Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination

Mathematics

Paper 2

Higher Level

2 hours 30 minutes

220 marks

Examination Number

Day and Month of Birth

For example, 3rd February
is entered as 0302

Centre Stamp

Do not write on this page

Instructions

There are **two** sections in this examination paper.

Section A   Concepts and Skills   120 marks   6 questions

Section B   Contexts and Applications   100 marks   4 questions

Answer questions as follows:
- any **four** questions from Section A   –   Concepts and Skills
- any **two** questions from Section B   –   Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet.
If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

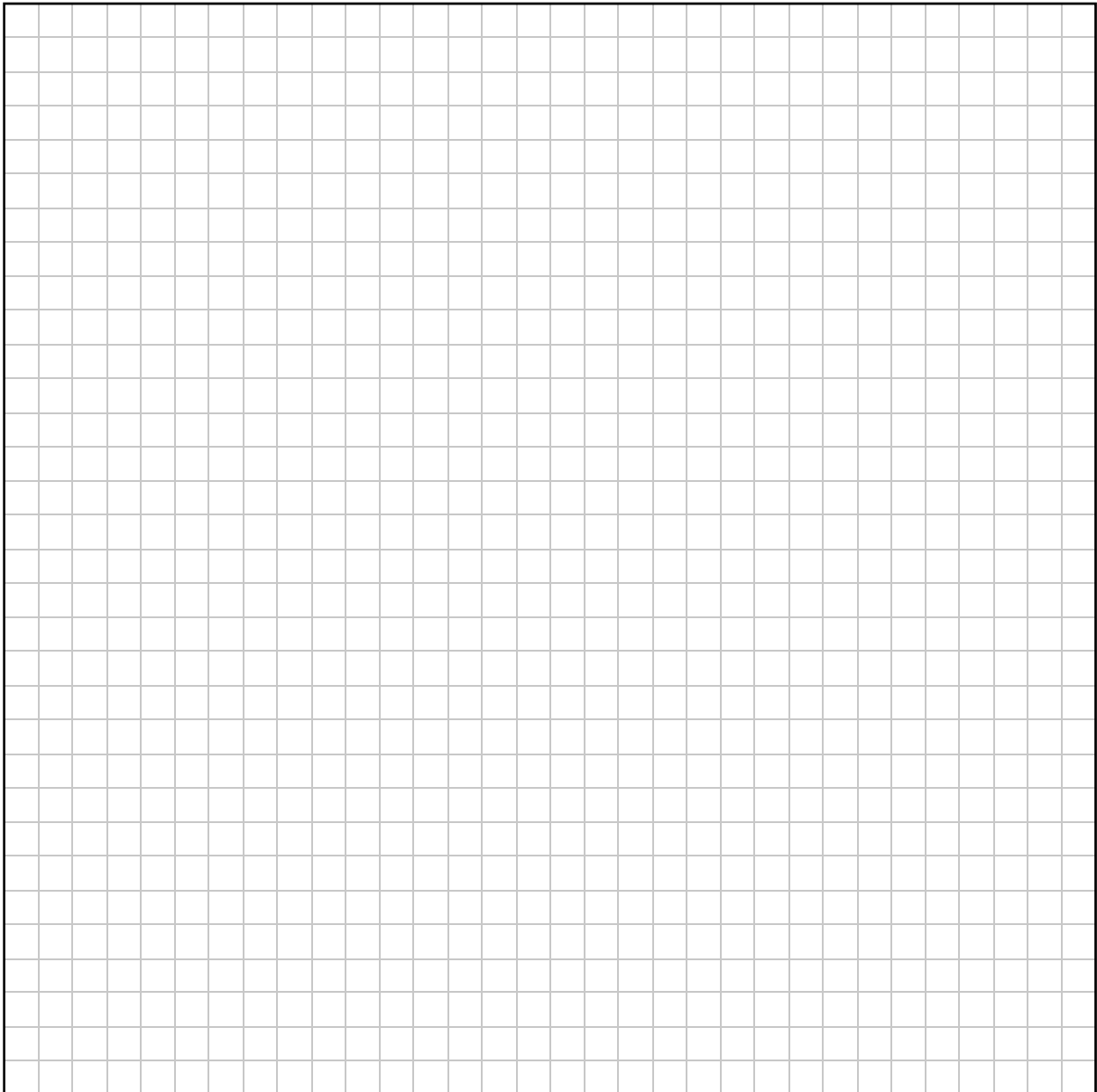Write the make and model of your calculator(s) here:

Answer **any four** questions from this section.

Question 1   **(30 marks)**

The number of matches in each of 53 boxes of matches was counted. The results are as follows:

Number of matches	41	42	43	44	45	46
Number of boxes	3	8	16	17	8	1

(a) Draw a suitable chart to show this data. Label your chart clearly.

(b) Find the mean, mode, and median number of matches per box.
Give the mean correct to 1 decimal place.

Mean =   Mode =   Median =

(c) A box is picked at random, the number of matches noted, and the box is replaced.
This process is then repeated a number of times.
Find the probability that the seventh box picked is the third box with exactly 42 matches.
Give your answer correct to 4 decimal places.

Question 2 (30 marks)

A, B, and C are three events. A and C are independent.

$$P(A) = \frac{1}{5}, \qquad P(B) = \frac{1}{6}, \qquad P(A \cap C) = \frac{1}{20}, \quad P(A \cup B) = \frac{1}{3}, \quad \text{and} \quad P(B \cup C) = \frac{3}{8}.$$

(a) **(i)** Show that $P(C) = \frac{1}{4}$.
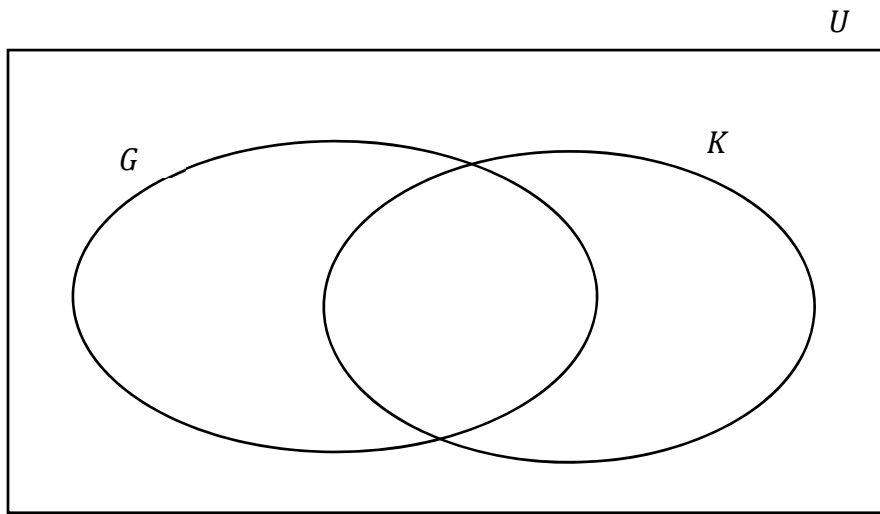
(ii) Find $P(A \cap B)$.

(iii) State whether B and C are **mutually exclusive** events, and justify your answer.

Answer:

Reason:

(b) Two events G and K are such that $P(G) = 0.56$, $P(K) = 0.48$, and $P(G|K) = 0.25$.

In the Venn diagram below, write into each of the four regions the probability that the event represented by that region occurs. Note: U is the universal set.
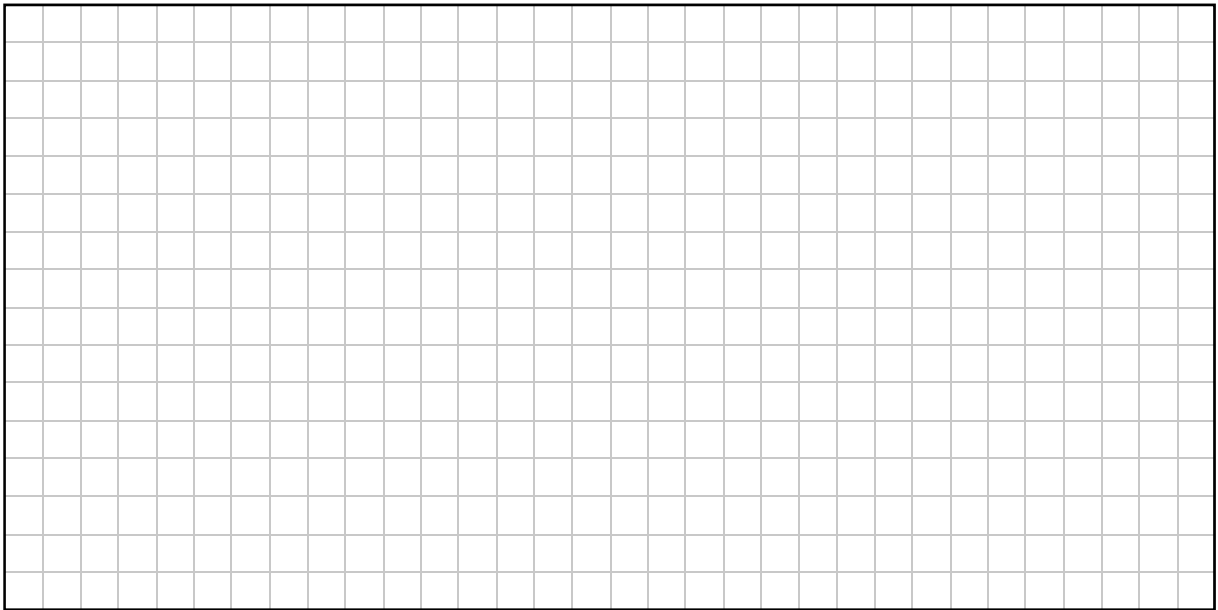
Question 3 **(30 marks)**

The line l has equation $x - 2y + 8 = 0$.

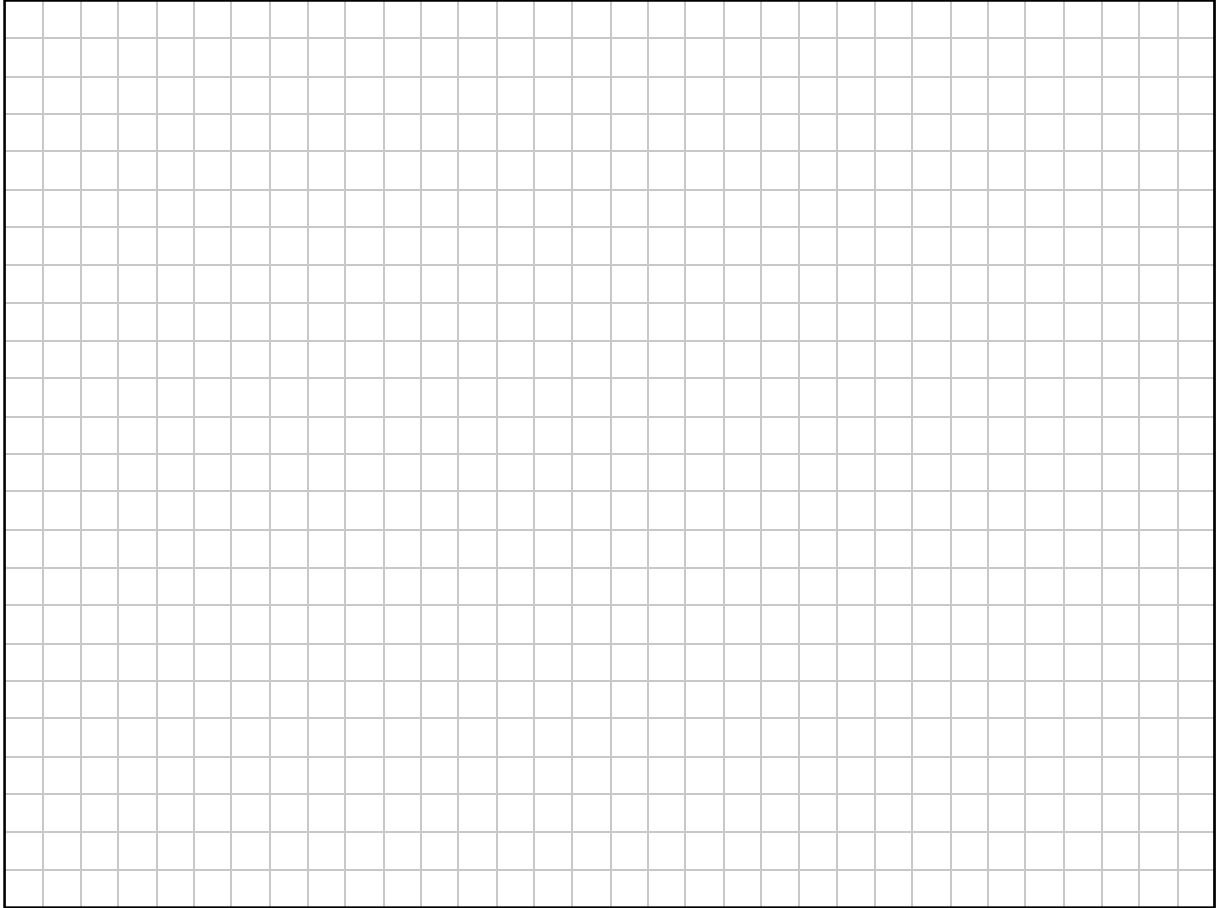The point P has co-ordinates $\left(k, \frac{k+8}{2}\right)$, where $k \in \mathbb{R}$.

(a) **(i)** Show that, for all values of k, the point P lies on l.

(ii) Another line passes through the point P and the point $A\,(-1, 1)$.
Find the slope of the line AP in terms of k.

(iii) The point B is at $(3, 3)$.
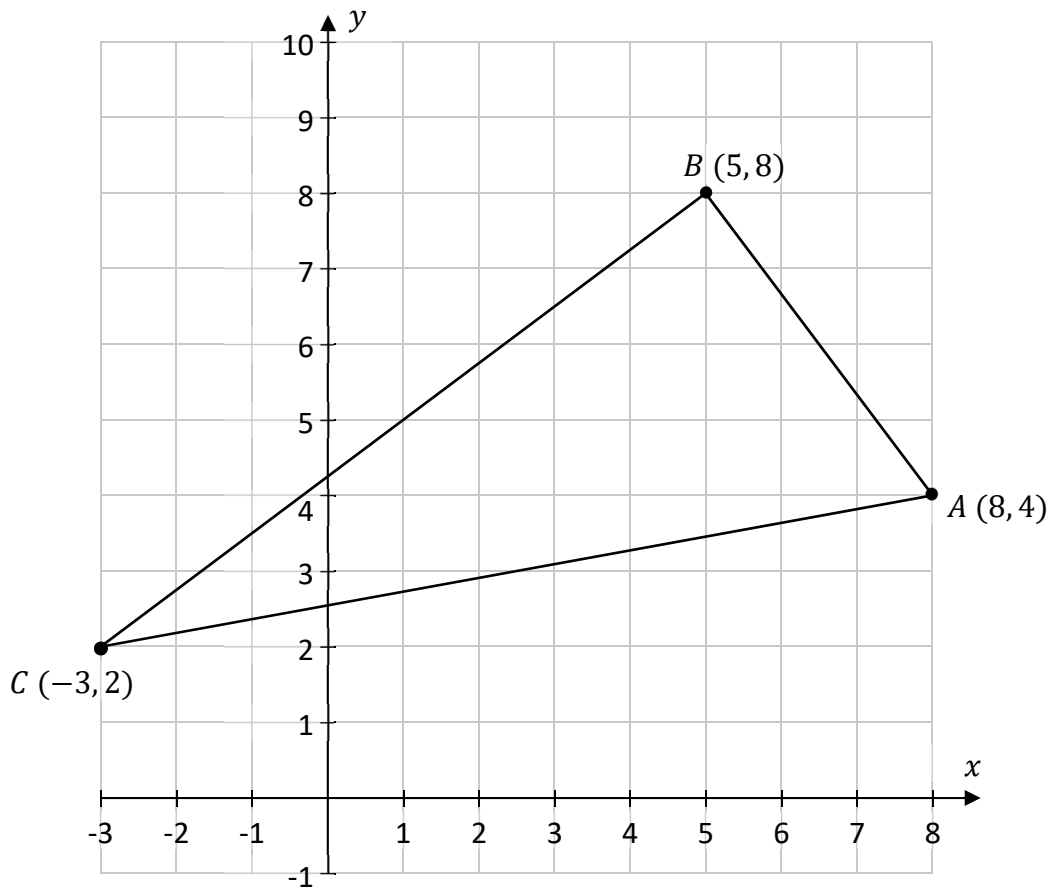Find the value of k such that $|\angle BAP| = 90°$.

(b) Find the area of the triangle with vertices $(-2, 5)$, $(4, 11)$, and $(7, -5)$.
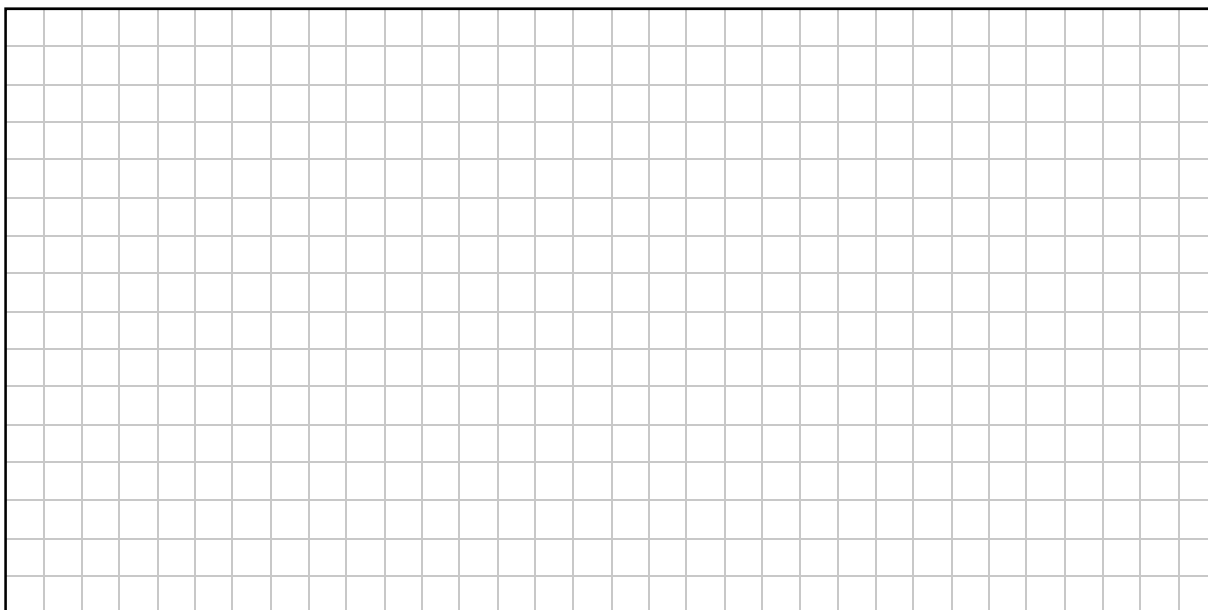
Question 4 (30 marks)

(a) The diagram below shows a triangle ABC (not to scale).
The co-ordinates of each point are given.

**(i)** Show that the circumcentre of the triangle ABC is the mid-point of $[AC]$.
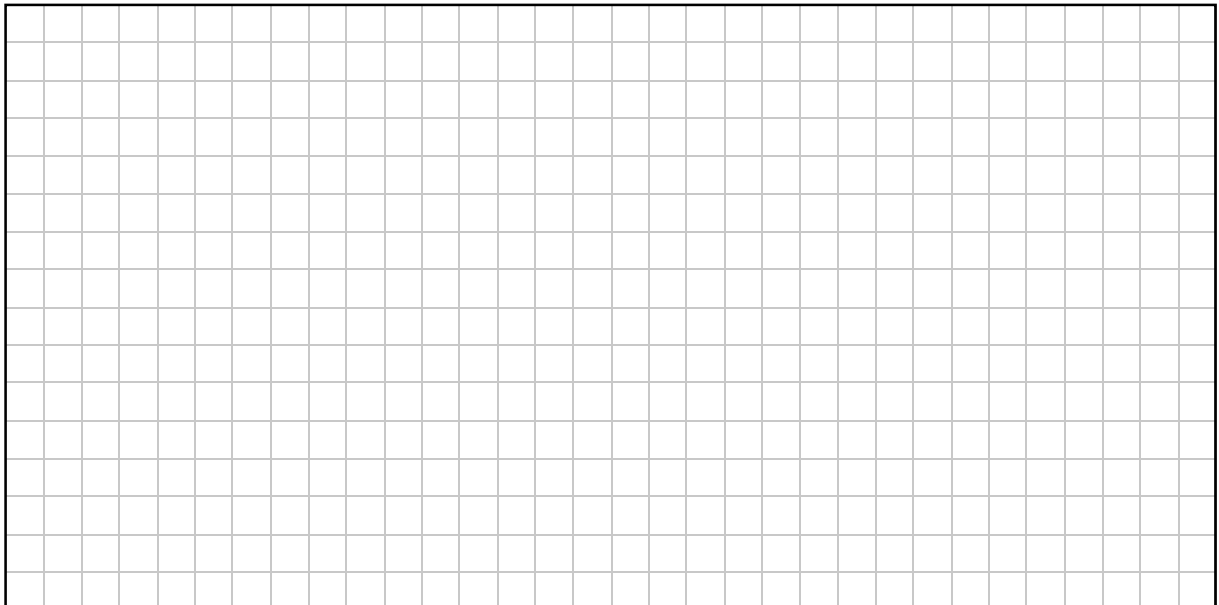
(ii) Find the equation of the circumcircle of the triangle ABC.
Give your answer in the form $x^2 + y^2 + ax + by + c = 0$, where $a, b, c \in \mathbb{R}$.

(b) A different circle, c, has a radius of 5 and a centre at $(-3, 1)$.
j is a horizontal line and k is a vertical line.
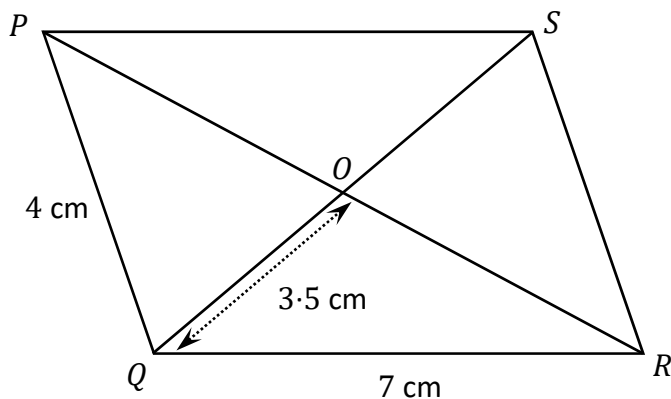The lines j and k are both tangents to the circle c.

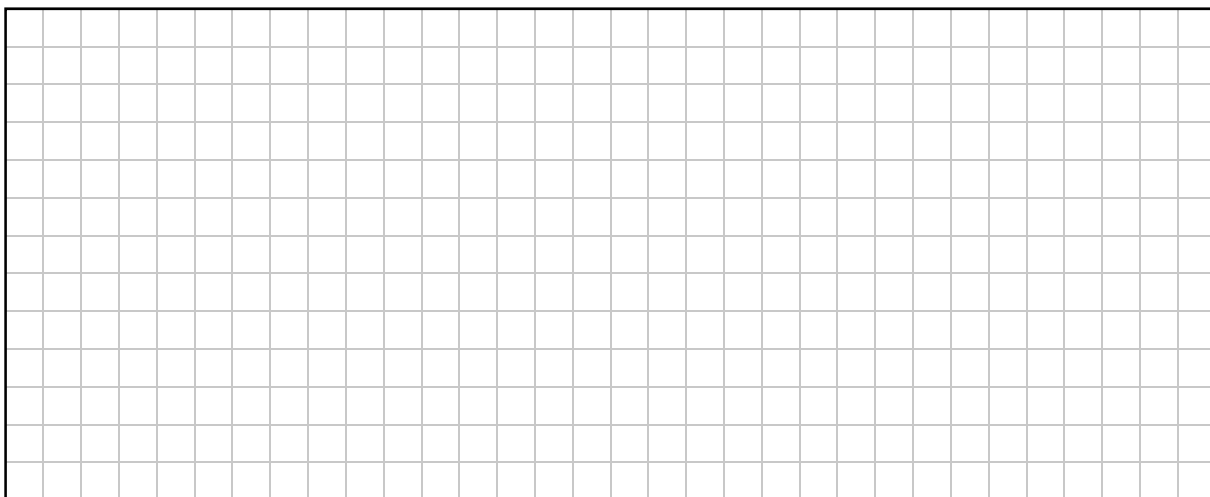Find one possible equation of the line j and one possible equation of the line k.

$j :$ _____ $k :$ _____
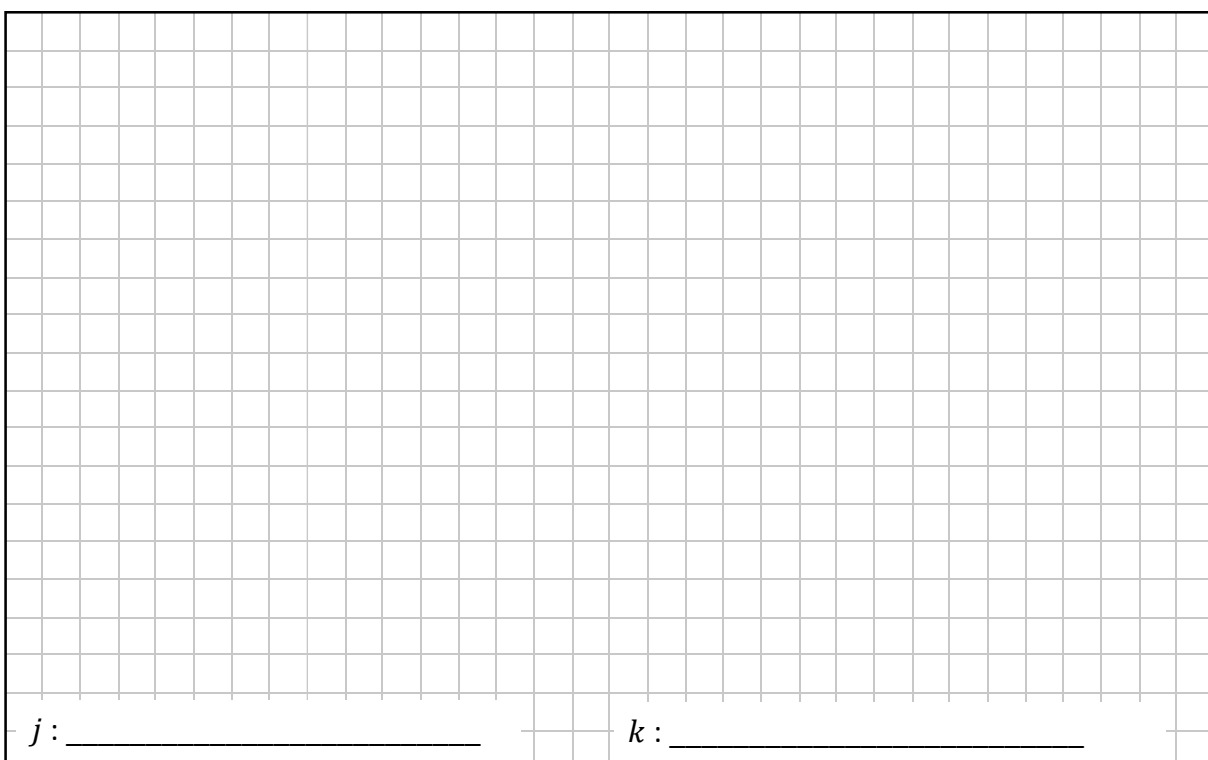
Question 5 (30 marks)
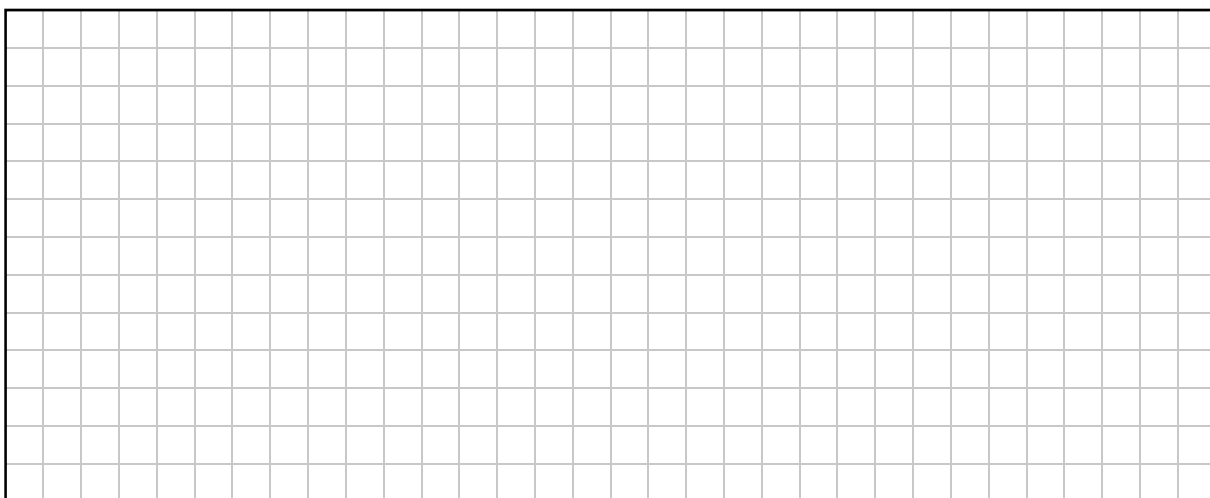
(a) $PQRS$ is a parallelogram, as shown in the diagram below (not to scale).
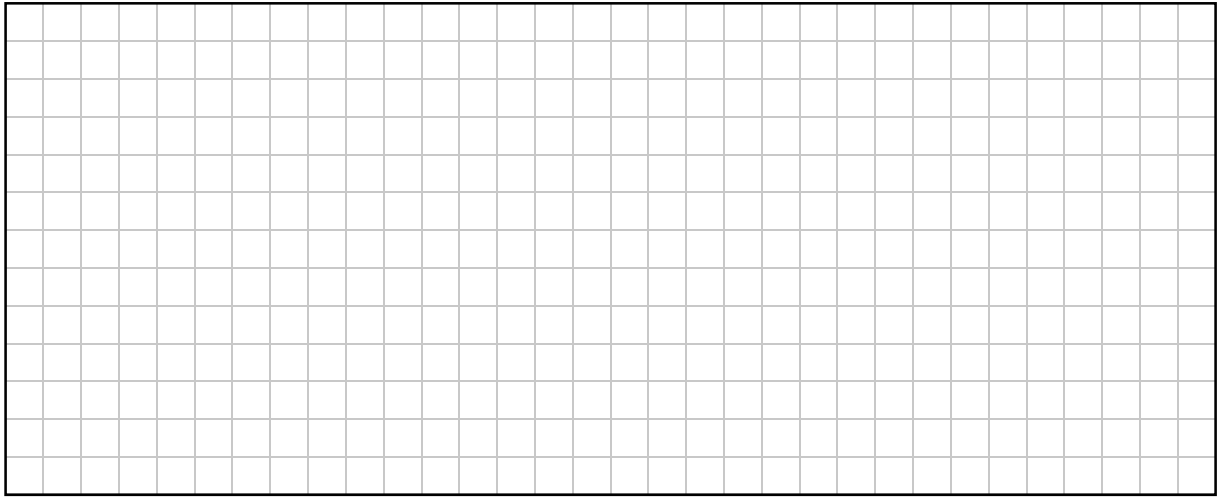PR and QS intersect at O. $|PQ| = 4$ cm, $|QR| = 7$ cm, and $|QO| = 3{\cdot}5$ cm, as shown.
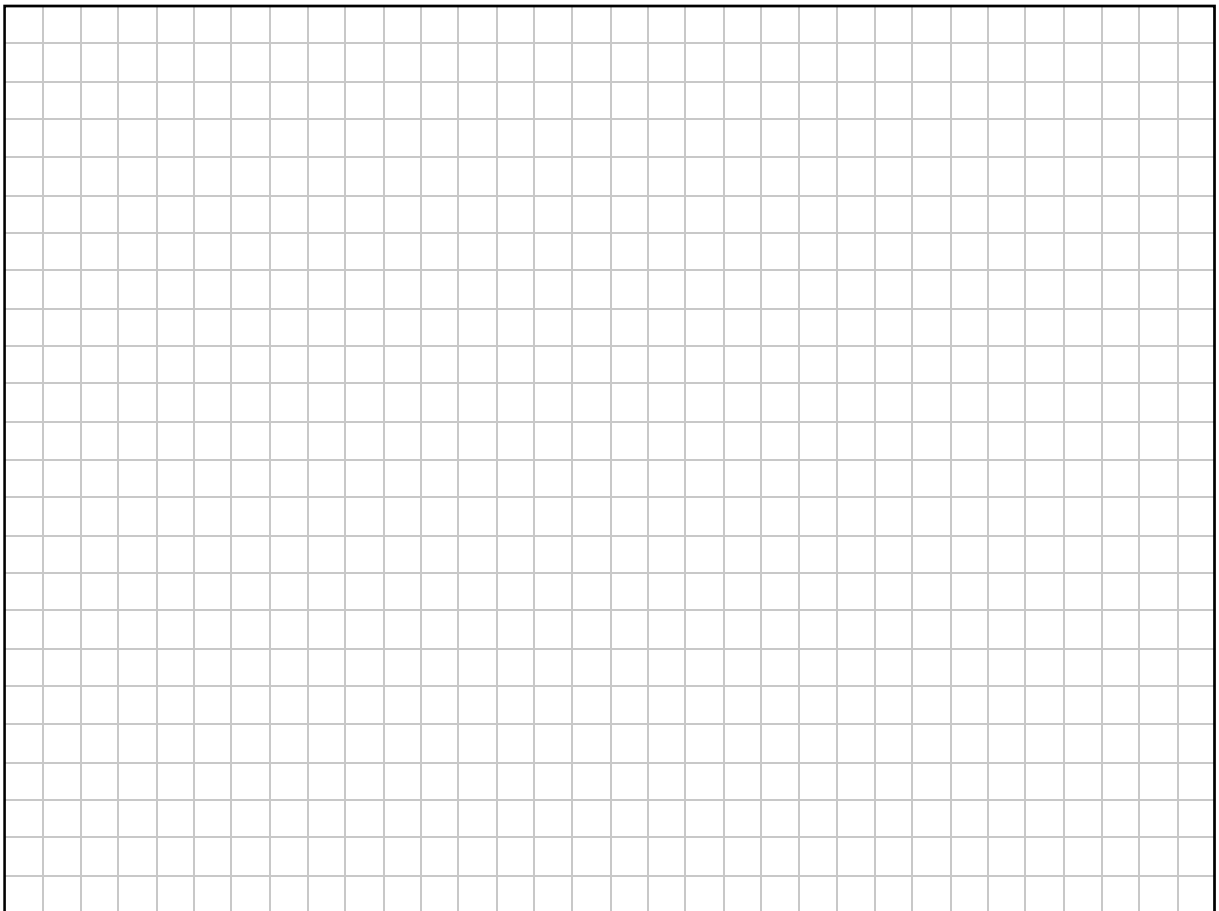

(i) Show that $\cos(\angle RSQ) = \dfrac{2}{7}$

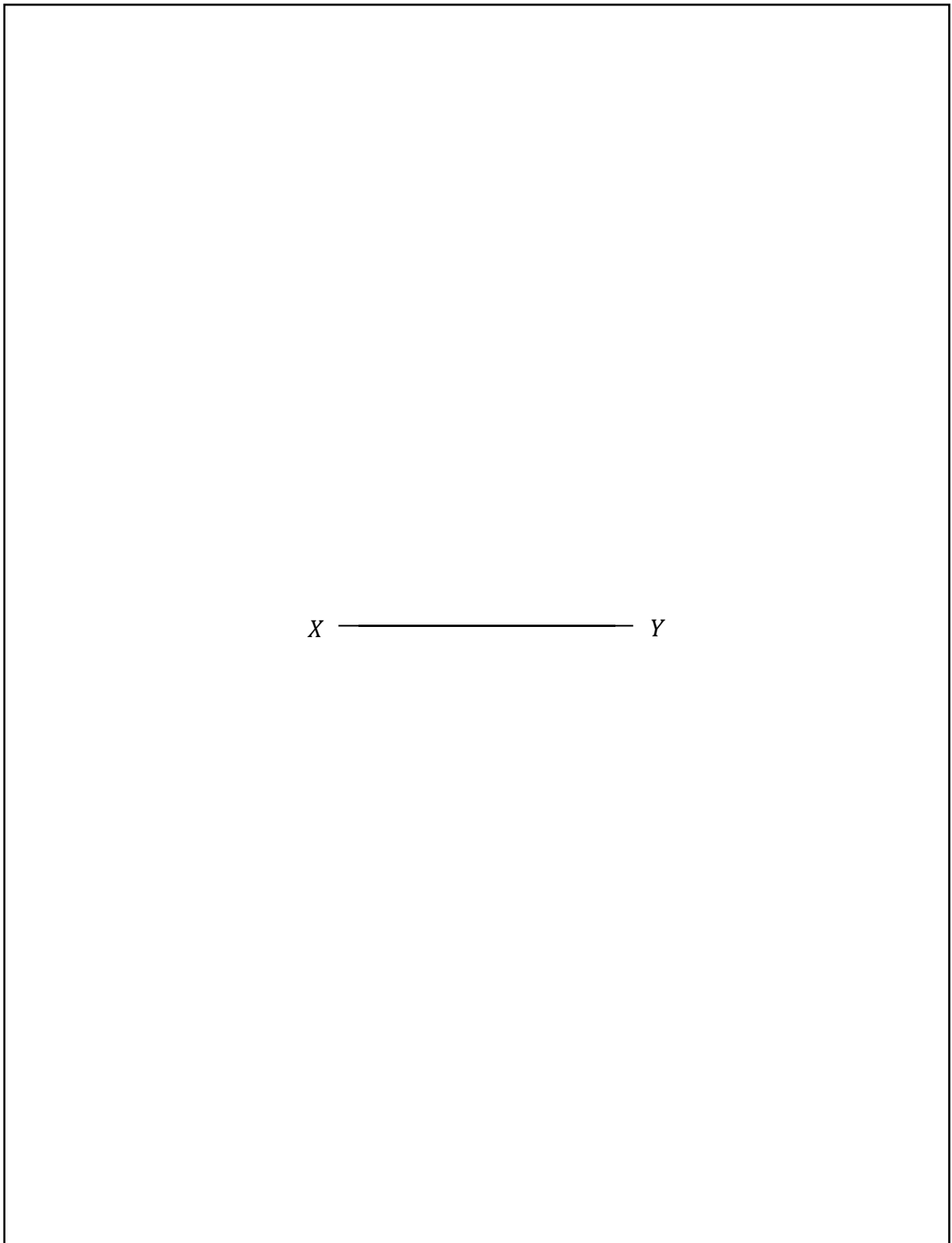(ii) Find $|PR|$.
There is space for more work on the next page.

**(b)** Find **all** the values of A for which $\tan A = -\sqrt{3}$, where $0 \leq A \leq 720°$.

Question 6 (30 marks)

(a) The diagram below shows a line segment $[XY]$ of length 1 unit.
Use this to construct a line segment of length $\sqrt{3}$ units in the space below.
Label this line segment clearly. Show all of your construction lines.

$$X \quad\rule{3cm}{0.4pt}\quad Y$$

(b) In the diagram below, the point D is on $[AB]$, and CD is perpendicular to AB.
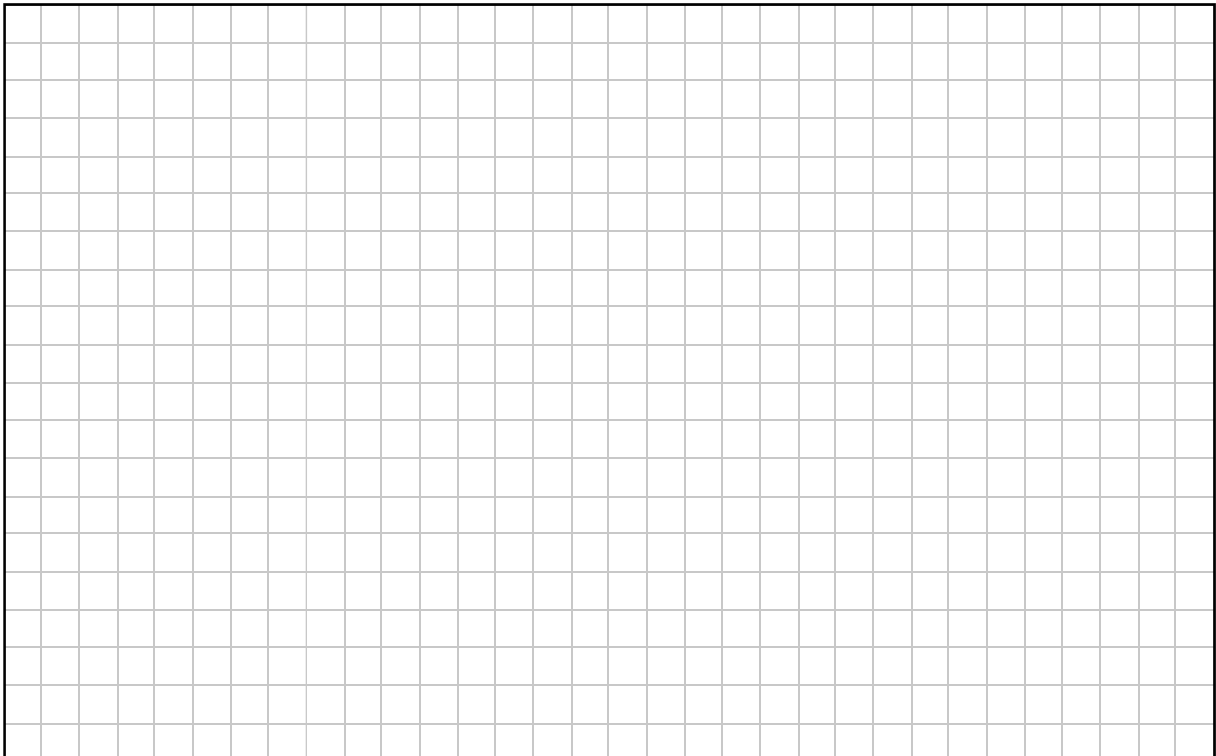The angle $\angle ACB$ is a right angle. The diagram is not to scale.
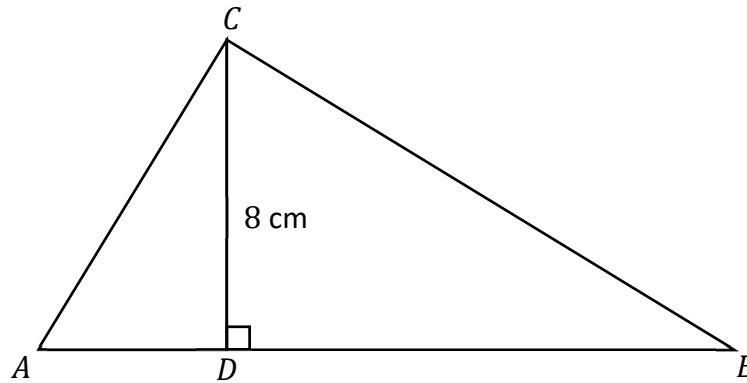
(i) From the diagram above, write down another angle that is equal in size to $\angle DCB$, **and** another angle that is equal in size to $\angle ACD$.

Angle equal in size to $\angle DCB =$ ⬜

Angle equal in size to $\angle ACD =$ ⬜

(ii) The ratio of $|AD|:|DB|$ is $2:5$. $|CD| = 8$ cm
Using similar triangles, find $|AB|$. Give your answer correct to 1 decimal place.

Answer **two questions** from this section.

Question 7   **(50 marks)**

Nuala and Mike are judges on a weekly talent show. There are ten acts in the competition each week.
The table below shows the judges' scores for each of the ten acts on a particular week.
A scatter plot of some of the scores is also shown.

| Act	Nuala's score	Mike's score
A	20	34
B	**35**	**34**
C	6	27
D	**49**	**38**
E	11	23
F	19	33
G	15	25
H	9	28
I	14	30
J	**32**	**32**
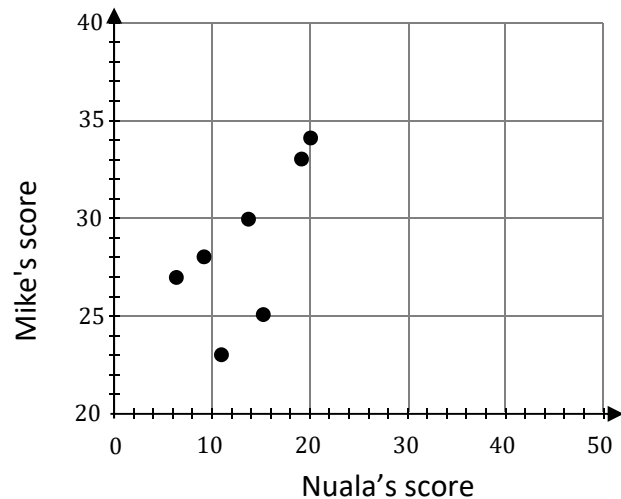
(a) The points for acts **B**, **D**, and **J** have not been included on the scatter plot.
Plot a point to represent each of these acts on the scatter plot, **and** label each one clearly.
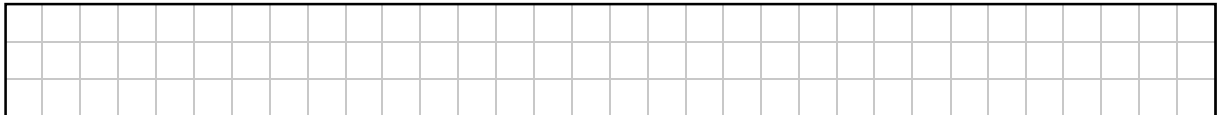
(b) Using a calculator, find the correlation coefficient between the scores of the two judges,
correct to 2 decimal places.

Answer:

(c) **(i)** On the scatter plot, draw the line of best fit, by eye.
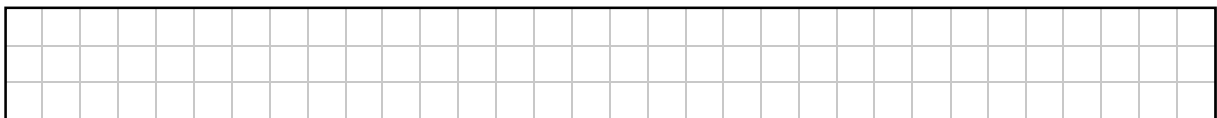
(ii) There was one more act, which is not in the table above.
Mike gave this act a score of 35. Use your line of best fit on the scatter plot to
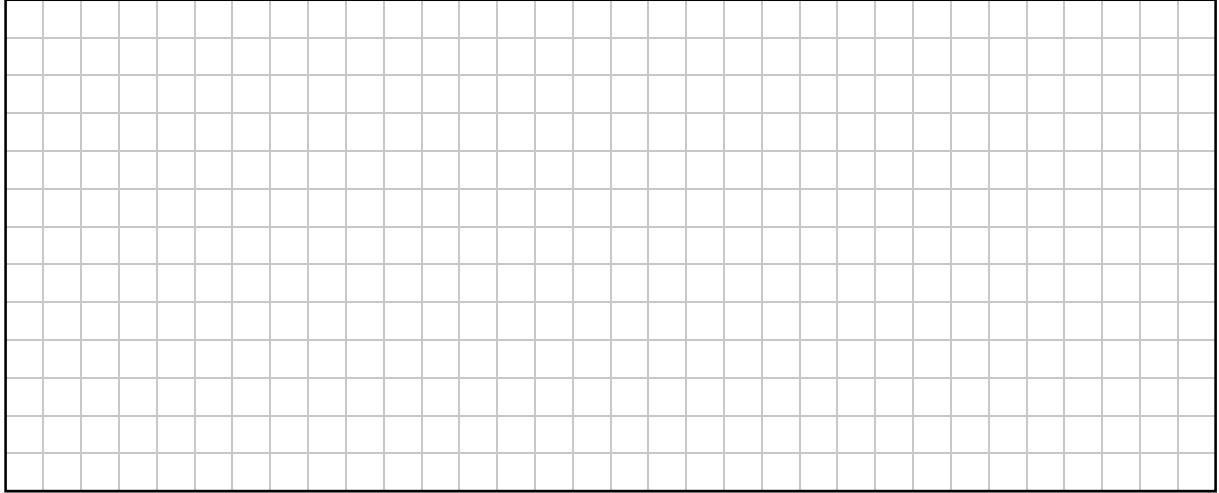estimate the score that Nuala gave this act. Show your work on the scatter plot.
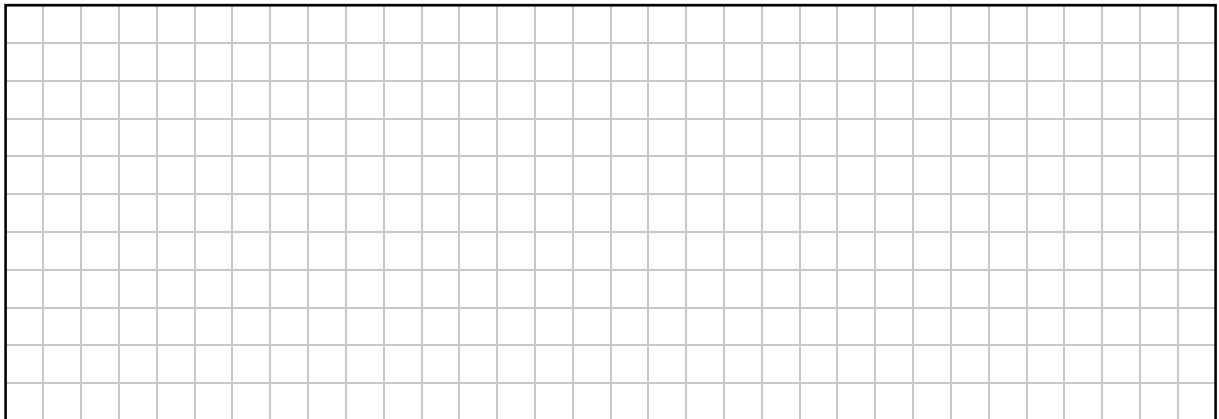
Answer:

(d) On a **different** week, the judges scored ten other acts, as shown on the scatter plot below. A line of best fit is also shown for these ten points.

(i) Using the diagram above, work out the equation of this line of best fit, taking x to be Nuala's score and y to be Mike's score.

(ii) Explain the meaning of the slope of this line, in the context of how Nuala and Mike scored the acts.

This question continues on the next page

(e) Over the first season of shows, Mike's scores were approximately normally distributed, with a mean of 31·6 and a standard deviation of 3·2.

(i) Twelve acts were picked at random from the first season of shows, and the scores given by Mike were recorded.

Work out the probability that the mean of these twelve scores is between 30 and 32.
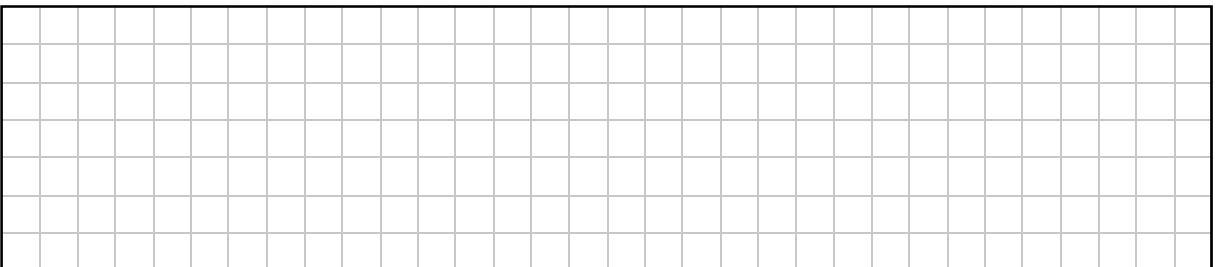
After the second season of shows, a website claims that Mike's scores in the second season have changed relative to the first season. A random sample of twenty of Mike's scores from the second season has a mean of 33·5.

(ii) Calculate the p-value for this sample mean, based on a two-tailed test.

(iii) Comment on what can be concluded from this p-value, in a two-tailed hypothesis test at the 5% level of significance, in relation to the website's claim.
Give a reason for your answer.

Question 8 **(50 marks)**

(a) A farmer employs 14 workers to sort, pack, and load egg boxes for delivery.
They work in groups of 5 at any one time.
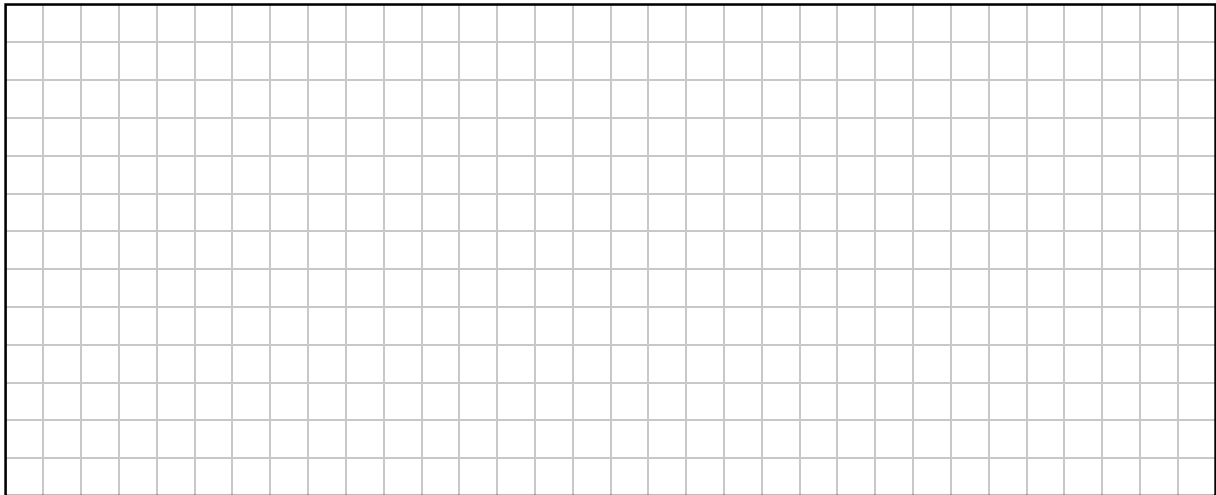Find how many different groups of 5 workers it is possible to form.

(b) The farmer has a coded lock on his office. He has forgotten the code.

The code consists of one letter followed by a 4-digit number, for example, A6465.
The farmer knows that the letter is a vowel (A, E, I, O, or U).
He also remembers that the 4-digit number begins with a 6, and is an odd number.

Find how many different possibilities there are for the farmer's code, given this information.

This question continues on the next page

(c) The farm shop sells boxes of eggs. The table below shows the price for each box, as well as the probability that a box of eggs that is sold is of a particular size.

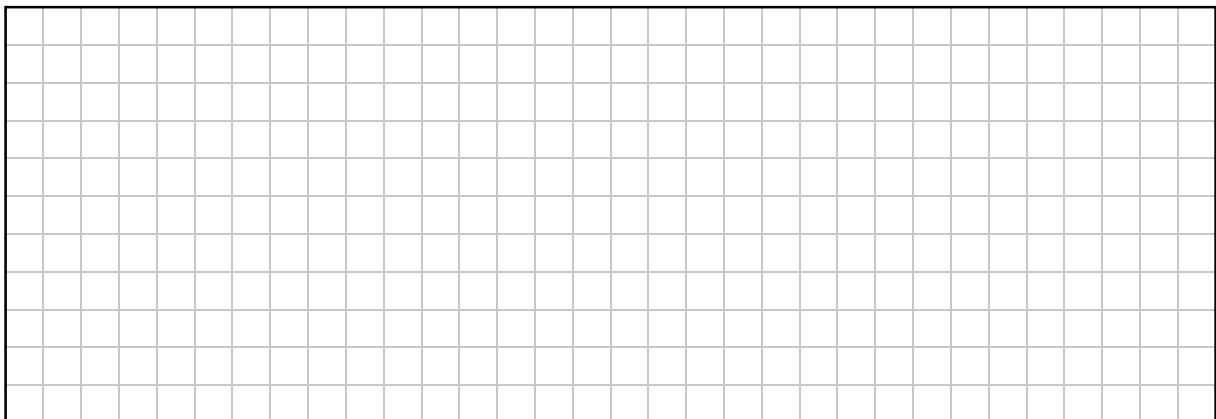| Size	small	medium	large	extra large
Price (€)	1·20	1·60	1·95	2·35
Probability of selling	0·16	0·31	0·34	0·19

Find the expected value of the shop's income from the sale of one box of eggs.
Give your answer in euro, correct to the nearest cent.

(d) The farmer's eggs are classified for sale into different sizes according to their weight.
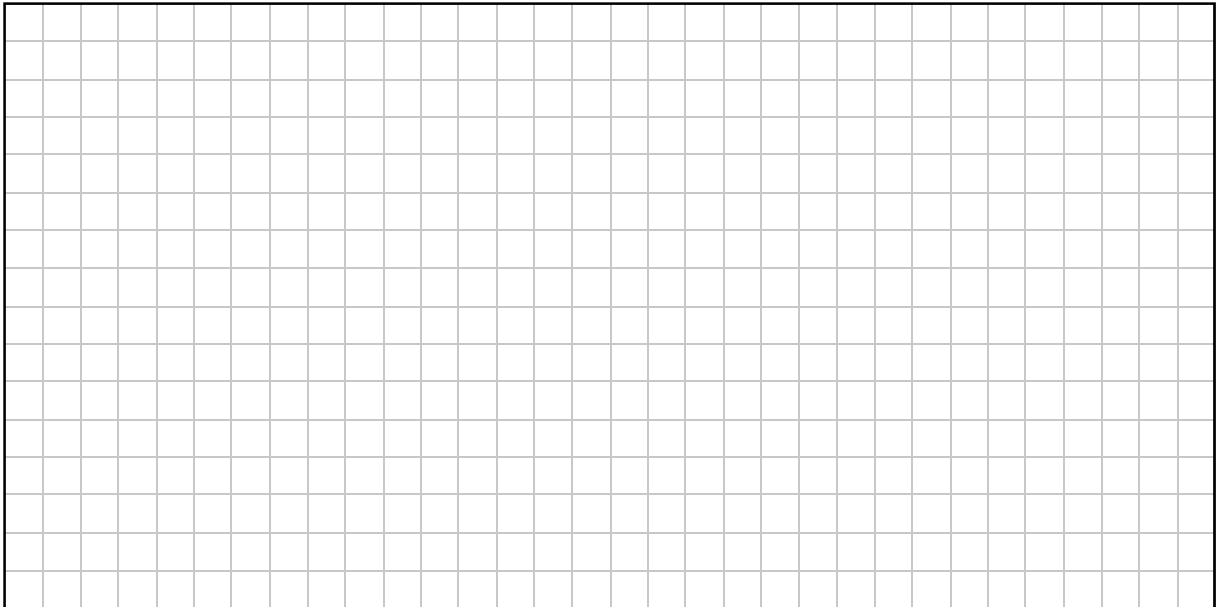Each egg is either **small**, **medium**, **large**, or **extra large**.

The weights of the eggs are normally distributed, with a mean weight of 60 g and a standard deviation of 15 g.

(i) An egg is small if it weighs less than 53 g. Work out the percentage of the farmer's eggs that are small. Give your answer correct to the nearest percent.

(ii) An egg is large if it weighs between 63 grams and N grams, for $N \in \mathbb{R}$.
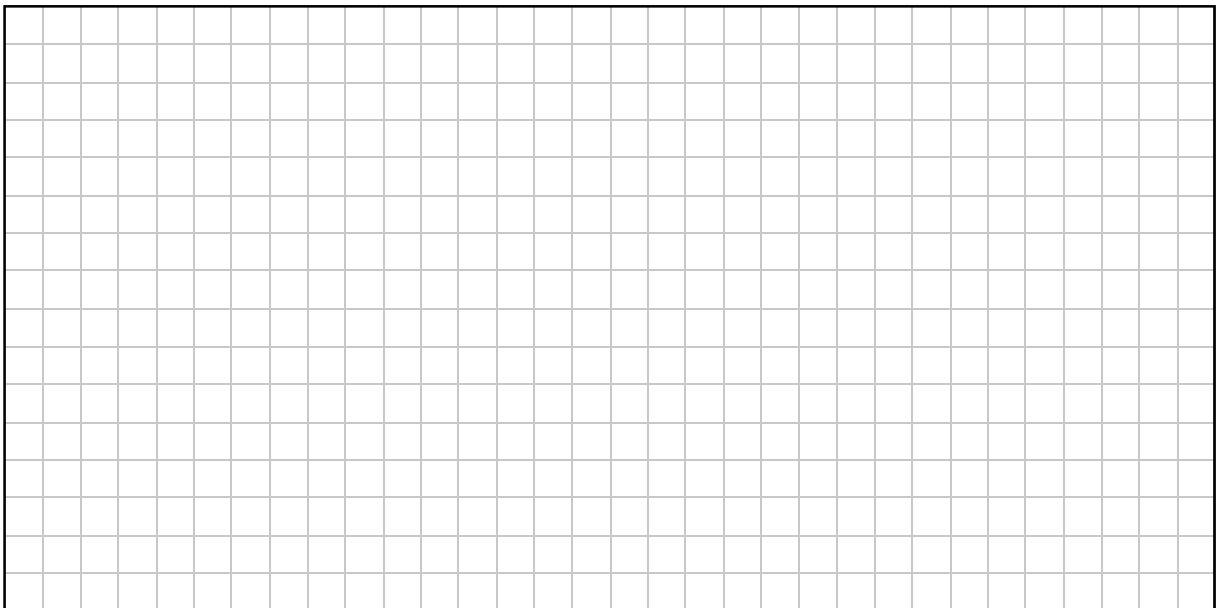23% of the farmer's eggs are large.

Work out the value of N, using the mean and standard deviation of the weights.
Give your answer correct to the nearest gram.

(iii) 19% of the farmer's eggs are **extra large**. 23% are **large**.

Due to a fault with a sorting machine, all of the **large** and **extra large** eggs in one batch are mixed up (at random) and boxed together. It is too expensive to remove all of these eggs from their boxes and sort them again, so the farmer instead labels all these boxes as **large**.

In such a box with 6 eggs, find the probability that at least 5 of the eggs will be **extra large**. Give your answer correct to 3 decimal places.
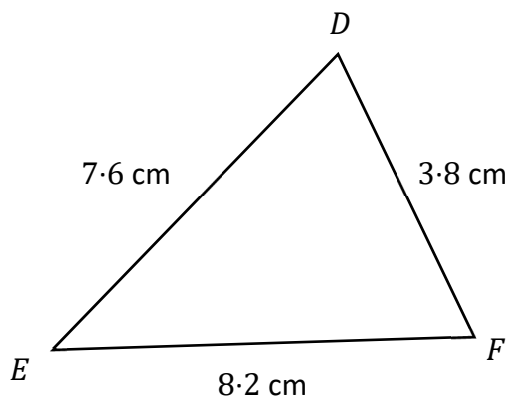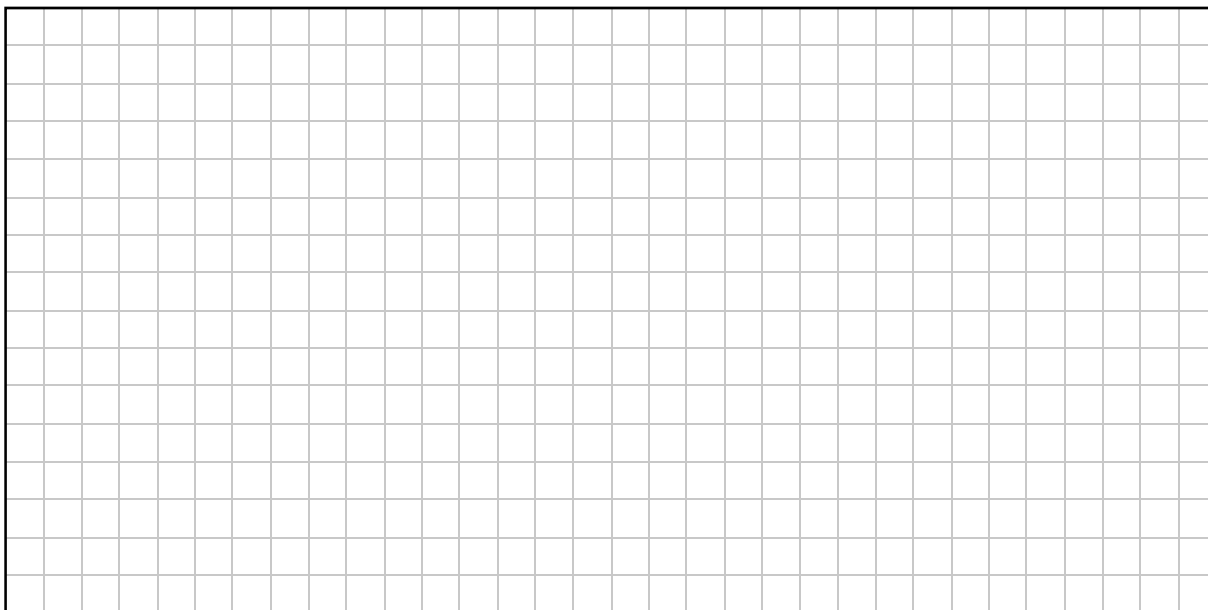
Question 9 (50 marks)

A team of surveyors is mapping mountains.

(a) The surveyors have a map showing the peaks of mountains D, E, and F.
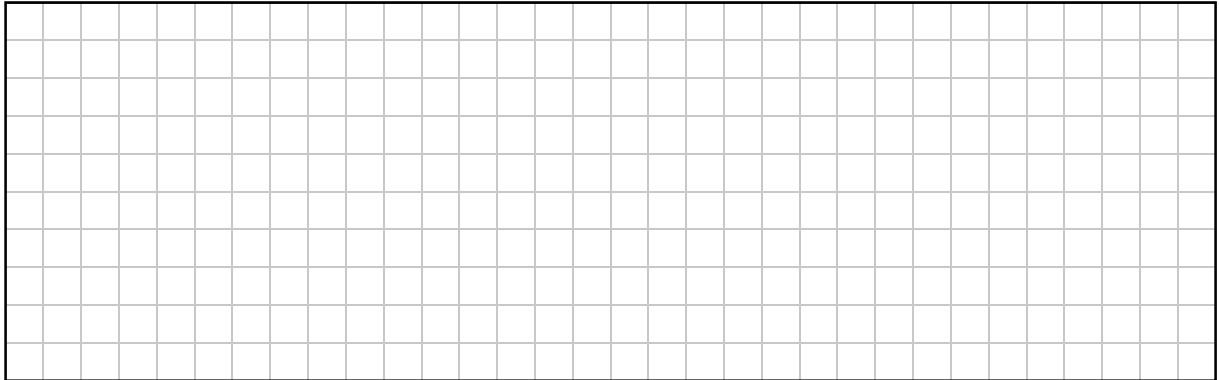On the map, $|DE| = 7 \cdot 6$ cm, $|EF| = 8 \cdot 2$ cm, and $|DF| = 3 \cdot 8$ cm, as shown below.

(i) Using only the information above, work out $|\angle DEF|$.
Give your answer in degrees, correct to 1 decimal place.

**(ii)** The surveyors measure the map and find that $|\angle FDE| = 85°$.

Using this value, or your answer from **part (a)(i)**, find the area of the triangle DEF on the map. Give your answer correct to 1 decimal place.

(iii) The scale on the map is $1 : 50\,000$.
Three other peaks form a triangle XYZ on the map of area 152 cm².
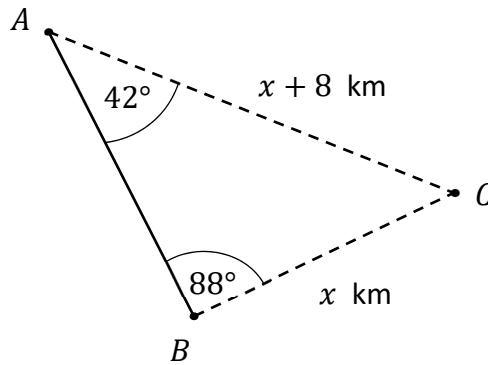
Find the actual area of the triangle formed by joining the peaks of the mountains $X, Y,$ and Z. Give your answer in the form $a \times 10^n$ km², where $1 \leq a < 10$ and $n \in \mathbb{N}$.
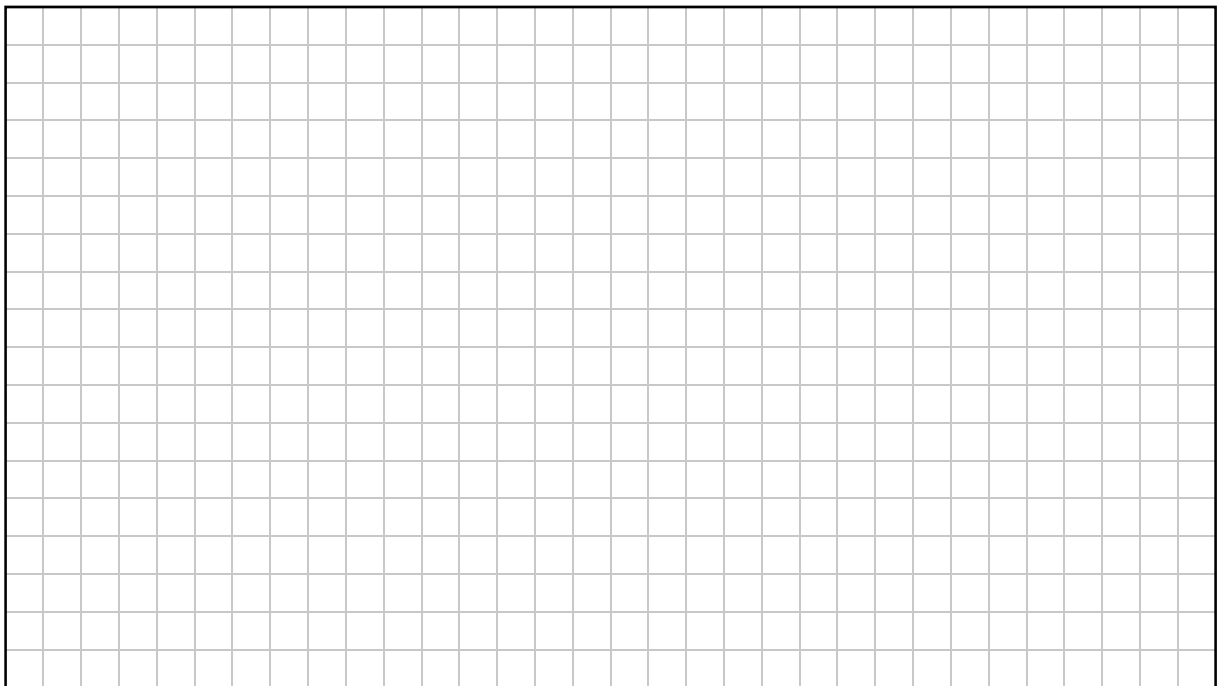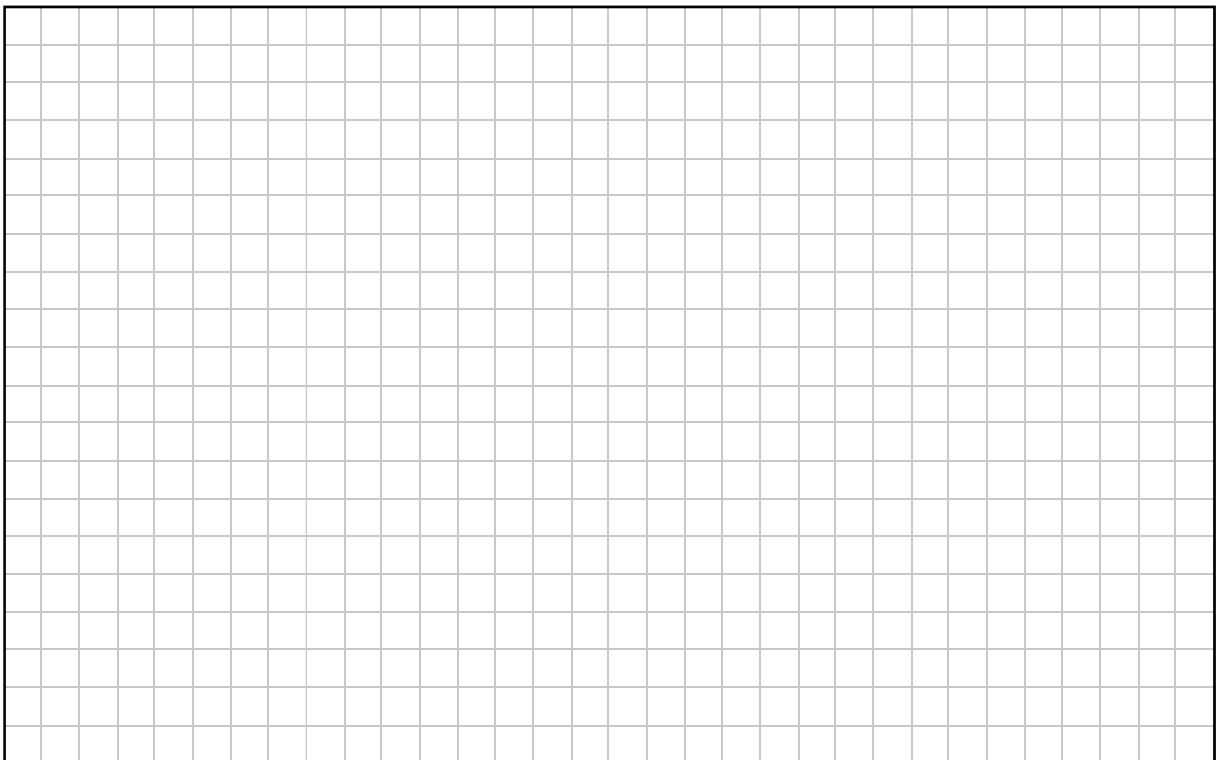
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The surveyors are also estimating the heights of some mountains that have not been mapped before, taking measurements from mountains that have already been mapped.

(b) The surveyors measure the angles to mountain C from mountains A and B, as shown below.
They also estimate that the distance $|AC|$ is 8 km more than the distance $|BC|$.
They let x km be the distance $|BC|$.

Find the value of x, correct to 1 decimal place.

(c) The surveyors know that the horizontal distance between mountains G and P is 8·3 km. They measure the angle of elevation of G from P, marked θ in the diagram below (not to scale). They use this to work out the vertical distance h.

For their calculations, the surveyors round the value of θ to the nearest degree, giving $6°$.

(i) Use $\theta = 6°$ to work out the surveyors' estimate for the value of h.
Give your answer in metres, correct to the nearest metre.

(ii) Find the maximum percentage error in the surveyors' estimate of the height h.
Give your answer as a percentage, correct to 1 decimal place.
Remember that the surveyors round the value of θ to the nearest degree.

Question 10 (50 marks)

Marc makes a grid on the co-ordinate plane, by drawing the following **gridlines**, for all **integers** k :
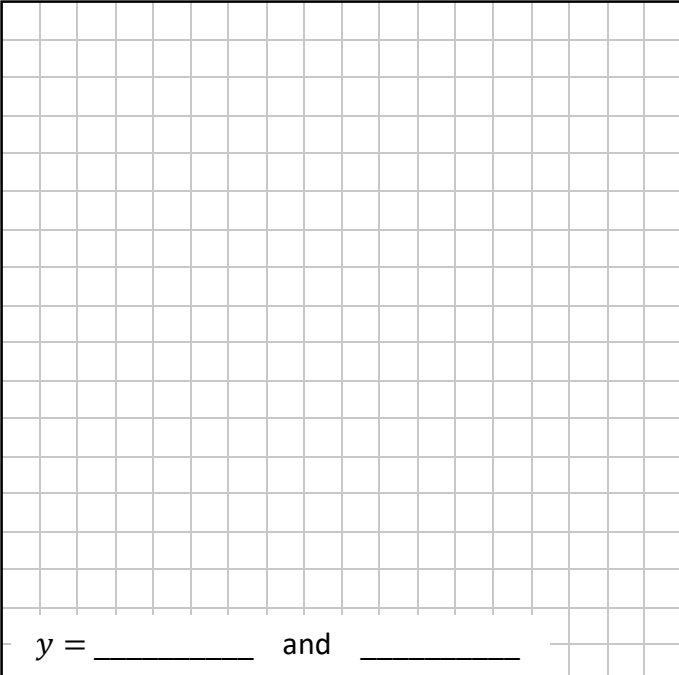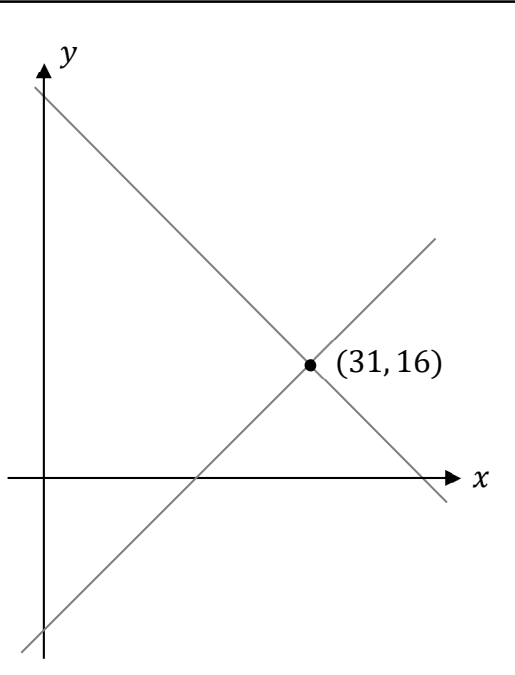
- He draws a line of slope 1 through each point of the form $(k, 0)$

- He also draws a line of slope -1 through each point of the form $(k, 0)$

The diagram below shows Marc's gridlines for some of the co-ordinate plane, as well as the x-axis and the y-axis.
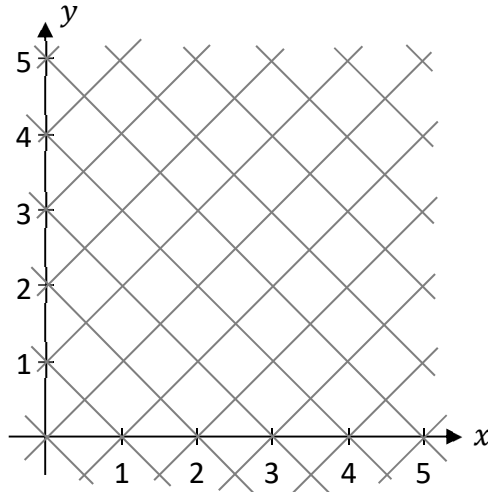
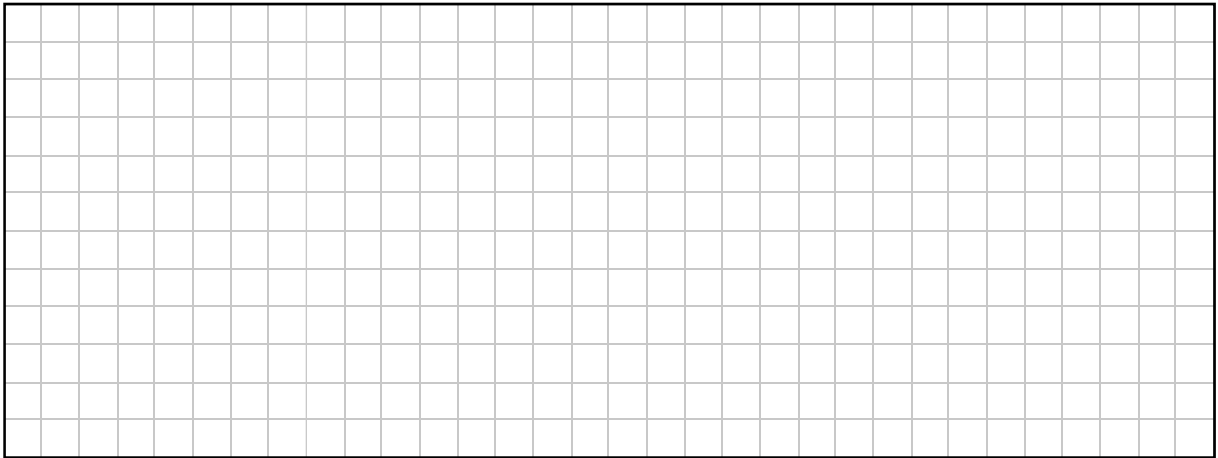**(a)** The point $(31, 16)$ lies on two of Marc's gridlines.

Work out the y-intercept of each of these two gridlines
(that is, the y-value for which each of these lines crosses the y-axis).

$y =$ _____ and _____

**(b)** l is one of the gridlines. It has the following equation, where $x, y \in \mathbb{R}$:

$$x + y = 12$$

Find the equation of the image of l under axial symmetry in the x-axis.

(c) k is a circle. The two gridlines that have a positive slope and that go through the points $A(-1, 0)$ and $B(5, 0)$, respectively, are tangents to the circle k.
The centre of k lies on the horizontal line $y = 22$.

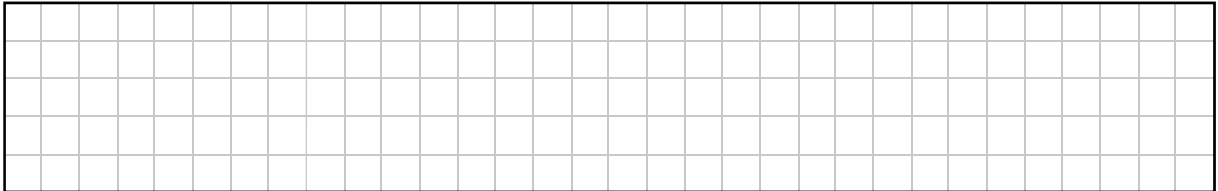Find the equation of the circle k. (Note that **neither** A **nor** B lie on the circle k.)

This question continues on the next page
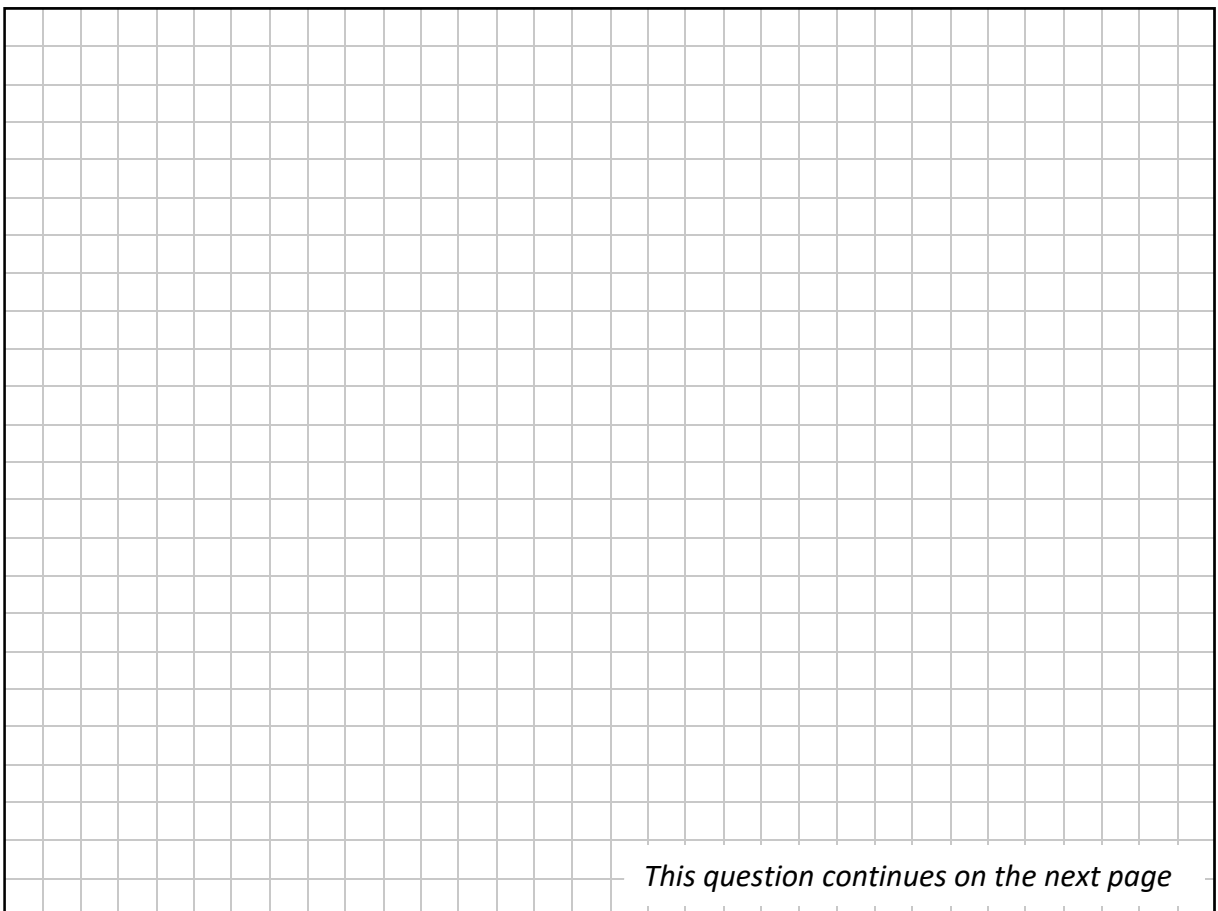
(d) Marc adds some horizontal and vertical gridlines, and shades in some of the resulting triangles.

The diagram on the right shows the pattern that he makes in the square for which $0 \leq x \leq 1$ and $0 \leq y \leq 1$.
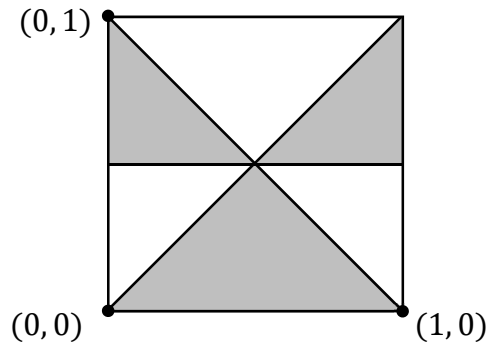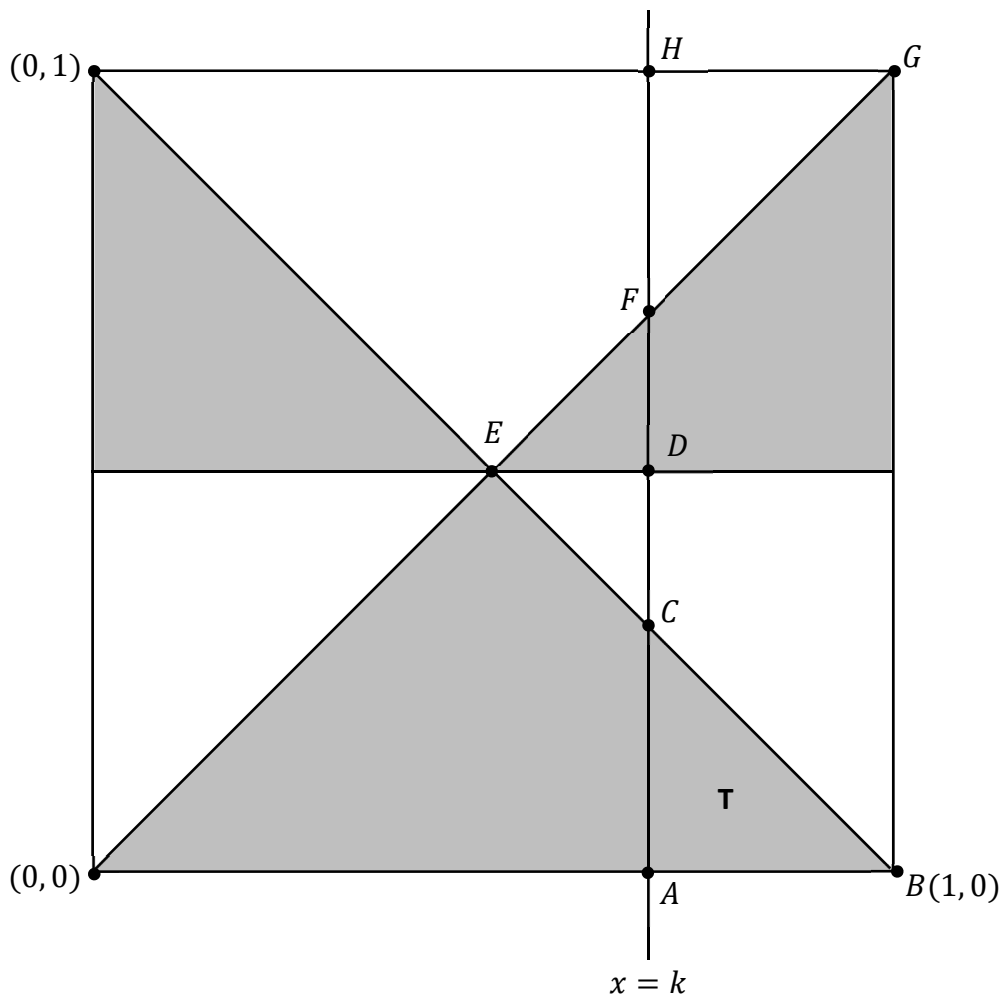
(i) What percentage of this square is shaded?

$(0,1)$

$(0,0)$ $(1,0)$

The diagram below shows the same square.
Marc draws a vertical line through this square at $x = k$, where $0 \cdot 5 < k < 1$ and $k \in \mathbb{R}$.
He marks in the points A to H. **T** is the shaded triangle ABC.

$(0,1)$ H G

F

E D

C

T

$(0,0)$ A $B(1,0)$

$x = k$

(ii) There is one triangle in this diagram that must be congruent to the triangle **T**.
State what triangle this is, **and** prove that this triangle is congruent to **T**.

Triangle that is congruent to **T**:

Proof that it is congruent to **T**:

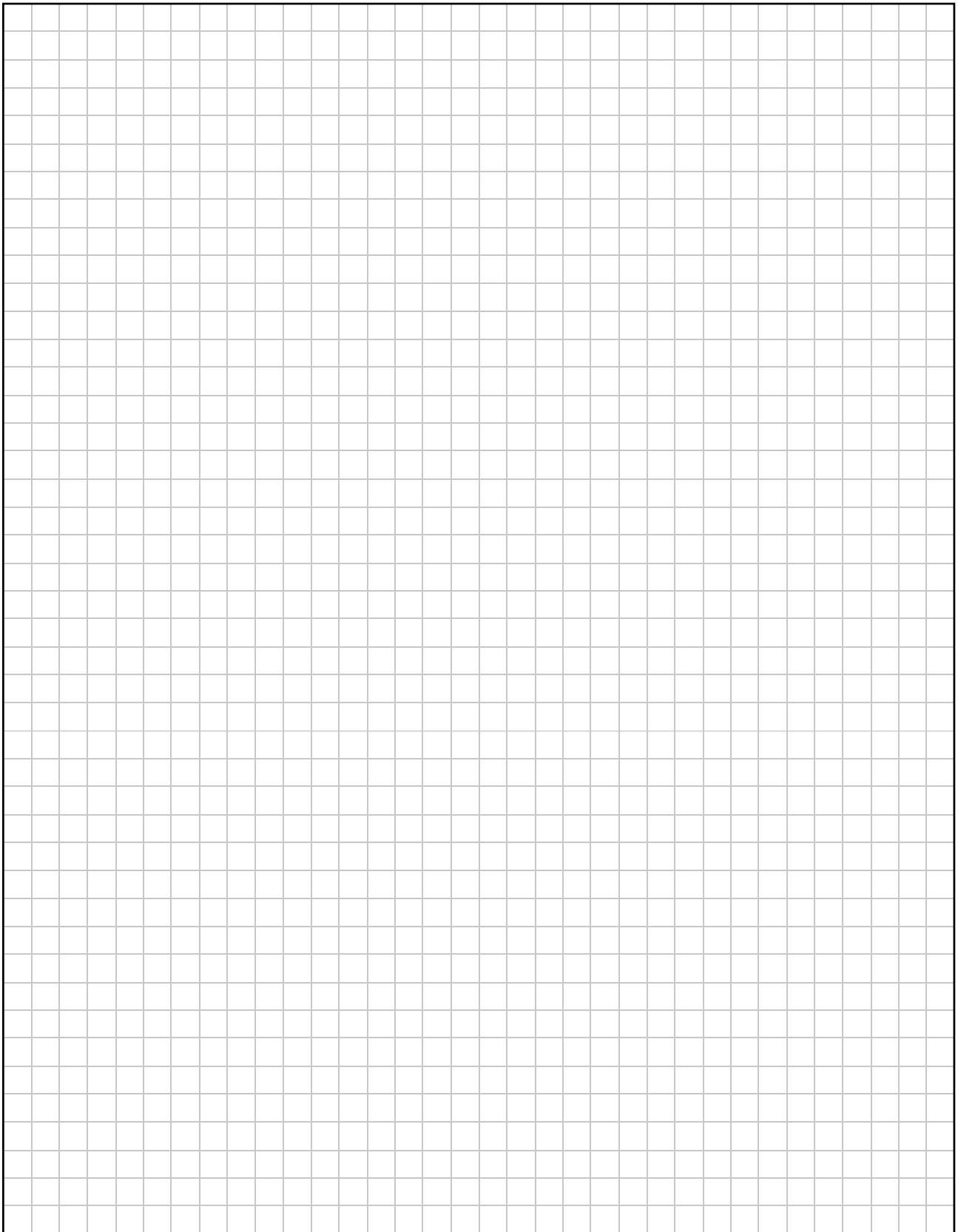**(iii)** Find the co-ordinates of the point C. Give your answer in terms of k.

(iv) Show that, within this square, exactly half of the line $x = k$ passes through shaded triangles.

Use your answer to **part (ii) or** your answer to **part (iii)** in your solution.
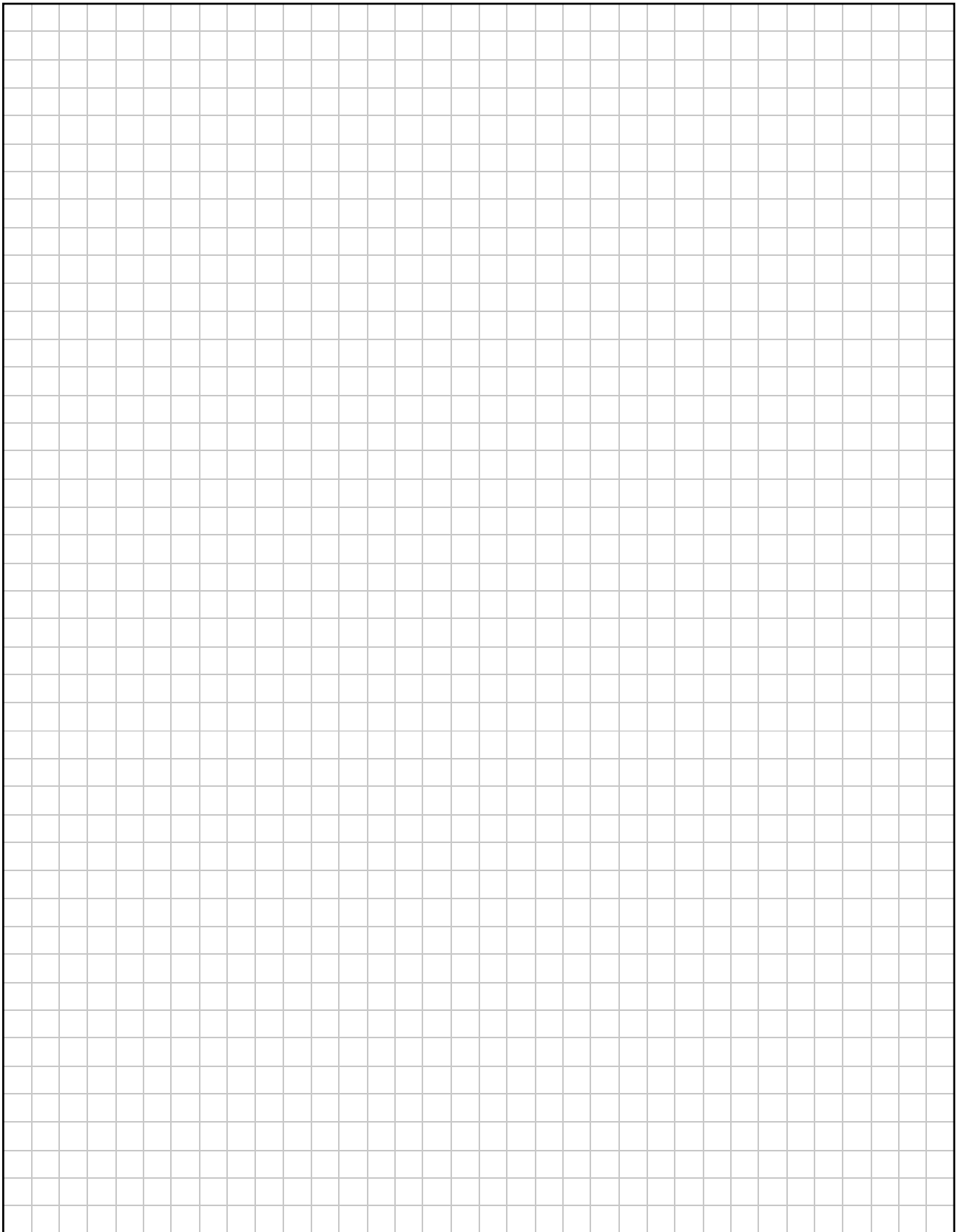
Page for extra work.
Label any extra work clearly with the question number and part.

Page for extra work.
Label any extra work clearly with the question number and part.

Leaving Certificate – Higher Level

Mathematics Paper 2

2 hours 30 minutes