2022.M28

2022L003G2EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2022

Mathematics

Paper 2

Ordinary Level

Monday 13 June Morning 9:30 - 12:00

220 marks

Examination Number

Day and Month of Birth

For example, 3rd February is entered as 0302

Centre Stamp

Do not write on this page

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	120 marks	6 questions
Section B	Contexts and Applications	100 marks	4 questions

Answer questions as follows:

- any **four** questions from Section A – Concepts and Skills
- any **two** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet.
If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

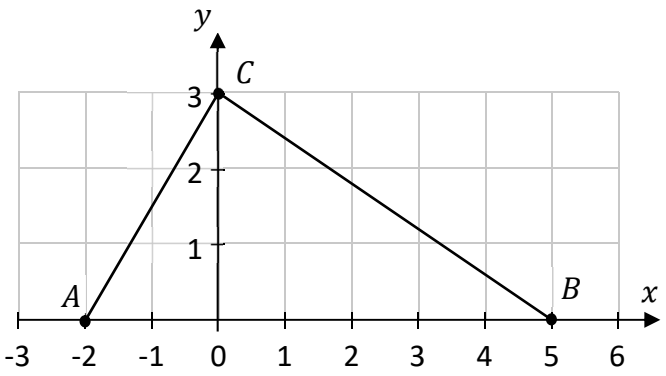
Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 120 marks

Answer **any four questions** from this section.

Question 1 **(30 marks)**

Parts of the lines AC and BC are shown in the co-ordinate diagram below (not to scale).



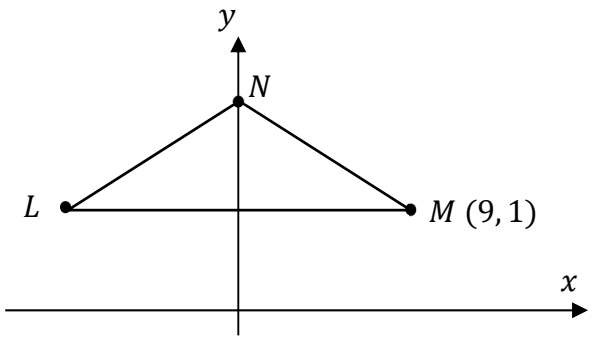
(a) **(i)** Find the slope of AC.

(ii) By using slopes, investigate if AC is perpendicular to BC. Justify your answer.

Answer: (Tick (✓) **one** box only)

AC **is** perpendicular to BC	AC is **not** perpendicular to BC
☐	☐

(b) The triangle LMN is shown on the co-ordinate diagram below (not to scale).
The point M has co-ordinates $(9, 1)$.
The triangle LMN is symmetrical about the y-axis.



(i) Find the length $|LM|$.

(ii) Write down the equation of the horizontal line LM.

(iii) The line NM has equation $x + 4y - 13 = 0$, where $x, y \in \mathbb{R}$.

Use this equation to find the co-ordinates of the point N.

$N = (\quad , \quad)$

Question 2 **(30 marks)**

(a) The circle k has equation $(x-4)^2+(y+2)^2=169$.

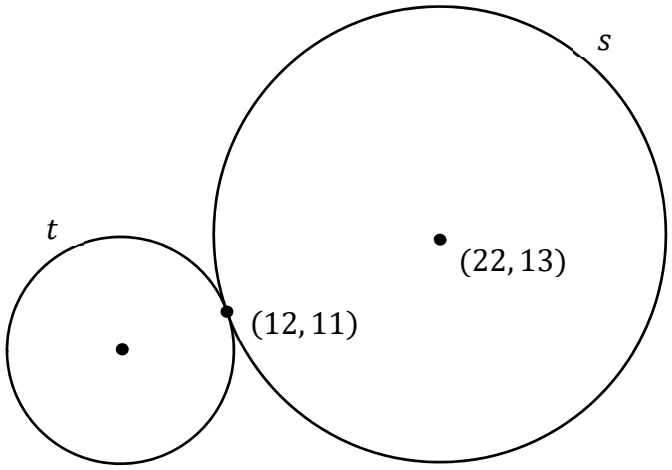
(i) Write down the centre and radius of the circle k.

Centre $=$ (,) Radius $=$ _____

(ii) Is the point $(11,10)$ on the circle k, inside the circle k, or outside the circle k? Show your working out.

Answer: (Tick (✓) **one** box only)	$(11,10)$ is **on** k ☐	$(11,10)$ is **inside** k ☐	$(11,10)$ is **outside** k ☐

(b) The diagram below shows two circles, s and t. The circle s has centre $(22, 13)$. The two circles touch at the point $(12, 11)$.



(i) Find the co-ordinates of another point on the circle s, **other than** $(12, 11)$.

(ii) The radius of the circle t is **half** the radius of s.
Find the co-ordinates of the centre of the circle t.

Question 3 **(30 marks)**

(a) A list of 7 numbers is shown below.

$$17, \quad 8, \quad 9, \quad 8, \quad 14, \quad 11, \quad 28$$

(i) Find the mean of these 7 numbers, correct to 1 decimal place.

(ii) Find the median of these 7 numbers.

(iii) One more number is added to the list. The new median is $10{\cdot}5$.
Find the number that was added to the list.

(b) Ben has to choose three subjects to study.
He must pick one subject from each of these three groups:

Group A (3 subjects)	**Group B** (4 subjects)	**Group C** (5 subjects)
French	Biology	Art
Spanish	Physics	Accounting
German	Chemistry	History
	Business	Geography
		Home Economics

(i) How many different choices of three subjects can Ben make, picking one from each group?

(ii) The school is going to add one extra subject to one of these groups (**A**, **B**, or **C**).
Which group should the extra subject be added to, in order to make the number of different choices of three subjects that Ben can make as **large** as possible?
Justify your answer.

Answer:

Justification:

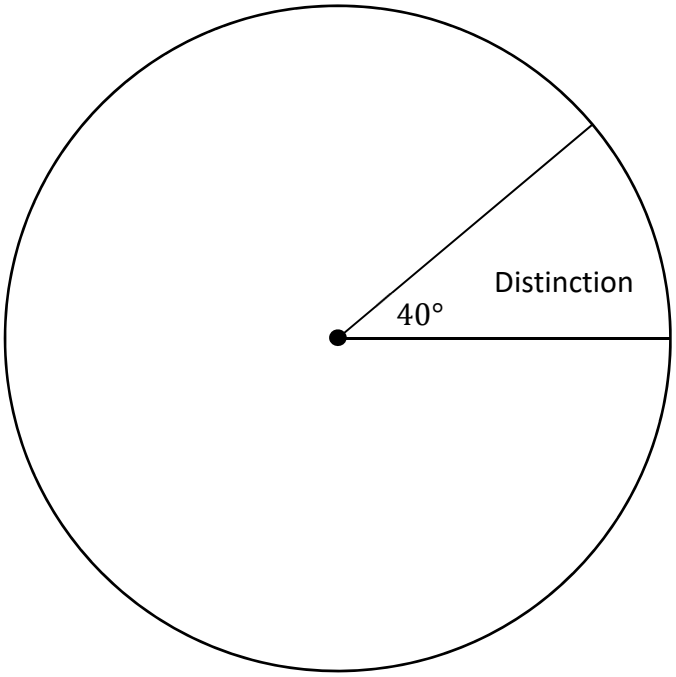
Question 4 **(30 marks)**

(a) A group of students sat an exam. Each student was given a grade.
The following table shows how many students got each grade.

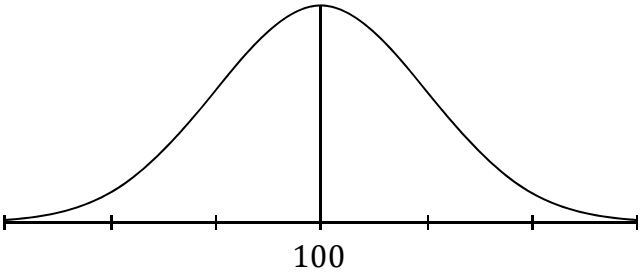
Grade	Distinction	High Merit	Merit	Achieved
Number of students	8	12	39	13

Complete the pie chart below to show this information.
Label each sector clearly. Show your working out.

The sector for Distinction is already given.
It has an angle of $40°$.



(b) A large group of people took a reading test.
The scores were normally distributed, with a mean of 100 and a standard deviation of 20.



Use the empirical rule to answer **parts (b)(i)** and **(b)(ii)**.

(i) What percentage of the people had scores between 80 and 120?

(ii) The top $2{\cdot}5\%$ of scores were given a grade of 'Exceptional'.
What was the least score that was needed to get this grade?

(c) The scores of six people on this test were as follows:

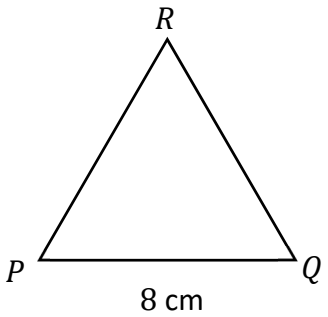
$$104, \quad 82, \quad 94, \quad 113, \quad 98, \quad 105$$

Find the range **and** the standard deviation of these numbers.
Give the standard deviation correct to 1 decimal place.

Range $=$ _____ Standard deviation $=$ _____

Question 5 **(30 marks)**

An equilateral triangle PQR has sides of length 8 cm.

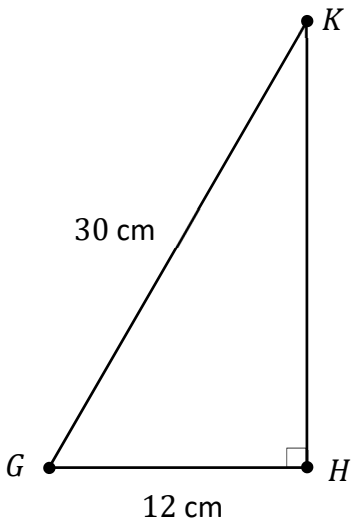


(a) **(i)** Write down the size of the angle $\angle PQR$.

(ii) Show that the area of the triangle PQR is $16\sqrt{3}$ cm^2.

(iii) Hence, or otherwise, find the perpendicular height of the triangle PQR, taking PQ as the base. Give your answer in the form $a\sqrt{b}$ cm, where $a, b \in \mathbb{N}$.

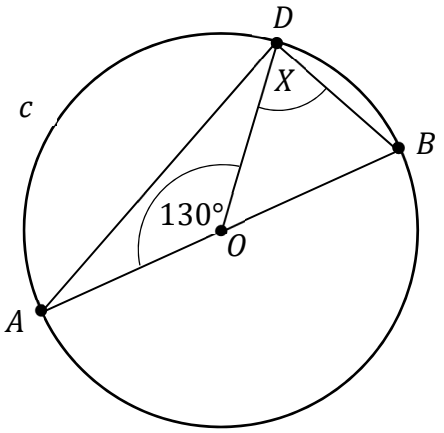
(b) GHK is a right-angled triangle. $|\angle GHK| = 90°$, $|GH| = 12$ cm, and $|GK| = 30$ cm.



Using the theorem of Pythagoras, find the distance $|HK|$.
Give your answer correct to 1 decimal place.

Question 6 **(30 marks)**

(a) The circle c is shown in the diagram below (not to scale). Its centre is at the point O. The points A, B, and D lie on the circle, and $[AB]$ is a diameter of the circle.



(i) Write down $|\angle ADB|$, the size of the total angle at the point D.

(ii) $|\angle AOD| = 130°$. Work out the size of the angle marked X in the diagram.

(iii) The radius of the circle is 18 cm. Find the length of the arc AD.
Give your answer in cm, in terms of π.

(b) Two statements, **A** and **B**, are shown below.

For each statement, put a tick (✓) in the correct box to show if it is true or false.
Give a reason for your answer in each case.

(i) **Statement A**: If two triangles are similar, then they **must** be congruent.

This statement is: (tick **one** box only)	true	false
	☐	☐

Reason:

(ii) **Statement B**: If two triangles are congruent, then they **must** be similar.

This statement is: (tick **one** box only)	true	false
	☐	☐

Reason:

Section B — Contexts and Applications — 100 marks

Answer **any two** questions from this section.

Question 7 **(50 marks)**

An animal shelter takes cats and dogs.

(a) The staff record the weight of each animal in the shelter on two different days: the day they are taken in to the shelter (Day **X**) and a number of days later (Day **Y**).

The table below shows the weights of 10 dogs (labelled **A** to **J**) on these two days.

Animal	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Weight Day X (kg)	4·5	4·9	5·3	5·3	5·5	5·7	6·7	6·9	7·3	7·4
Weight Day Y (kg)	4·8	5·4	5·5	6·1	6·3	6·0	7·0	7·6	8·1	7·9

(i) Draw a back-to-back stem-and-leaf plot to show this information.

Day X					**Day Y**			
				4				
				5				
				6				
				7				
				8				

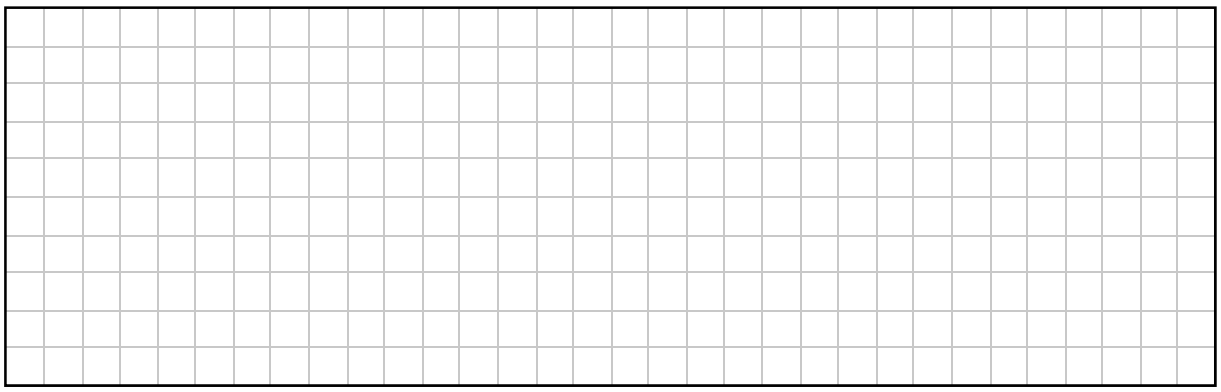
Key:
$6 \mid 0 = 6{\cdot}0$ kg

(ii) What does the stem-and-leaf plot show about the weights of the dogs on Day **X** and Day **Y**?

(iii) r is the correlation coefficient between the weight on Day **X** and the weight on Day **Y**. Based on the data in the table, pick the most likely value of r from the list below.

Give a reason for your answer, based on the data in the table.

Most likely value of r:	$-0{\cdot}9$	$-0{\cdot}2$	$0{\cdot}2$	$0{\cdot}9$
(tick **one** box only)	☐	☐	☐	☐



This question continues on the next page.

The table below shows the breakdown of the animals in the shelter, by type of animal (cat or dog) and by sex (male or female), on a particular Monday.

(b) Complete the table, by filling in the four missing values.

	Male	**Female**	**Total**
Cats	5	9	**14**
Dogs	11		
Total			**40**

Three different animals were picked at random from the animals in the shelter on this Monday.

(c) **(i)** Find the probability that the first animal picked was a cat.

(ii) Find the probability that all three animals picked were male dogs.
Give your answer correct to 3 decimal places.

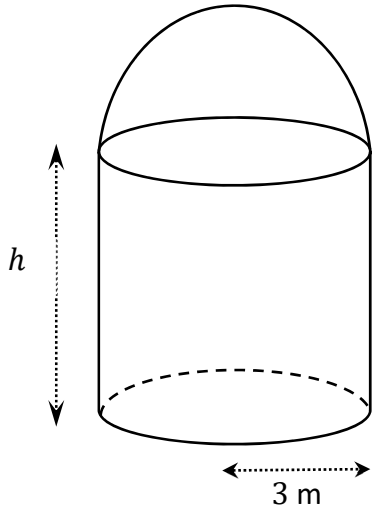
(d) The 9 female cats were put in 9 separate pens.
Work out the number of ways in which this could have been done
(that is, the number of different possible arrangements).

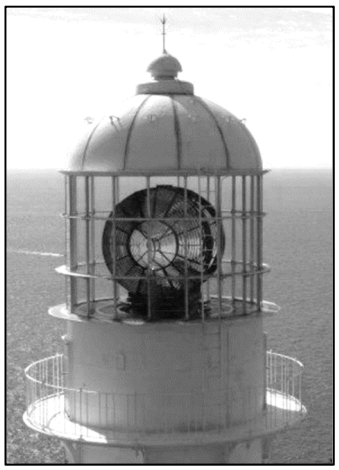
(e) By the end of this week, 10 of the animals had left the shelter, and **no** new animals had been taken in. If an animal was picked at random at the end of this week, the probability of picking a dog would be $\frac{11}{15}$.

Work out how many cats left the shelter during this week.

Question 8 **(50 marks)**

The top of a particular lighthouse is in the shape of a hemisphere on top of a cylinder.
The hemisphere and the cylinder both have a radius of 3 m.



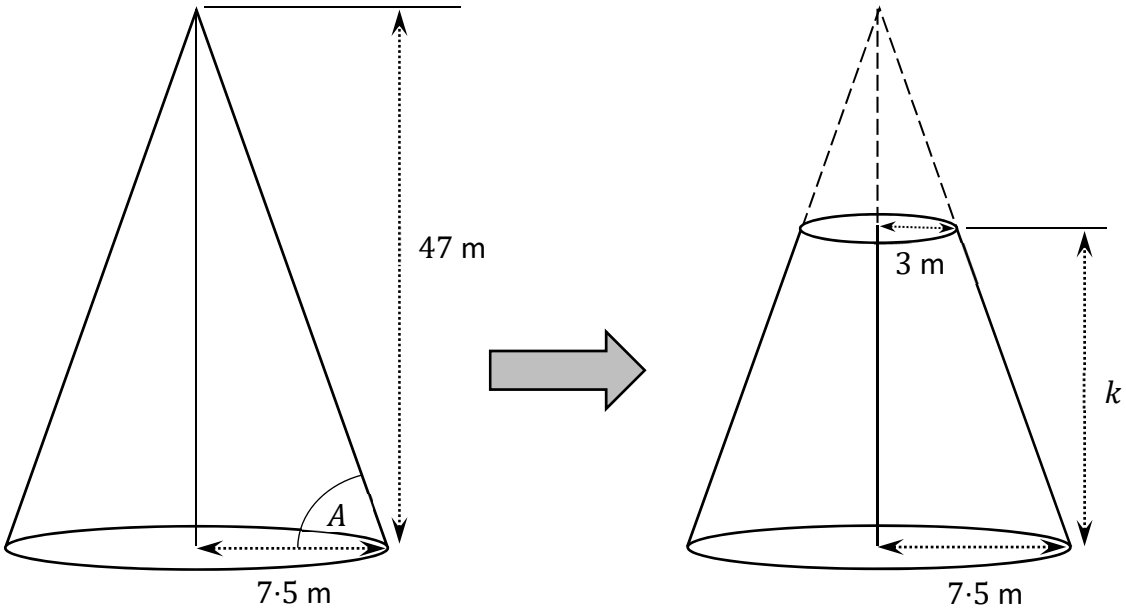


(a) **(i)** Find the volume of the **hemisphere**. Give your answer in m^3 in terms of π.

(ii) The volume of the **cylinder** is 36π m^3.
Work out h, the height of the cylinder.

Source of image: https://cdn.4travel.jp. Altered

(b) The diagram on the right below shows part of the base of the lighthouse (not to scale). It is in the shape of a cone of radius $7{\cdot}5$ m, from which the top part has been removed, leaving a horizontal circle of radius 3 m.



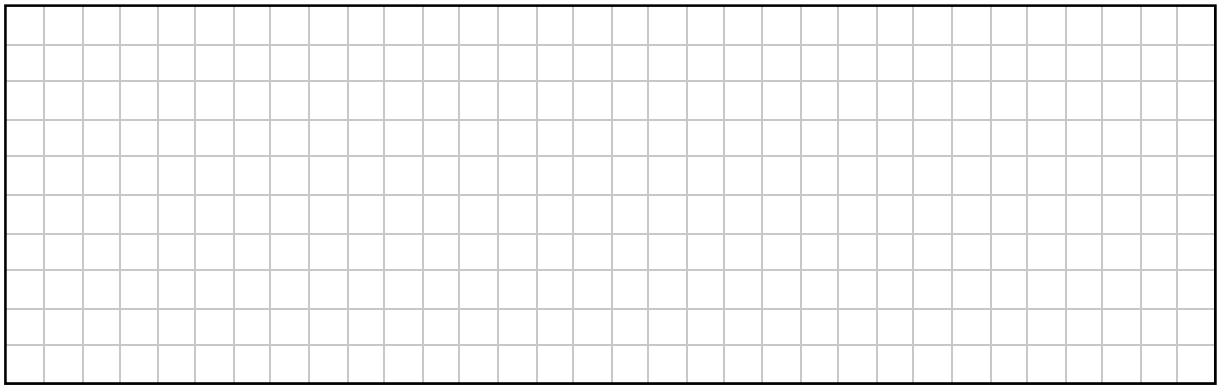
(i) The height of the cone **before** the top part is removed is 47 m.
Work out the size of the angle at the base of the cone, marked A in the diagram above.
Give your answer correct to the nearest degree.

(ii) Find the distance marked k on the diagram, the height **after** the top part is removed.

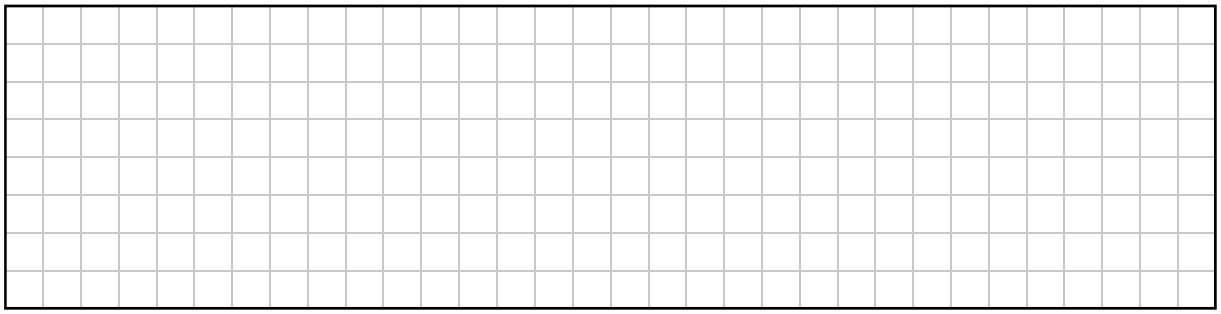
This question continues on the next page.

(c) Assume that the Fastnet lighthouse can be seen from anywhere within a circle of radius 50 km.

(i) Work out the area of the circle within which the Fastnet lighthouse can be seen.
Give your answer correct to the nearest km^2.

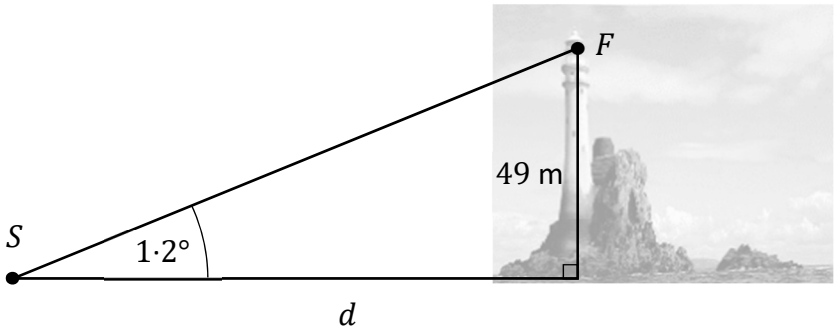


(ii) 50 km $= 27$ nautical miles.
Use this to work out how many km are in 1 nautical mile.
Give your answer correct to 4 significant figures.



(d) The top of the Fastnet lighthouse, F, is 49 m above sea level.
The angle of elevation of the top of the lighthouse from a ship S is $1{\cdot}2°$, as shown in the diagram below (not to scale).

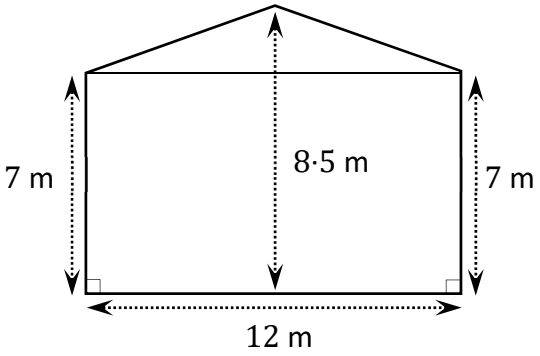
Find the horizontal distance marked d below, from the ship to the base of the lighthouse.
Give your answer in **kilometres**, correct to 2 decimal places.



Source of image: roaringwaterjournal.com. Altered.

Question 9 **(50 marks)**

Seán has built a shed. The diagrams below show the dimensions of Seán's shed.
The shed is in the shape of a prism. Its front face is in the shape of a triangle on top of a rectangle.
Its highest point is directly above the centre of its base.



Front face of shed

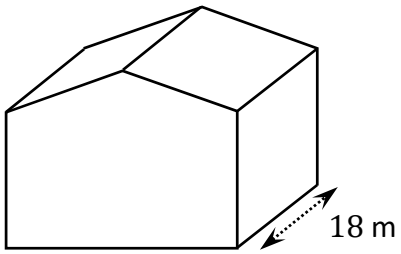


Diagram of whole shed

(a) State which of the following statements is most likely to be true, **and** write down a possible height of Seán that would support your answer. Tick (✓) **one** box only.

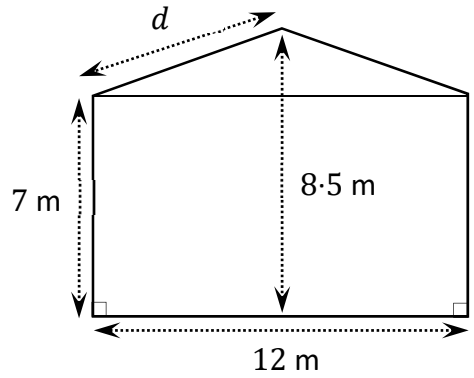
The shed at the highest point is 3 times as high as Seán.	The shed at the highest point is 5 times as high as Seán.	The shed at the highest point is 8 times as high as Seán.
☐	☐	☐

Possible height of Seán:

(b) Seán says that his shed has a capacity of over one million litres, where $1\ \text{m}^3 = 1000$ litres.
Work out the volume of Seán's shed, to show that he is correct.

There is space for more work on the next page.

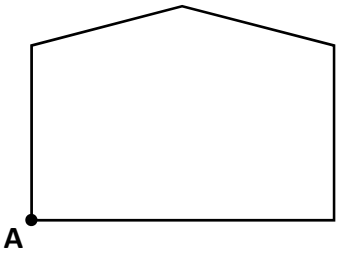
(c) Use the theorem of Pythagoras to find the length of the distance marked d in the diagram below, the slant length of the roof. Give your answer in metres, correct to 1 decimal place.



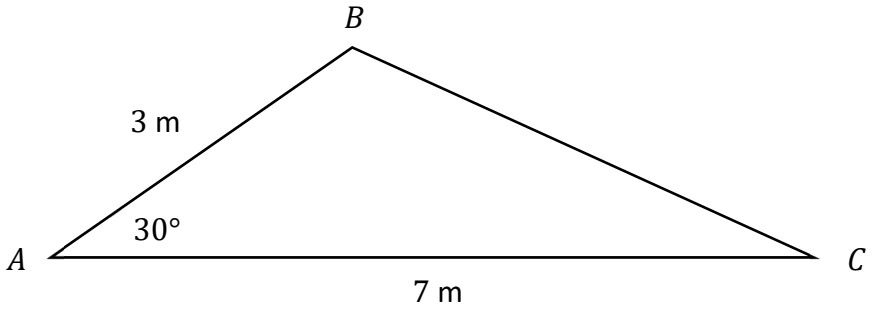
This question continues on the next page.

(d) A scale diagram of the front of the shed is shown below.
One point on the diagram is marked **A**.

Construct an enlargement of the diagram below, with centre **A** and a scale factor of 3.
Show all of your construction lines clearly.



(e) The diagram below shows part of the roof of a smaller shed (not to scale).
Some measurements are marked on the diagram.



(i) Show that $|BC| = 4{\cdot}65$ m, correct to 2 decimal places.

(ii) Find $|\angle ACB|$, the angle that the roof makes at the point C.
Give your answer correct to the nearest degree.
Remember that $|BC| = 4{\cdot}65$ m, correct to 2 decimal places.

Question 10 **(50 marks)**

A survey was carried out to investigate the amount and type of exercise that adults in Ireland are taking in 2022. This survey was carried out on a sample of adults in Ireland.

(a) Write down one advantage and one disadvantage of carrying out a survey on a sample instead of a population.

Advantage:

Disadvantage:

(b) **(i)** A random sample of 1500 people took part in the survey.
Show that the margin of error for the survey is $2{\cdot}6\%$, correct to 1 decimal place.

(ii) 71% of the sample said that they walk for recreation.
Find the number of people from the sample who said that they walk for recreation.

(iii) Use the percentages given in **(b)(i)** and **(b)(ii)** to write down a 95% confidence interval for the percentage of all adults in Ireland who walk for recreation, in 2022.

(iv) According to the *2019 Irish Sport Monitor Report*, 65% of the adults in Ireland walked for recreation.

Carry out a Hypothesis Test, at the 5% level of significance, to find out if this figure of 65% has changed in 2022, based on the results of the above survey. Clearly state your conclusion **and** give a reason for your conclusion.

Conclusion:

Reason:

(c) Assume that 20% of adults in Ireland jog for recreation.
Three adults are picked at random.
Find the probability that exactly one of these adults jogs for recreation.

This question continues on the next page.

Sinéad is joining a gym. She will be able to use the gym, and also go to classes.

(d) She could pay for individual classes, at €6 per class. Sinéad estimates that there is:

- a 30% chance she will go to no classes in the year,
- a 60% chance she will go to 1 class a week, so 52 classes in a year, and
- a 10% chance she will go to 2 classes a week, so 104 classes a year.

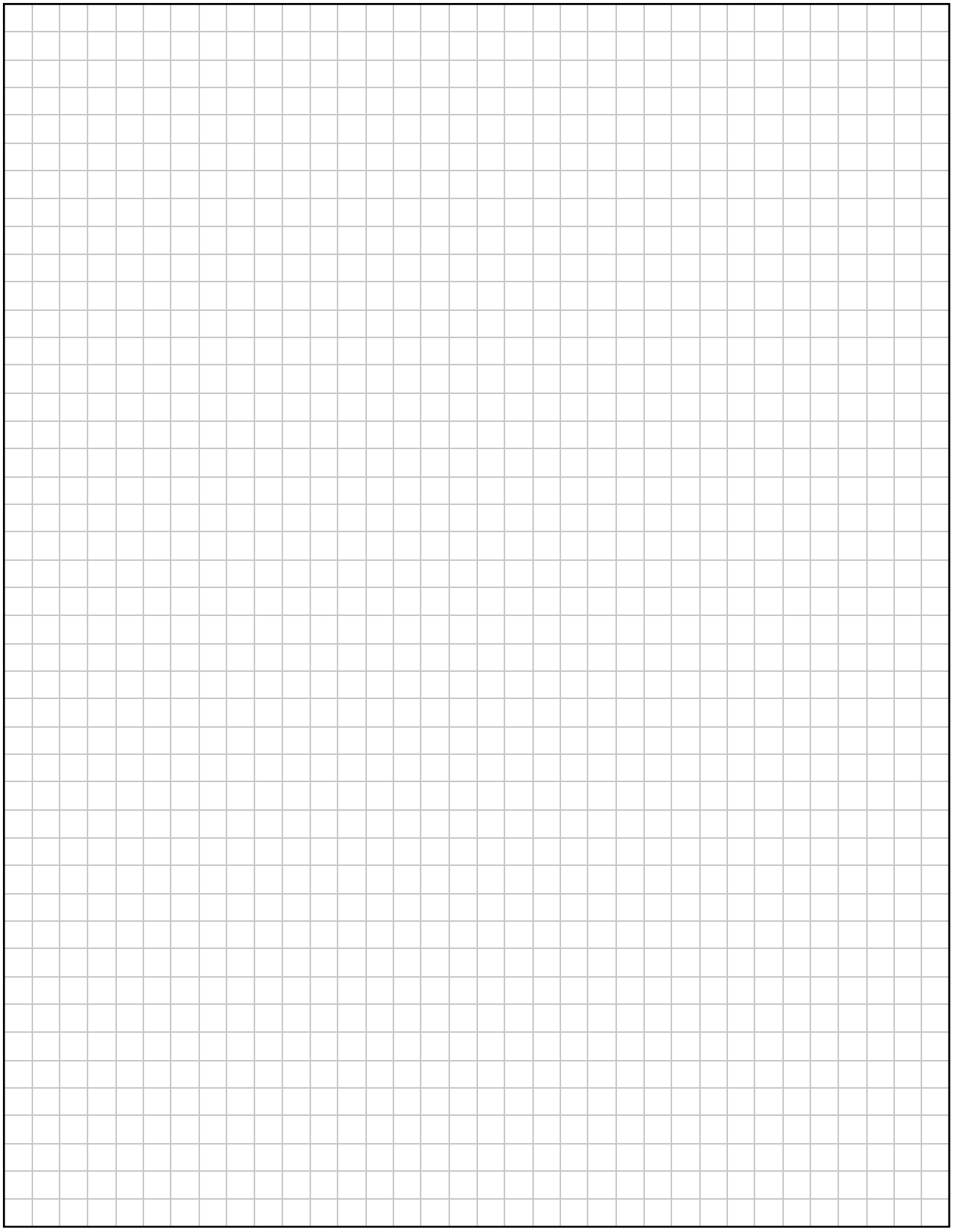
Use these figures to work out the expected value of the cost of the classes for a year.

(e) There are two price plans for the gym, **Silver** and **Gold**, as follows:

Silver	Gold
€420 for the year. Pay €6 for each class.	€670 for the year. All classes are free.

Work out the least number of classes that Sinéad would have to go to in a year, so that the **Gold** plan would cost less than the **Silver** plan.

Page for extra work.
Label any extra work clearly with the question number and part.



Do not write on this page

Leaving Certificate – Ordinary Level

Mathematics Paper 2

Monday 13 June

Morning 9:30 - 12:00