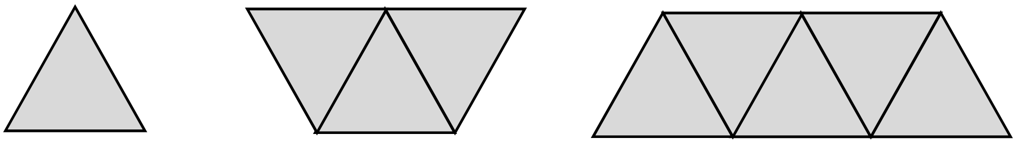
(c) **(i)** Find a formula for S_n, the **total** number of matchsticks required to make the first n patterns.

(ii) Find the total number of complete patterns in the sequence that can be made using 820 matchsticks.

This question continues on the next page.

(d) **(i)** The table below shows the number of triangles formed by each pattern for the first two patterns. Complete the table to show the number of triangles formed for patterns three to six.

Pattern Number	1	2	3	4	5	6
Number of Triangles	1	3				



Pattern 1 **Pattern 2** **Pattern 3**

(ii) The area of each triangle is $4\sqrt{3}$ cm^2.
Find, correct to the nearest cm^2, the **combined total area** covered by the first 15 patterns in the sequence.

Page for extra work.
Label any extra work clearly with the question number and part.

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Leaving Certificate – Ordinary Level

Mathematics Paper 1

2 hours 30 minutes