Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2023
Deferred Examinations

Marking Scheme

Mathematics

Higher Level

v2

Note to teachers and students on the marking schemes for the deferred examinations

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. However, it should be noted that the marking schemes for the deferred examinations may not necessarily be as detailed as the corresponding marking schemes for the main sitting of an examination, which serve to ensure consistency across a large team of examiners.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination, and the need to maintain consistency in standards between the main sitting and the deferred sitting and from year to year. In the case of the deferred examinations, this means that the level of detail may vary by question, as the marking scheme will only have been finalised for the questions attempted by the candidates who sat these examinations.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with a senior examiner when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes (whether for the main examinations or the deferred examinations) should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination concerned. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination will not necessarily be the same for the deferred sitting as for the main sitting or from one year to the next.

Leaving Certificate Examination

Deferred Exam 2023

Mathematics

Higher Level

Paper 1

Marking Scheme

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scales		0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10 mark scales		0, 5, 10	0, 4, 7, 10	0, 3, 5, 8, 10
15 mark scales			0, 6, 12, 15	0, 4, 8, 12, 15
20 mark scales				0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

NOTE: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.
Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A		***Section B***	
Question 1		Question 7	
(a)	15D	(a)	20D
(b)(i)	10C	(b)	10D
(b)(ii)	5D	(c)	5C
		(d)(i)	5C
		(d)(ii)	5C
		(e)	5D
Question 2		Question 8	
(a)	10D	(a)(i)	10C
(b)(i)	10B	(a)(ii)	10C
(b)(ii)	5C	(a)(iii)	10D
(b)(iii)	5D	(a)(iv)	5C
		(b)(i)	10D
		(b)(ii)	5D
Question 3		Question 9	
(a)	15D	(a)(i) + (a)(ii)	15C
(b)	10C	(a)(iii)	5C
(c)	5D	(a) (iv)	5C
		(a)(v)	10C
		(b) (i)	5C
		(b)(ii)	5C
		(b)(iii)	5B
Question 4		Question 10	
(a)	10D	(a)(i)	10C
(b) (i)	5C	(a)(ii)	10C
(b)(ii)	15D	(a)(iii)	10D
		(b)	5D
		(c)(i)	10C
		(c)(ii)	5D
Question 5			
(a)(i) + (a)(ii)	15D		
(a)(iii)	10D		
(b)	5D		
Question 6			
(a)	15D		
(b)(i)	10D		
(b)(ii)	5D		

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	$\begin{cases} x + y = 2 \\ y + 2z = 12 \end{cases}$ $x + 2y + 2z = 14$ $\begin{cases} x + 2y + 2z = 14 \\ -10x - 14y - 2z = -26 \end{cases}$ $\begin{cases} 9x + 12y = 12 \\ 9x + 9y = 18 \end{cases}$ $3y = -6$ $\Rightarrow y = -2$ $x - 2 = 2$ $\Rightarrow x = 4$ $(-2) + 2z = 12$ $2z = 14$ $z = 7$ (4, –2, 7)	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit*: • Effort to eliminate one variable *Mid Partial Credit*: • 2 equations in 2 unknowns *High Partial Credit*: • 2 unknowns found

(b) (i)	Third term = $\binom{10}{2}\left(\frac{k}{x}\right)^8 (x)^2 = \frac{45k^8}{x^6}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit*: • $\binom{10}{2}$ or x^2 or $\left(\frac{k}{x}\right)^8$ • Effort at binomial term *High Partial Credit*: • T_4 found correctly *Full Credit -1* • 3rd term found but in decreasing powers of x **Note**: Accept $\binom{10}{2}\left(\frac{k}{x}\right)^8 (x)^2$
(b) (ii)	$120k^3 = 7680$ $k^3 = \frac{7680}{120}$ $k^3 = 64$ $k = 4$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit*: • Effort at binomial term • $\left(\frac{k}{x}\right)^{10-n} (x)^n$ or similar *Mid Partial Credit*: • T_7 identified *High Partial Credit*: • T_7 formulated correctly • $120k^3$

Q2	Model Solution – 30 Marks	Marking Notes
(a)	$(3x + 1)(x - 1) \leq 6(x - 1)^2$ $3x^2 - 2x - 1 \leq 6(x^2 - 2x + 1)$ $3x^2 - 2x - 1 \leq 6x^2 - 12x + 6$ $-3x^2 + 10x - 7 \leq 0$ $3x^2 - 10x + 7 \geq 0$ $(3x - 7)(x - 1) \geq 0$ $\{x < 1\} \cup \left\{x \geq \frac{7}{3}\right\}$	**Scale 10D(0, 3, 5, 8, 10)** *Low Partial Credit*: • Use of $(x - 1)^2$ • One correct linear inequality *Mid Partial Credit*: • Multiplication completed correctly • 2 correct linear inequalities • One correct linear inequality solved *High Partial Credit*: • $3x^2 - 10x + 7$
(b) (ii)	$a = 4$ $b = 3$	**Scale 10B (0, 5, 10)** *Partial Credit*: • a **or** b correct
(b) (ii)	$(\log_2 3)\left(\frac{\log_2 4}{\log_2 3}\right)$ $= \log_2 4$ $= 2$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit*: • $\frac{\log_2 4}{\log_2 3}$ • some work with logs *High Partial Credit*: • $(\log_2 3)\left(\frac{\log_2 4}{\log_2 3}\right)$ *Full credit* –1: • $\log_2 4$
(b) (iii)	$(\log_2 3) \cdots \frac{\log_2 n + 1}{\log_2 n} = 11$ $\log_2 n + 1 = 11$ $n = 2048 - 1$ $n + 1 = 2^{11}$ $n = 2047$ **or** $n = 2^{11} - 1$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit*: • One term written in terms of $\log_2$ *Mid Partial Credit*: • *LHS* of expression written in terms of $\log_2$ *High Partial Credit*: • $\log_2(n + 1) = 11$

Q3	Model Solution – 30 Marks	Marking Notes
(a)	$z_1 = \frac{25(3-i)(7+i)}{(7-i)(7+i)}$ $= \frac{25(22-4i)}{50}$ $z_1 = 11 - 2i$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit*: • $(7+i)$ • Some correct multiplication *Mid Partial Credit*: • $\frac{25(3-i)(7+i)}{(7-i)(7+i)}$ *High Partial Credit*: • $\frac{25(22-4i)}{50}$
(b)	$z_1 + \overline{z_1} = -a \quad z_1\overline{z_1} = b$ $4 = -a \quad (2-i)(2+i) = b$ $a = -4 \quad b = 5$ **or** $(z-2+i)(z-2-i)$ $= z^2 - 4z + 5$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit*: • Mention of sum of roots **or** product of roots • Mentions conjugate • A factor written • Some correct substitution *High Partial Credit*: • a or b identified • $(2+i)$ /$(2-i)$ substituted and multiplied correctly • Some multiplication of correct factors • Two relevant equations in a and b

(c)	$z = \left(4 + 4\sqrt{3}i\right)^{\frac{1}{3}}$ $r = \sqrt{16 + 48} = 8$ $\tan\theta = \sqrt{3}$ $\theta = \frac{\pi}{3}$ $\left(4 + \sqrt{3}i\right)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left(\cos\left(\frac{\pi}{3} + 2n\pi\right) + i\sin\left(\frac{\pi}{3} + 2n\pi\right)\right)$ $n = 0$ $z_1 = 2\left(\cos\left(\frac{\pi}{9}\right) + i\sin\left(\frac{\pi}{9}\right)\right)$ $n = 1$ $z_2 = 2\left(\cos\left(\frac{7\pi}{9}\right) + i\sin\left(\frac{7\pi}{9}\right)\right)$ $n = 2$ $z_3 = 2\left(\cos\left(\frac{13\pi}{9}\right) + i\sin\left(\frac{13\pi}{9}\right)\right)$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit*: • $\sqrt{16 + 48}$ • $\tan\theta = \sqrt{3}$ • Relevant diagram drawn *Mid Partial Credit*: • r **or** θ found *High Partial Credit*: • One solution in required form

Q4	Model Solution – 30 Marks	Marking Notes
(a)	Test for n = 1 P(1): $13^1 - 1$ = 12 which is divisible by 12 Assume true for $n = k$ P(k): $13^k - 1$ is divisible by 12 Prove true for $n = k + 1$ P($k+1$): $13^{k+1} - 1$ is divisible by 12 $13^{k+1} - 1 = 13^k(13) - 1$ $= 13(13^k - 1) + 12$ Is divisible by 12 true for n = 1 and true for $n = k + 1$ if true for $n = k$ ⇒ by induction true for all $n \in \mathbb{N}$	**Scale 10D (0, 3, 5, 8, 10)** Note: Accept *Step* $P(1)$*, Step* $P(k)$*, Step* $P(k+1)$ in any order *Low Partial Credit:* • Step $P(1)$ *Mid Partial Credit:* • *Step* $P(k)$ or Step $P(k+1)$ *High Partial Credit:* • Uses *Step* $P(k)$ to prove *Step* $P(k+1)$ *Full credit* –1: • Omits conclusion but otherwise correct

(b) (i)	$f(k) = k^3 - (k^2 + 1)k + k$ $= k^3 - k^2 - k + k$ $= 0$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit*: • Some work of merit, for example, some correct substitution into $f(x)$ *High Partial Credit:* • Fully correct substitution
(b) (ii)	$f(10) = 10^3 - 101(10) + 10 = 0$ $x - k$ is a factor of $f(x)$ $x - 10 \overline{)\, x^3 - 101x + 10}$ $x^2 + 10x - 1$ $x^3 - 10x^2$ $10x^2 - 101x$ $10x^2 - 100x$ $-x + 10$ $-x + 10$ 0 $x = \frac{-10 \pm \sqrt{10^2 + 4}}{2(1)}$ $x = \frac{-10 \pm \sqrt{104}}{2}$ Other roots: $\{-5 + \sqrt{26},\ -5 - \sqrt{26}\}$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit*: • Some correct substitution into the given equation • Division by $x - 10$ formulated *Mid Partial Credit:* • $f(10) = 0$ shown • First cycle of long division correct *High Partial Credit*: • $f(10) = 0$ shown **and** $x^2 + 10x - 1$ *Full credit* –1: • Only 1 of the irrational roots listed but otherwise correct

Q5	Model Solution – 30 Marks	Marking Notes
(a) (i) & (ii)	**(i)** $q - p = \frac{1}{4} - q \quad q = \frac{4p+1}{8}$ $2q = p + \frac{1}{4}$ **(ii)** $\frac{\frac{1}{4}}{p} = \frac{q}{\frac{1}{4}}$ $16qp = 1$ $q = \frac{1}{16p}$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit*: • Some correct substitution into T_n formula *Mid Partial Credit:* • Correct equation in in terms of p, q and d • r in terms of either p or q *High Partial Credit:* • (i) **or** (ii) correct • Correct equations in p and q in **both** (i) and (ii)
(a) (iii)	$\frac{1}{16p} = \frac{4p+1}{8}$ $16p(4p+1) = 8$ $8p^2 + 2p - 1 = 0$ $(4p-1)(2p+1) = 0$ $p = \frac{1}{4}$ or $-\frac{1}{2}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit*: • Some use of previous answers *Mid Partial Credit*: • $\frac{1}{16p} = \frac{4p+1}{8}$ *High Partial Credit*: • $8p^2 + 2p - 1 = 0$
(b)	**(a):** $u_{n+1} = (n+1)(n-1)!$ **(b):** $(n-1)u_n + (n-1)!$ $= (n-1)n\,(n-2)! + (n-1)!$ $= (n-1)!n + (n-1)!$ $= (n-1)!\,(n+1)$ $= u_{n+1}$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit*: • Effort at u_{n+1} from $n\,(n-2)!$ • Effort at u_{n+1} from $(n-1)u_n + (n-1)!$ • Finds u_n for some $n \geq 2$ • Finds u_{n+1} in terms of u_n for some $n \geq 2$ *Mid Partial Credit*: • $u_{n+1} = (n+1)(n-1)!$ from **(a)** • Proves true for a particular case *High Partial Credit*: • $u_{n+1} = (n+1)(n-1)!$ from **(b)**

Q6	Model Solution – 30 Marks	Marking Notes
(a)	$g'(x) = 12x^2 - 80x + 77 = 0$ $x = \frac{80 \pm \sqrt{80^2 - 4(12)(77)}}{2(12)}$ $x = \frac{80 \pm 52}{24}$ $x = \frac{11}{2}$ or $\frac{7}{6}$ $g: x \mapsto 4x^3 - 40x^2 + 77x$ $g\left(\frac{11}{2}\right) = 4\left(\frac{11}{2}\right)^3 - 40\left(\frac{11}{2}\right)^2 + 77\left(\frac{11}{2}\right)$ $= -121$ $g''(x) = 24x - 80 = 0$ $x = \frac{80}{24} = \frac{10}{3}$ $g\left(\frac{10}{3}\right) = 4\left(\frac{10}{3}\right)^3 - 40\left(\frac{10}{3}\right)^2 + 77\left(\frac{10}{3}\right)$ $= \frac{-1070}{27}$ Local minimum: $\left(\frac{11}{2}, -121\right)$ Point of inflection: $\left(\frac{10}{3}, \frac{-1070}{27}\right)$	**Scale 15D (0, 5, 8, 12, 15)** *Low Partial Credit:* • Any correct differentiation *Mid Partial Credit:* • $g'(x)$ and $g''(x)$ found • $g'(x) = 0$ solved • $g''(x) = 0$ solved *High Partial Credit:* • $g'(x) = 0$ and $g''(x) = 0$ solved • Minimum Point **or** inflection point found

(b) (i)	$f(a) = \left[\frac{x^3}{3} - \frac{2ax^2}{2} + a^2x\right]_0^1$ $f(5) = \int_0^1 x^2 - 10x + 25\,dx$ $\left[\frac{x^3}{3} - \frac{10x^2}{2} + 25x\right]_0^1 = \frac{61}{3}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit*: • Any correct integration • Any correct substitution *Mid Partial Credit:* • Fully correct substitution • Fully correct integration *High Partial Credit*: • $\frac{x^3}{3} - \frac{10x^2}{2} + 25x$
(b) (ii)	$f(a) = \int_0^1 x^2 - 2ax + a^2\,dx$ $\left[\frac{1}{3}x^3 - ax^2 + a^2x\right]_0^1$ $\frac{1}{3} - a + a^2$ $f'(a) = -1 + 2a$ $-1 + 2a = 0 \quad a = \frac{1}{2}$ $f\left(\frac{1}{2}\right) = \frac{1}{3} - \frac{1}{2} + \left(\frac{1}{2}\right)^2$ $= \frac{1}{3} - \frac{1}{2} + \frac{1}{4} = \frac{4-6+3}{12} = \frac{1}{12}$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit*: • Some correct integration Some correct substitution of both limits • Mentions derivative or $f'(a)$ *Mid Partial Credit*: • Fully correct integration *High Partial Credit*: • $a = \frac{1}{2}$

Q7	Model Solution – 50 Marks	Marking Notes
(a)	<table><tr><th rowspan="2">Pay No</th><th rowspan="2">Monthly Payment</th><th colspan="2">€389·56</th><th rowspan="2">Balance (€)</th></tr><tr><th>Int</th><th>Reduced</th></tr><tr><td>0</td><td></td><td></td><td></td><td>20 000</td></tr><tr><td>1</td><td>389·56</td><td>105·20</td><td>284·36</td><td>19715·64</td></tr><tr><td>2</td><td>389·56</td><td>103·7</td><td>285·86</td><td>19429·78</td></tr><tr><td>3</td><td>389·56</td><td>102·2</td><td>287·36</td><td>19142·42</td></tr><tr><td>4</td><td>389·56</td><td>100·69</td><td>288·87</td><td>18853·55</td></tr></table>	**Scale 20D (0, 5, 10, 15, 20)** *Low Partial Credit*: • One correct (relevant) new entry in table *Mid Partial Credit*: • 3 correct new entries *High Partial Credit*: • 6 correct new entries
(b)	$L = \frac{389{\cdot}56}{1{\cdot}00526} + \frac{389{\cdot}56}{1{\cdot}00526^2} + \cdots + \frac{389{\cdot}56}{1{\cdot}00526^{35}} + \frac{389{\cdot}56}{1{\cdot}00526^{36}}$ $L = \frac{\frac{389{\cdot}56}{1{\cdot}00526}\left[1 - \left(\frac{1}{1{\cdot}00526}\right)^{36}\right]}{1 - \frac{1}{1{\cdot}00526}}$ $= €12\,745{\cdot}92$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit*: • $0{\cdot}526\%$ converted to a decimal • One correct present value, for example, $\frac{389{\cdot}56}{1{\cdot}00526}$ *Mid Partial Credit*: • Series correctly formulated *High Partial Credit*: • S_n formula fully substituted
(c)	$(1 + 0{\cdot}00526)^{12} = 1{\cdot}065$ rate = 6·5%	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $0{\cdot}00526$ **or** 12 *High Partial Credit:* • 0·065 • Formula fully substituted *Full Credit -1* • 0·1
(d) **(i)**	$(1 + i)^{12} = 1{\cdot}0553$ $i = \sqrt[12]{1{\cdot}0553} = 1{\cdot}004495498$ $i = 0{\cdot}45\%$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $1{\cdot}0553$ *High Partial Credit:* • Formula fully substituted *Full Credit -1* • $\sqrt[12]{1{\cdot}0553} - 1$

(d) (ii)	$A = P\ \dfrac{i(1+i)^t}{(1+i)^t - 1}$ $A = \dfrac{8761{\cdot}77(0{\cdot}0045)(1{\cdot}0045)^{12}}{1{\cdot}0045^{12} - 1}$ $A = €751{\cdot}68$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit*: • Some relevant substitution into correct formula e.g. $0{\cdot}0045$ **or** 12 **or** $8761{\cdot}77$ • Some relevant substitution into present value formula *High Partial Credit*: • Formula fully substituted
(e)	$2A = A(1{\cdot}0675)^t$ $2 = 1{\cdot}0675^t$ $t = \dfrac{\log 2}{\log 1{\cdot}0675} = 10{\cdot}61164 = 11$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit*: Note: Accept correct answer verified • 6.75% written as a decimal • Some correct substitution into the future value formula • $1{\cdot}0675$ or $(x)^t$ *Mid Partial Credit*: • The value in terms of A and t of the investment after t years • $2 = 1{\cdot}0675^t$ *High Partial Credit*: • Formula in t (without indices)

Q8	Model Solution – 50 Marks	Marking Notes
(a) (i)	$A = \frac{1}{2}(m)(l)$ $= \frac{1}{2}ml$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit*: • Some correct substitution into a relevant formula • Recognition of 90° angle in semi-circle • $A = \frac{1}{2}(d)(h)$ *High Partial Credit:* • Correct area in terms of l and m and some other unknown, for example, $A = \frac{1}{2}ml\sin C$
(a) (ii)	$P = l + m + d$ $l + m = P - d$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit*: • Some understanding of perimeter • Some relevant substitution *High Partial Credit* • $P = l + m + d$
(a) (iii)	**(a):** $l^2 + m^2 = d^2$ **(b):** $l^2 + m^2 = (l + m)^2 - 2lm$ $= (P - d)^2 - 4A$ $= P^2 - 2Pd + d^2 - 4A$ $d^2 = P^2 - 2Pd + d^2 - 4A$ $2Pd = P^2 - 4A$ $\Rightarrow d = \frac{P^2 - 4A}{2P}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit*: • Some use of Pythagoras • Some use of given identity *Mid Partial Credit*: • Given identity fully substituted and in terms of P, d and a • $l^2 + m^2 = d^2$ • d written in terms of P, l and m *High Partial Credit*: • $d^2 = P^2 - 2Pd + d^2 - 4A$
(a) (iv)	$d = \frac{P^2 - 4A}{2P}$ $d = \frac{60^2 - 4(120)}{2(60)}$ $d = 26$ Radius $= 13$ cm	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Any relevant substitution *High Partial Credit:* • Formula fully substituted *Full Credit -1* • $d = 26$ and stops

(b) (i)	Area $= \frac{1}{2}r^2\beta = 16$ $2r + r\beta = 20$ $r = \frac{20}{\beta + 2}$ $\beta = \frac{20 - 2r}{r}$ $\frac{1}{2}r^2\left(\frac{20 - 2r}{r}\right) = 16$ $20r - 2r^2 = 32$ $2r^2 - 20r + 32 = 0$ $r^2 - 10r + 16 = 0$ $(r - 2)(r - 8) = 0$ $r = 2$ **or** $r = 8$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Relevant substitution into area of sector formula • Relevant substitution into perimeter of sector formula *Mid Partial Credit:* • solved correctly without being established from given information. • $r^2 - 10r + 16 = 0$ solved correctly without being established from the given equation • $\frac{1}{2}r^2\beta = 16$ **or** • $2r + r\beta = 20$ *High Partial Credit:* • Any two from the MPC list • $2r^2 - 20r + 32 = 0$
(b) (ii)	$r = 2 \Rightarrow \beta = \frac{20-4}{r} = 8,$ $r = 8 \Rightarrow \beta = \frac{20-16}{r} = \frac{1}{2}$ $\beta = \frac{1}{2}$ 8 radians is not valid as it is more than 1 full circle	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Effort to find β from either root *Mid Partial Credit:* • One value of β found *High Partial Credit:* • Both values of β found but no reason

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i) & (ii)	**(i)**: $N(0) = \frac{3000}{1+6{\cdot}5e^{\circ}}$ $= 400$ **(ii)**: $N(10) = \frac{3000}{1 + 6{\cdot}5e^{-8{\cdot}14\times10}} = 2994$	**Scale 15C (0, 6, 12, 15)** *Low Partial Credit:* • Some correct substitution into function *High Partial Credit:* • (i) or (ii) correct
(a) (iii)	$\lim_{t\to\infty} N(t) = \lim_{t\to\infty} \frac{3000}{1 + 6{\cdot}5e^{-0{\cdot}814t}}$ $= 3000$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $t \to \infty$ • Attempts at showing the limit by substituting at least 2 values for n *High Partial Credit:* • $e^{-\infty} = 0$ (stated or implied)
(a) (iv)	$\frac{3000}{1 + 6{\cdot}5e^{-0{\cdot}814n}} > 2650$ $3000 > 2650 + 17225e^{-0{\cdot}814n}$ $\frac{350}{17225} > e^{-0{\cdot}814n}$ $\ln\frac{350}{17225} > -0{\cdot}814n \ln e$ $\frac{-3{\cdot}896183941}{-0{\cdot}814} < n$ $n > 4{\cdot}7864$ $n = 5$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $\frac{3000}{1+6{\cdot}5e^{-0{\cdot}814n}} >$ (**or** $=$) 2650 *Mid Partial Credit:* • $\frac{350}{17225} > e^{-0{\cdot}814n}$ *High Partial Credit:* $\ln\frac{350}{17225} > -0{\cdot}814n \ln e$ • $n = 4{\cdot}7864$ *Full Credit -1:* $n > 4{\cdot}786$ …

(a) **(v)**	$N(t) = \dfrac{3000}{1 + 6{\cdot}5e^{-0{\cdot}814t}}$ $N'(t) = \dfrac{(1 + 6{\cdot}5e^{-0{\cdot}814t})(0) - 3000\{-5{\cdot}291e^{-0{\cdot}814t}\}}{(1 + 6{\cdot}5e^{-0{\cdot}814t})^2}$ $N'(t) = \dfrac{15873e^{-0{\cdot}814t}}{(1 + 6{\cdot}5e^{-0{\cdot}814t})^2}$ **or** $N'(t) = 3000(1 + 6{\cdot}5e^{-0{\cdot}814t})^{-2} \times 5{\cdot}291e^{-0{\cdot}814t}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Any correct differentiation *High Partial Credit:* • Quotient rule correctly applied but with one error • Chain rule correctly applied but with one error

(b) (i)

t	0	1	2	3	4	5	6	7	8
$P(t)$	400	1312	1904	2289	2538	2700	2805	2874	2918

Scale 5C (0, 2, 3, 5)

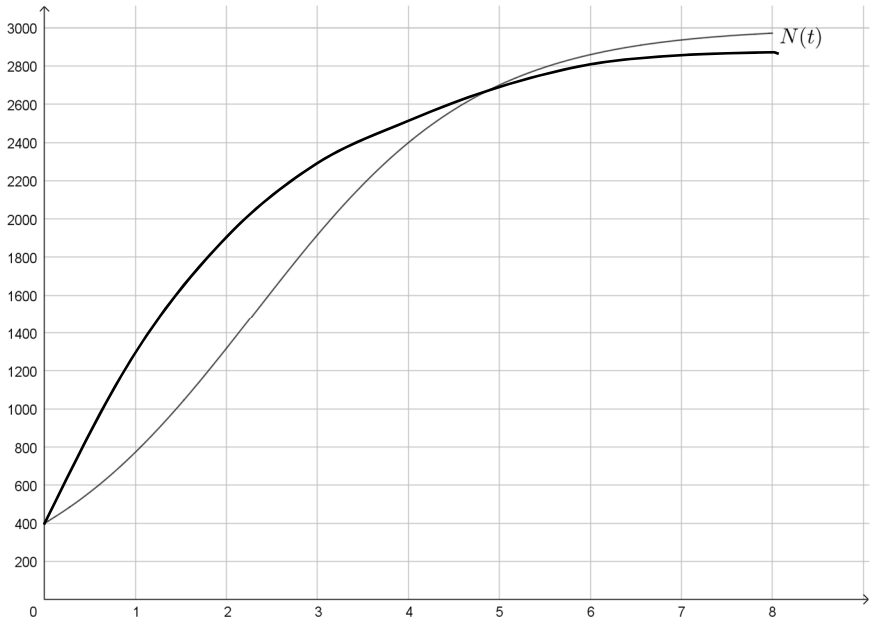
Low Partial Credit:

- One correct new entry

High Partial Credit:

- *4 correct new entries*

(b) (ii)



Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- *One correct new plot*

High Partial Credit:

- *4 correct new plots*

(b) (iii)

Similarity: both models have a positive rate of increase
Difference: $P(t)$ increases at a greater rate than $N(t)$ $0 < t < 5$
While $N(t)$ increases at a greater rate than $P(t)$, $t > 5$

Scale 5B (0, 2, 5)

Partial Credit:

- One correct similarity **or** difference

Q10	Model Solution – 50 Marks	Marking Notes
(a) (i)	$\|BD\|^2 = 25^2 + 6^2 = 661$ $\|BD\| = \sqrt{661} = 25{\cdot}71$ m	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Recognises Pythagoras *High Partial Credit:* • Pythagoras fully substituted
(a) (ii)	$\|BE\| = \sqrt{x^2 + 49}$ $\|ED\| = \sqrt{13^2 + (25 - x)^2}$ $l = \sqrt{x^2 + 49} + \sqrt{x^2 - 50x + 794}.$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Some correct substitution into Pythagoras *High Partial Credit:* • $\|\boldsymbol{BE}\|$ **or** $\|\boldsymbol{ED}\|$ correct
(a) (iii)	$\|BD\|^2 = \|BE\|^2 + \|ED\|^2$ $661 = x^2 + 49 + x^2 - 50x + 794$ $2x^2 - 50x + 182 = 0$ $x^2 - 25x + 91 = 0$ $x = \frac{25 \pm \sqrt{625 - 364}}{2} = \frac{25 \pm \sqrt{261}}{2}$ $x = 20{\cdot}58$ or $4{\cdot}42$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • $x^2 + 49$ **or** $x^2 - 50x + 794$ **or** 661, • $\|BD\|^2 = \|BE\|^2 + \|ED\|^2$ *Mid Partial Credit:* • $661 = x^2 + 49 + x^2 - 50x + 794$ *High Partial Credit:* • $2x^2 - 50x + 182 = 0$

(b)	$\lvert BE\rvert^2 = 7^2 + x^2$ $\lvert BE\rvert = \sqrt{49 + x^2} = (49 + x^2)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}\left((49 + x^2)^{-\frac{1}{2}}\right) 2x$ $= \frac{x}{\sqrt{49 + x^2}}$ $\frac{x}{\sqrt{49 + x^2}} = \frac{dy}{dt} \cdot \frac{1}{4}$ $\frac{dy}{dt} = \frac{4x}{\sqrt{49 + x^2}}$ When $x = 7$, $\frac{dy}{dt} = \frac{28}{\sqrt{98}} = 2{\cdot}8$ m/s	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Any correct differentiation • Any relevant derivative • $x = 7$ • $\frac{dx}{dt} = 4$ • $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ *Mid Partial Credit:* • $\frac{dy}{dx} = \frac{1}{2}\left((49 + x^2)^{-\frac{1}{2}}\right) 2x$ • Any two of ($x = 7$, $\frac{dx}{dt} = 4$, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$) *High Partial Credit:* • $\boldsymbol{\frac{dy}{dx}}$ correctly substituted
(c) **(i)**	$x^2 + y^2 = 12^2$ $x^2 = 144 - y^2$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Recognises Pythagoras *High Partial Credit:* • $x^2 + y^2 = 12^2$ (or equivalent)
(c) **(ii)**	$V = \pi x^2 2y$ $V = \pi(144 - y^2)2y$ $V = \pi(288y - 2y^3)$ $\frac{dv}{dy} = 288\pi - 6\pi y^2 = 0$ $6y^2 = 288$ $y = 4\sqrt{3}$ $V = \pi\left(288\left(4\sqrt{3}\right) - 2(4\sqrt{3})^3\right)$ $V = 768\pi\sqrt{3}$ $V = 4179$ cm^3	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $V = \pi x^2 2y$ • Some correct substitution into volume of cylinder formula *Mid Partial Credit:* • $V = \pi(144 - y^2)2y$ *High Partial Credit:* • $y = 4\sqrt{3}$ *Full Credit -1:* • $V = 768\pi\sqrt{3}$

Leaving Certificate Examination

Deferred Exam 2023

Mathematics

Higher Level

Paper 2

Marking Scheme

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	B	C	D
No of categories	3	4	5
5 mark scales	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10 mark scales	0, 5, 10	0, 4, 7, 10	0, 3, 5, 8, 10
15 mark scales		0, 5, 10, 15	0, 4, 8, 12, 15
20 mark scales			0, 6, 12, 17, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

Summary of mark allocations and scales to be applied

Section A

Question 1	
(a)	10C
(b)	15C
(c)	5D
Question 2	
(a)(i)	10C
(a)(ii)	15D
(b)	5C
Question 3	
(a)	15C
(b)	5D
(C)	5D
(d)	5D
Question 4	
(a)(i) &(a)(ii)	20D
(b)(i)	5C
(b)(ii)	5C
Question 5	
(a)(i)	10C
(a)(ii)	10C
(b)	5C
(c)	5C
Question 6	
(a)(i)	10B
(a)(ii)	5C
(a)(iii)	5C
(b)(i)	5B
(b)(ii)	5C

Section B

Question 7	
(a)(i)	5B
(a)(ii)	15C
(a)(iii)	10D
(a)(iv)	5D
(a)(v)	5C
(a)(vi)	5D
(b)	5C
Question 8	
(a)(i)	5C
(a)(ii)	10C
(a)(iii)	15D
(b)(i)	10D
(b)(ii)	5D
(b)(iii)	5D
Question 9	
(a)	20D
(b)	5D
(c)(i)	10C
(c) (ii)	20D
(d)(iii)	5C
Question 10	
(a)(i)	10C
(a)(ii)	10D
(a)(iii)	5D
(b)(i)	10C
(b)(ii)	5D
(c)	10D

Detailed marking notes

Model Solutions & Marking Notes

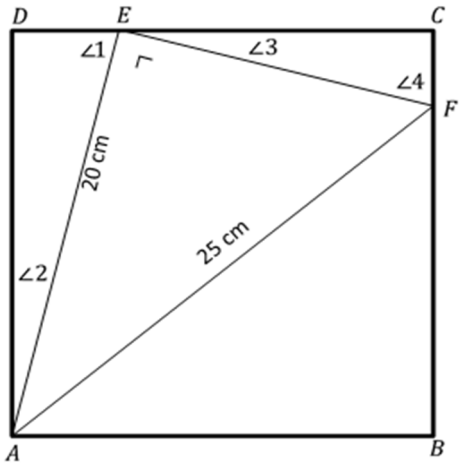
Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	Midpoint of $AB = (\frac{7}{2}, \frac{5}{2})$ Slope of $AB = \frac{2-3}{0-7} = \frac{-1}{-7} = \frac{1}{7}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Some correct substitution into either relevant formula *High Partial Credit:* • Midpoint or slope correct. • Work of merit in both parts
(b)	Perp. slope $= -7$ Eq. of perp: $y - \frac{5}{2} = -7\,(x - \frac{7}{2})$ $y - \frac{5}{2} = -7x + \frac{49}{2}$ $7x + y - 27 = 0$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* • Some correct substitution into line formula • Slope of perpendicular *High Partial Credit:* • Formula fully substituted.

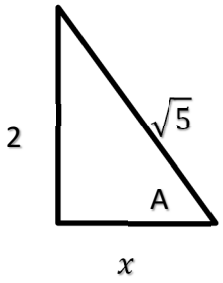
(c)	Midpoint of $CB = (2, -2)$ Slope of $CB = \frac{2+6}{0-4} = \frac{8}{-4} = -2$ $\Longrightarrow$ Perp slope $= \frac{1}{2}$ $\Longrightarrow$ Eq of perp: $y + 2 = \frac{1}{2}(x - 2)$ $\Longrightarrow 2y + 4 = (x - 2)$ $6 = x - 2y$ $7x + y = 27$ $14x + 2y = 54$ $x - 2y = 6$ $15x = 60$ $\Longrightarrow x = 4,$ $\Longrightarrow y = -1$ $\therefore$ Centre $(4, -1)$.	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit towards finding equation of perpendicular bisector of *BC* or *AC*, for example, midpoint *BC*, slope *BC* or equivalent *Mid Partial Credit:* • Equation of perpendicular bisector fully substituted. *High Partial Credit:* • System reduced to 1 variable.

Q2	Model Solution – 30 Marks	Marking Notes
(a) (i)	Centre *O* (5, –1), $R = \sqrt{25 + 1 + 26} = \sqrt{52} = 2\sqrt{13}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit towards finding x ordinate **or** y ordinate of centre *High Partial Credit:* • Centre or radius correct
(a) (ii)	$2x - 3y + c = 0$ radius $= 2\sqrt{13}$ $\Longrightarrow \frac{\lvert 2(5) - 3(-1) + c \rvert}{\sqrt{4+9}} = 2\sqrt{13}$ $\Longrightarrow \lvert 13 + c \rvert = 26$ $\Longrightarrow$ 13 + $c = 26$ **or** 13 + $c = -26$ $\Longrightarrow c = 13$ **or** $c = -39$	**Scale 15D (0, 4, 8, 12, 15)** **3 steps** **1.** $\frac{\lvert 2(5) - 3(-1) + c \rvert}{\sqrt{4+9}}$ **2.** $\frac{\lvert 2(5) - 3(-1) + c \rvert}{\sqrt{4+9}} = 2\sqrt{13}$ **3.** Find values of *c* *Low Partial Credit:* • Work of merit, for example some substitution into relevant formula, or draws diagram with relevant data, (radius, centre marked, and one or both tangents) *Mid Partial Credit* • One step correct. *High Partial Credit:* • 2 steps correct.

(b)	$3x + 4y = 7$ Centre $(h, k) = (\frac{2+8}{2}, k)$ $= (5, k)$ $(5, k) \in 3x + 4y = 7$ $\Longrightarrow 15 + 4k = 7$ $\Longrightarrow 4k = 8$ $k = -2$ $\therefore$ Centre $(5, -2)$, R $= 3$ $\Longrightarrow$ Equation: $(x-5)^2 + (y+2)^2 = 9$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit towards finding radius or centre of circle *High Partial Credit:* • $h = 5$ and radius $= 3$ • Relevant equation in k (only) and radius $= 3$

Q3	Model Solution – 30 Marks	Marking Notes
(a)	Pythagoras $\Longrightarrow \ \|EF\|^2 = 25^2 - 20^2$ $\|EF\|^2 = 225$ $\Longrightarrow \ \|EF\| = 15$ cm	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* • $\|EF\|^2$ **or** 25^2 **or** 20^2 *High Partial Credit:* • Pythagoras fully substituted. *Full Credit -1:* • No units or incorrect units
(b)	Mark angles 1, 2, 3 and 4  $90^o = 90^o$ **(i)** ΔAED and ΔEFC are right angled $\|\angle 1\| + \ \|\angle 2\| + 90^o = 180°$ (3 angles in Δ add to 180^o) $\|\angle 1\| + \ \|\angle 2\| = 90^o$ $\|\angle 1\| + 90° + \ \|\angle 3\| = 180^o$ (straight angle) $\Longrightarrow \ \|\angle 1\| + \|\angle 2\| = \ \|\angle 1\| + \ \|\angle 3\|$ $\Longrightarrow \ \|\angle 2\| = \ \|\angle 3\|$ **(ii)** Both ΔAED and ΔEFC are right angled and $\|\angle 2\| = \|\angle 3\|$ $\Longrightarrow$ remaining angles are equal i.e. $\|\angle 1\| = \|\angle 4\|$ **(iii)** $\Longrightarrow \ \Delta AED$ and ΔEFC are similar	**Scale 5D (0, 2, 3, 4, 5)** Consider the solution as having 4 elements: 3 steps and a conclusion: **Steps 1, 2 & 3:** 3 correct statements for similar triangles (with justifications) **and** a conclusion *Low Partial Credit:* • Work of merit, for example, $\|\angle EDA\| = \|\angle ECF\|$ indicated *Mid Partial Credit:* • 1 correct step with justification • 2 correct steps (no justifications) *High Partial Credit:* • 3 correct steps with at least one justification • All 3 steps correct and conclusion stated (no justifications)

(c)	ΔADE is right angled $\Longrightarrow 20^2 = \lvert AD\rvert^2 + \lvert ED\rvert^2$ $\Longrightarrow 400 = \lvert AD\rvert^2 + \frac{\lvert AD\rvert^2}{16} = \frac{17\lvert AD\rvert^2}{16}$ $\Longrightarrow \frac{6400}{17} = \lvert AD\rvert^2 \Longrightarrow \lvert AD\rvert = \frac{80}{\sqrt{17}}$ cm	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit, for example $20^2 = \lvert AD\rvert^2 + \lvert ED\rvert^2$ *Mid Partial Credit:* • $20^2 = \lvert AD\rvert^2 + (\frac{\lvert AD\rvert}{4})^2$ *High Partial Credit:* • $400 = \frac{17\lvert AD\rvert^2}{16}$ *Full Credit -1:* • No units or incorrect units
(d)	$\frac{\lvert CF\rvert}{\lvert DE\rvert} = \frac{15}{20} = \frac{3}{4} \quad \Longrightarrow \lvert CF\rvert = \frac{3}{4}\lvert DE\rvert$ $\Longrightarrow \lvert CF\rvert = \frac{3}{4}\left(\frac{1}{4}\lvert AD\rvert\right) = \frac{3}{16}\lvert AD\rvert$ $\Longrightarrow \lvert FB\rvert = \frac{13}{16}\lvert AD\rvert$ $\Longrightarrow$ Area $\Delta AFB = \frac{1}{2}\lvert AD\rvert\left(\frac{13}{16}\lvert AD\rvert\right)$ $= \frac{13}{32}\lvert AD\rvert^2 = \frac{13}{32}\left(\frac{6400}{17}\right) = \frac{2600}{17}$ [cm^2 ***or*** $\lvert AD\rvert^2 = \frac{6400}{17}$ Area $\Delta AFE = \frac{1}{2}(20 \times 15) = 150$ Area $\Delta AED = \frac{1}{2}\lvert AD\rvert \times \left(\frac{1}{4}\lvert AD\rvert\right) = \frac{800}{17}$ $\lvert CF\rvert^2 = 15^2 - \left(\frac{3}{4} \times \frac{80}{\sqrt{17}}\right)^2 = \frac{225}{17}$ Area $\Delta EFC = \frac{1}{2}\left(\frac{3}{4}\left\lvert\frac{80}{\sqrt{17}}\right\rvert \times \left\lvert\frac{15}{\sqrt{17}}\right\rvert\right) = \frac{450}{17}$ $\frac{6400}{17} - \frac{800}{17} - \frac{450}{17} - 150 = \frac{2600}{17}$ [cm^2	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $\lvert CF\rvert = \frac{3}{4}\lvert DE\rvert$ *Mid Partial Credit:* • $\lvert CF\rvert = \frac{3}{16}\lvert AD\rvert$ *High Partial Credit:* • $\frac{1}{2}\lvert AD\rvert\left(\frac{13}{16}\lvert AD\rvert\right)$ **Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Area of 1 relevant shape found. *Mid Partial Credit:* • Area of 2 relevant shapes found. *High Partial Credit:* • Area of 3 relevant shapes found.

Q4	Model Solution – 30 Marks	Marking Notes
(a) (i) & (a) (ii)	cos (A – B) = cos A cos B + sin A sin B **now** cos [90 – (A + B)] = cos [(90 – A) – B] = cos (90 – A) cos B + sin (90 – A) sin B **but** cos(90 – A) = sin A and sin(90 – A) = cos A  $\sqrt{5}^2 = 2^2 + x^2 \Rightarrow x = 1$ sin 2A = 2 sin A cos A $= 2\left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right)$ $= \frac{4}{5}$ cos 2A = $\cos^2 A - \sin^2 A$ $= \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2 = -\frac{3}{5}$	**Scale 20D (0, 6, 12, 17, 20)** Consider the solution as having 3 elements: 1. Proof of trig identity 2. sin 2A 3. cos 2A *Low Partial Credit:* • Work of merit in either a(i) or (a)(ii) for example, cos [90 – (A + B)] in (a)(i) or 1 identified as third side of triangle in (a)(ii) *Mid Partial Credit* • cos (90 – A) cos B + sin (90 – A) sin B • sin 2A or cos 2A found *High Partial Credit* • Proof of (a)(i) and work of merit in both parts of (a)(ii) • sin2A found with work of merit for cos 2A and work of merit in (a)(i) • Any 2 elements completed

b) **(i)**	$\cos \propto = \frac{3}{6} = \frac{1}{2} \Longrightarrow \propto = 60^{o}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Adjacent **or** hypotenuse of $\propto$ identified (3 or 6) *High Partial Credit:* • $\cos \propto = \frac{3}{6}$
(b) **(ii)**	Area of rectangle – Area of sector $= (9 \times 6) - \pi(6)^2 \left(\frac{120}{360}\right)$ $= 54 - 36\pi \left(\frac{1}{3}\right) = (54 - 12\pi)$ cm^2	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Area of rectangle **or** area of **circle** found. *High Partial Credit:* • Area of sector found. • Area of rectangle found with work of merit towards area of sector

Q5	Model Solution – 30 Marks	Marking Notes
(a) (i)	P(All born in same month) $= 1 \times \frac{1}{12} \times \frac{1}{12} \times \frac{1}{12}$ $= \frac{1}{1728} = 0{\cdot}0006$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit, for example, use of 12 *High Partial Credit:* • $\frac{12}{12}$ used/ implied and $\frac{1}{12}$ used *Full Credit -1 :* • No rounding or incorrect rounding
(a) (ii)	P (at least two entered the same day) $= 1 -$ P (all on different days) $= 1 - \frac{30}{30} \times \frac{29}{30} \times \frac{28}{30} \times \frac{27}{30} \times \frac{26}{30}$ $= 1 - \frac{2639}{3750} = \frac{1111}{3750} = 0{\cdot}2963$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • 1 – P (all on different days) • $\frac{x}{30}$ where $x \in \{26, 27, 28, 29, 30\}$ *High Partial Credit:* • $1 - \frac{30}{30} \times \frac{29}{30} \times \frac{28}{30} \times \frac{27}{30} \times \frac{26}{30}$ *Full Credit -1 :* • No rounding or incorrect rounding

(b)	$P(A' \mid B) = \frac{P(A' \cap B)}{P(B)}$ $\Longrightarrow P(A'\|B) = \frac{0{\cdot}33}{0{\cdot}55} = \frac{3}{5} = 0{\cdot}6$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example $\frac{P(A' \cap B)}{P(B)}$ or $P(B) = 0{\cdot}55$ *High Partial Credit* • Formula fully substituted.
(c)	$P(C \cup D) = P(C) + P(D) - P(C \cap D)$ A and B are independent $\Longrightarrow P(C \cap D) = P(C) \times P(D)$ $P(C \cap D) = 0{\cdot}5\, P(D)$ $\therefore 0{\cdot}85 = 0{\cdot}5 + P(D) - 0{\cdot}5\, P(D)$ $\Longrightarrow 0{\cdot}35 = P(D) - 0{\cdot}5\, P(D)$ $0{\cdot}35 = 0{\cdot}5\, P(D)$ $\Longrightarrow P(D) = \frac{0{\cdot}35}{0{\cdot}5} = 0{\cdot}7$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $P(C \cup D) = P(C) + P(D) - P(C \cap D)$ • $P(C \cap D) = P(C).P(D)$ *High Partial Credit:* • $0{\cdot}85 = 0{\cdot}5 + P(D) - 0{\cdot}5\, P(D)$

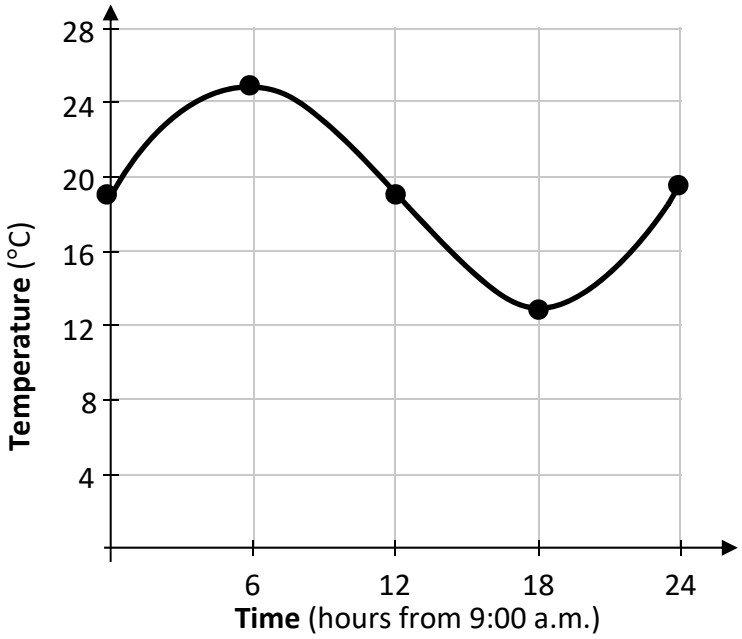
Q6	Model Solution – 30 Marks	Marking Notes
(a) (i)	EDUCATION – 9 letters in total. $9! = 362\,880$ – Total arrangements	**Scale 10B (0, 5, 10)** *Partial Credit:* • Some use of 9
(a) (ii)	Vowels together (EUAIO)CTDN $= \ 5! \times 5! \ = \ 14\,400$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, one arrangement shown *High Partial Credit:* • 5! calculated
(a) (iii)	(ACTION) $\times 3 \times 2 \times 1 \ = 6$ $3 \times$ (ACTION) $\times 2 \times 1 = 6$ $3 \times 2 \times$ (ACTION) $\times 1 \ = 6$ $3 \times 2 \times 1 \times$ (ACTION) $= 6$ $= 24$ **or** $4! = 24$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • 1 arrangement shown. *High Partial Credit:* • 4 arrangements shown

(b) (i)	8M + 8W $= 16$ $\Longrightarrow \binom{16}{12}$ or $\binom{16}{4}$ or 1820	**Scale 5B (0, 2, 5**) *Partial Credit:* • Work of merit, for example, reference to 16
(b) (ii)	More women than men $\Longrightarrow$ 8W and 4M $\Longrightarrow \binom{8}{8} \times \binom{8}{4} = 70$ or 7W and 5M $\Longrightarrow \binom{8}{7} \times \binom{8}{5} = 448$ Total $= 518$ $\therefore P(\text{more women than men}) = \frac{518}{1820}$ $= \frac{37}{130}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, indicates 8W & 4M **or** 7W & 5M *High Partial Credit:* • $\binom{8}{8} \times \binom{8}{4}$ or/and $\binom{8}{7} \times \binom{8}{5}$

Q7	Model Solution – 50 Marks	Marking Notes
(a) (i)	ΔOAX and ΔOYA are congruent by RHS $\|\angle OAX\| = \|\angle OAY\|$ $\|\angle CAB\| = 60^o$ (equilateral) $\Longrightarrow \|\angle OAY\| = 30^o$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example one pair of correct sides or angles indicated.
(a) (ii)	$\tan 30^o = \frac{\|OY\|}{\|AY\|} = \frac{4}{\|AY\|}$ $\Longrightarrow \frac{1}{\sqrt{3}} = \frac{4}{\|AY\|}$ $\Rightarrow \|AY\| = 4\sqrt{3}$ [cm]	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* • Some substitution into trigonometric formula *High Partial Credit:* • $\frac{1}{\sqrt{3}} = \frac{4}{\|AY\|}$
(a) (iii)	$\|AB\| = 2\|AY\| + 4$(radius) $= 2(4\sqrt{3})$ + (4 + 8 + 4) $= (8\sqrt{3} + 16)$ cm $= 29{\cdot}8564$ $\backsimeq 29{\cdot}9$ cm	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit, for example, listing some of the components of |AB| *Mid Partial Credit:* • Some components of |AB| calculated where at least one is $\|AY\|$ *High Partial Credit*: • $2(4\sqrt{3})$ + (4 + 8 + 4) or equivalent and stops *Full Credit* -1: • Incorrect rounding or no rounding • No units or incorrect units

(a) (iv)	Area of $\Delta ABC = \frac{1}{2}(29{\cdot}9)(29{\cdot}9)\sin 60^o$ Volume Prism $= (h)\frac{1}{2}(29{\cdot}9)(29{\cdot}9)\sin 60^o$ $= 387{\cdot}12h$ cm^3 Volume **not** occupied: $387{\cdot}12h - 6\,(\pi(4)^2)h$ $387{\cdot}12h - 96\pi h = 422{\cdot}5$ $85{\cdot}41h = 422{\cdot}5$ $h = \frac{422{\cdot}5}{85{\cdot}41} = 4{\cdot}94672 = 5$ [cm]	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Area of triangle formula with some substitution *Mid Partial Credit:* • $387{\cdot}12h$ • Volume of one cylinder *High Partial Credit:* • $387{\cdot}12h - 6\,(\pi(4)^2)h$ *Full Credit* -1: • Incorrect rounding or no rounding
(a) (v)	equation of $[AB]$: $y = 0$ slope of $[AC] = \tan 60° = \sqrt{3}$ Equation of $[AC]$: $y = \sqrt{3}x$ Slope $[BC] = \tan 120 = -\sqrt{3}$ Point on $[BC]$: $\left(16 + 8\sqrt{3}, 0\right)$ equation of $[BC]$: $y - 0 = -\sqrt{3}\left(x - \left(16 + 8\sqrt{3}\right)\right)$ $\sqrt{3}x + y = 24 + 16\sqrt{3}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • 1 correct equation *High Partial Credit:* • 2 correct equations

(a) (vi)	Centre $= (4\sqrt{3}+8, b)$ $r = 4$ tangent $\quad \sqrt{3}x - y = 0$ $4 = \dfrac{\lvert\sqrt{3}(4\sqrt{3}+8) - b\rvert}{\sqrt{3+1}} = \dfrac{\lvert 12 + 8\sqrt{3} - b\rvert}{2}$ $\Rightarrow \quad \dfrac{\lvert 12 + 8\sqrt{3} - b\rvert}{2} = 4$ $12 + 8\sqrt{3} - b = 8$ $4 + 8\sqrt{3} = b$ **or** $12 + 8\sqrt{3} - b = 8$ $4 + 8\sqrt{3} = b$ $k: (x - 4\sqrt{3} - 8)^2 + (y - 8\sqrt{3} - 4)^2 = 16$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit, for example indicates midpoint of AB *Mid Partial Credit:* • Identifies $r = 4$ or x ordinate equals $4\sqrt{3} + 8$ *High Partial Credit:* • $4 = \frac{\lvert\sqrt{3}(4\sqrt{3}+8)-b\rvert}{\sqrt{3+1}}$
(b)	$\frac{4}{3}\pi r^3 = \pi x^2 3r$ $4r^3 = 9x^2 r$ $x^2 = \frac{4}{9}r^2$ $x = \frac{2}{3}r$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $\pi(R)^2 h$ with some substitution *High Partial Credit:* • $\frac{4}{3}\pi r^3 = \pi x^2 3r$

Q8	Model Solution – 50 Marks	Marking Notes
(a) (i)	Range of $f(t)$: $[19-6, 19+6] = [13, 25]$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Use of 19 **or** 6 *High Partial Credit:* • 13 **or** 25
(a) (ii)		**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • 1 correct plot *High Partial Credit:* • 3 correct plots

(a) (iii)	$f(t) = 21 \Rightarrow 19 + 6\sin\left(\frac{\pi t}{12}\right) = 21$ $\Rightarrow 6\sin\left(\frac{\pi t}{12}\right) = 2$ $\Rightarrow \sin\left(\frac{\pi t}{12}\right) = \frac{1}{3}$ $\frac{\pi t}{12} = 0{\cdot}3398369095$ $\pi t = 4{\cdot}078042913$ $\Rightarrow t = 1{\cdot}297559$ = 1 hour 18 mins $\Rightarrow$ Temp = 21°C at 10:18	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • Work of merit, for example $f(t) = 21$ *Mid Partial Credit:* • $19 + 6\sin\left(\frac{\pi t}{12}\right) = 21$ or equivalent *High Partial Credit:* • Non trig equation in t i.e. $\frac{\pi t}{12} = 0{\cdot}3398369095$ *Full Credit* -1: • Incorrect rounding or no rounding

(b) (i)	Equidistant from O at t hours after 9 a.m. $\Rightarrow \lvert OA\rvert = \lvert OB\rvert$ $\Rightarrow 100t = 120(t - \frac{1}{3})$ $\Rightarrow 100t = 120t - 40$ $\Rightarrow 20t = 40$ $\Rightarrow t = 2$ hours i.e at 11.00 a.m.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit, for example, $\frac{\text{dist}}{\text{time}}$ *Mid Partial Credit:* • $100t$ or $(t - \frac{1}{3})$ *High Partial Credit:* • $100t = 120(t - \frac{1}{3})$
(b) (ii)	At 10:15 A is travelling for 75 mins. $\Rightarrow \lvert OA\rvert = 100 \times \frac{75}{60} = \frac{750}{6} = 125$ km At 10:15am B is travelling for 55 mins having passed O. $\Rightarrow \lvert OB\rvert = 120 \times \frac{55}{60} = 110$ km Cosine Rule: $\Rightarrow d^2 = 125^2 + 110^2 - 2(125)(110)\cos 60^o$ $\Rightarrow d^2 = 15625 + 12100 - 13750$ $\Rightarrow d^2 = 13\,975$ $\Rightarrow d = 118{\cdot}2159$ [km] $\approx 118{\cdot}2$ [km]	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit towards finding how far A had travelled in 75 minutes. *Mid Partial Credit:* • Distance travelled by one, and work of merit towards finding the other distance *High Partial Credit:* • Cosine rule fully substituted *Full Credit* -1: • Incorrect rounding or no rounding

(b) (iii)	$\lvert OA\rvert = 100t$ (in t hours) $\lvert OB\rvert = 120(t - \frac{1}{3})$ $\lvert OB\rvert = 120t - 40$ Cosine Rule: $d^2 = \lvert OA\rvert^2 + \lvert OB\rvert^2 - 2\lvert OA\rvert\lvert OB\rvert$ Cos 60° $d^2 = 52^2$ $\Rightarrow 52^2 = (100t)^2 + (120t - 40)^2$ $-2(100t)(120t - 40)$cos 60° $\Rightarrow 10000t^2 + 14400t^2 - 9600t + 1600 -$ $12000t^2 + 4000t = 2704$ $\Rightarrow 12400t^2 - 5600t - 1104 = 0$ $(\div by\ 16)$ $\Rightarrow 775t^2 - 350t - 69 = 0$ $\Rightarrow (5t - 3)(155t + 23) = 0$ $\Rightarrow t = \frac{3}{5}$ hours $= 36$ mins $(t = -\frac{23}{115}$ not possible) ∴ Exact time is $9:36$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $120(t - \frac{1}{3})$ or $100t$ *Mid Partial Credit:* • Cosine rule fully substituted *High Partial Credit:* • $12400t^2 - 5600t - 1104 = 0$ Note: Apply F*(−1) if 2nd solution is not discarded

Q9	Model Solution – 50 Marks	Marking Notes
(a)	$\sum f(x) = 40 + 240 + 500 + 420 + 180$ $= 1380$ Mean $= \frac{1380}{30} = 46$	**Scale 20D (0, 6, 12, 17, 20)** *Low Partial Credit:* • Work of merit, for example, 1 mid interval found or sum of f formulated. *Mid Partial Credit:* • 3 correct $x.f(x)$ values found *High Partial Credit:* $\sum f(x) = 1380$
(b)	$P(\text{correct}) = \frac{1}{5}$ $P(\text{Not correct}) = \frac{4}{5}$ Expected Value: $10\left(\frac{1}{5}\right) - 2\left(\frac{4}{5}\right) = \frac{2}{5}$ expected score $= \frac{2}{5}(15) = 6$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $\frac{1}{5}$ **or** $\frac{4}{5}$ *Mid Partial Credit:* • $10\left(\frac{1}{5}\right)$ **or** $2\left(\frac{4}{5}\right)$ *High Partial Credit:* • Expected value fully formulated

(c) (i) & (c) (ii)

Goals scored	0	1	2	3	4	5
Probability $P(x)$	$\frac{1}{243}$	$\frac{10}{243}$	$\frac{40}{243}$	$\frac{80}{243}$	$\frac{80}{243}$	$\frac{32}{243}$

(c) (ii)

$\sum x.P(x) = 0\left(\frac{1}{243}\right) + 1\left(\frac{10}{243}\right) + 2\left(\frac{40}{243}\right)$

$+3\left(\frac{80}{243}\right) + 4\left(\frac{80}{243}\right) + 5\left(\frac{32}{243}\right)$

$= \frac{10}{3}$

expected score $= 3{\cdot}333$

Scale 20D (0, 6, 12, 17, 20)

Low Partial Credit:

- One correct new entry in (c)(i)
- One correct $x.P(x)$

Mid Partial Credit:

- Work of merit in both parts
- Two correct new entries in (c)(i)
- Expression fully substituted in (c)(ii)

High Partial Credit:

- One part correct

(d)

$x =$ No. Male, $y =$ No. Female

$$\frac{71x + 73y}{x + y} = 71{\cdot}8$$

$$71x + 73y = 71{\cdot}8x + 71{\cdot}8y$$

$$1{\cdot}2y = 0{\cdot}8x$$

$$\frac{x}{y} = \frac{1{\cdot}2}{0{\cdot}8} = \frac{3}{2}$$

ratio Males to Females is $3 : 2$

Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- Work of merit, for example $71x + 73y$

High Partial Credit:

- $\frac{71x+73y}{x+y} = 71{\cdot}8$

Q10	Model Solution – 50 Marks	Marking Notes
(a) (i)	$P(2 \text{ Contacts}) = \binom{10}{2}\left(\frac{2}{25}\right)^2\left(\frac{23}{25}\right)^8$ $= 0{\cdot}1478$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit, for example, $\binom{10}{2}$ **or** $\frac{23}{25}$ or equivalent *High Partial Credit:* • Formula fully substituted
(a) (ii)	P(7th interviewed is the 2nd) $=\left[\binom{6}{1}\left(\frac{2}{25}\right)\left(\frac{23}{25}\right)^5\right]\times\left(\frac{2}{25}\right) = 0{\cdot}02531$ $\approx 0{\cdot}0253$	**Scale 10D (0, 3, 5, 8, 10)** Consider the solution as being the product of four terms: $\binom{6}{1}$, $\frac{2}{25}$, $\left(\frac{23}{25}\right)^5$ and $\frac{2}{25}$ *Low Partial Credit:* • One of the above values *Mid Partial Credit:* • Product of two correct terms evaluated • Product of three correct terms *High Partial Credit:* • $\left[\binom{6}{1}\left(\frac{2}{25}\right)\left(\frac{23}{25}\right)^5\right]\times\left(\frac{2}{25}\right)$ *Full Credit -1* • Incorrect rounding or no rounding
(a) (iii)	P (at least 1 person wearing contacts) = 1 – P (none wear contacts) $\Longrightarrow 1 - \left(\frac{23}{25}\right)^n \geq 0{\cdot}9$ $\Longrightarrow 0{\cdot}1 \geq (0{\cdot}92)^n$ **or** $(0{\cdot}92)^n \leq 0{\cdot}1$ $\Longrightarrow n \times \ln(0{\cdot}92) \geq \ln(0{\cdot}1)$ $\Longrightarrow n \geq \frac{\ln(0{\cdot}1)}{\ln(0{\cdot}92)}$ $\Longrightarrow n \geq 27{\cdot}615$ $\Longrightarrow$ n (least) $= 28$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit, for example, some use of 1 –…, or reference to 0·9 *Mid Partial Credit:* • $1 - \left(\frac{23}{25}\right)^n$ or equivalent *High Partial Credit:* • $1 - \left(\frac{23}{25}\right)n \geq 0{\cdot}9$

(b) (i)	$z = \frac{x-\mu}{\sigma} = \frac{9{\cdot}85-10}{0{\cdot}18} = -0{\cdot}833333$ $\text{P}(z \leq -0 \cdot 83333) = 1 - 0{\cdot}7967$ $= 0{\cdot}2033$ $= 20[\%]$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Formula with some substitution *High Partial Credit:* • Formula fully substituted
(b) (ii)	P $(9{\cdot}85 < x < k) = 0{\cdot}55$ $\Longrightarrow (x < k) = 0{\cdot}2 + 0{\cdot}55$ $= 0{\cdot}75$ $\Longrightarrow k = ?$ $z = 0{\cdot}675$ **or** $0{\cdot}67$ **or** $0{\cdot}68$ Since $\text{P}(z \leq 0{\cdot}675) = 0{\cdot}75$ $\Longrightarrow \frac{k-10}{0{\cdot}18} = 0{\cdot}675$ $\Longrightarrow k - 10 = 0{\cdot}1215 \Longrightarrow$ $k = 10{\cdot}12$ [ml]	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Use of 0·2 or 0·55 *Mid Partial Credit:* • $\text{P}(x < k) = 0{\cdot}2 + 0{\cdot}55$ *High Partial Credit:* • $\frac{k-10}{0{\cdot}18} = 0{\cdot}675$ *Full Credit -1*: • Incorrect rounding or no rounding

(c)	$H_0: \mu = 10$ ml $H_1: \mu \neq 10$ ml $\bar{x} = 9{\cdot}96$ $\sigma = 0{\cdot}18$ $N = 50$ Test statistics $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{N}}}$ $Z = \frac{9{\cdot}96 - 10}{\frac{0{\cdot}18}{\sqrt{50}}}$ $\Rightarrow Z = -1{\cdot}57 > -1{\cdot}96$ Since $-1{\cdot}57 > -1{\cdot}96 \Rightarrow$ the result is not significant so we accept H_0. The mean contents has not changed	**Scale 10D (0, 3, 5, 8, 10)** **Solution has 4 steps** 1. Hypotheses 2. Calculations (sufficient to support a conclusion 3. Conclusion (not considered without some relevant calculations 4. Reason (must match calculations) *Low Partial Credit:* • Work of merit, for example, one hypothesis stated correctly, some relevant calculation indicated, identifies μ or σ, hypotheses swapped *Mid Partial Credit:* • Calculations correct *High Partial Credit:* • Calculations correct **and** one other step