6069.M30C

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Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination

Mathematics

Paper 2

Higher Level

2 hours 30 minutes

300 marks

Examination Number

Day and Month of Birth

For example, 3rd February is entered as 0302

Centre Stamp

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- any **five** questions from Section A – Concepts and Skills
- any **three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Diagrams are generally not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

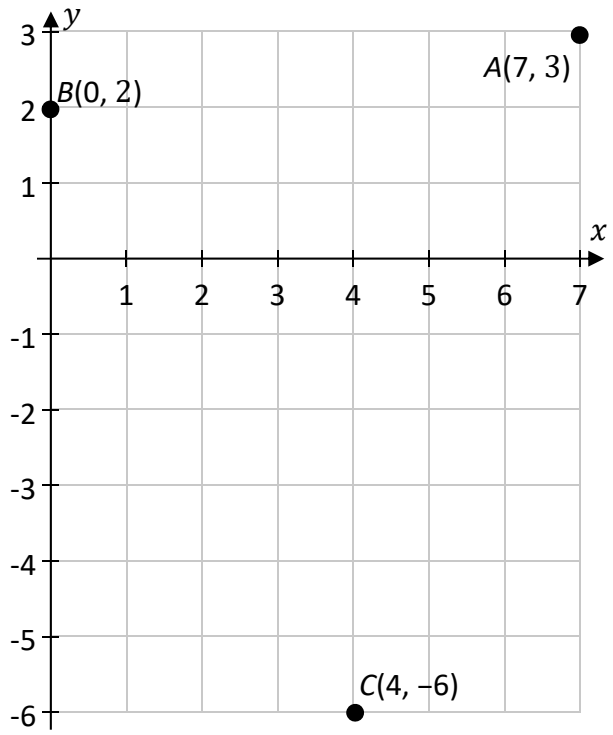
Write the make and model of your calculator(s) here: | |

Section A — Concepts and Skills — 150 marks

Answer **any five** questions from this section.

Question 1 **(30 marks)**

The points $A(7, 3)$, $B(0, 2)$, and $C(4, -6)$ are shown in the diagram.



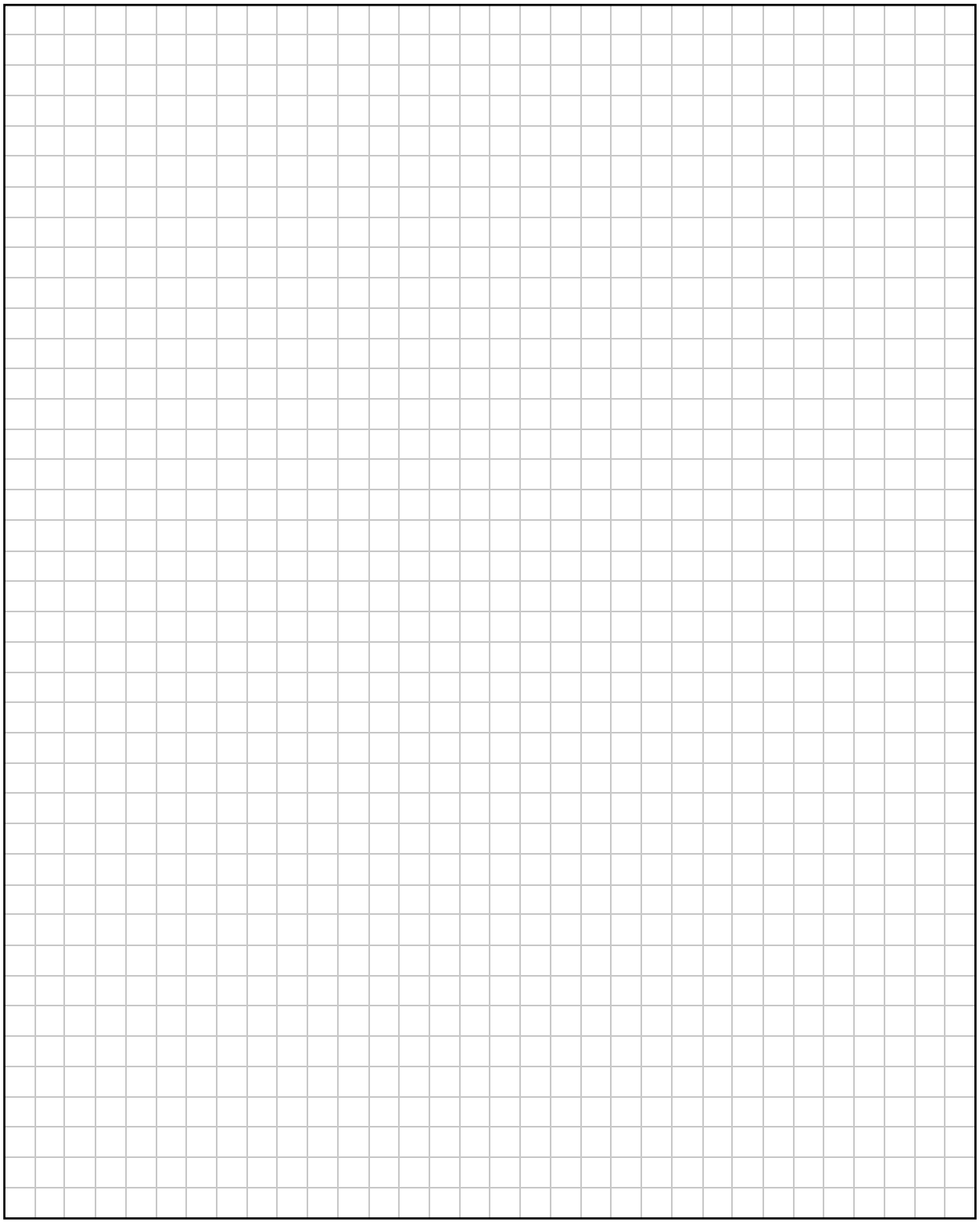
(a) Find the midpoint **and** slope of $[AB]$.

Midpoint:

Slope:

(b) Find the equation of the perpendicular bisector of $[AB]$.
Give your answer in the form $ax + by + c = 0$, where a, b, and $c \in \mathbb{Z}$.

(c) Use your answer from **part (b)** to find the co-ordinates of O, the circumcentre of ΔABC.



Question 2 **(30 marks)**

(a) The equation of a circle, s, is: $x^2 + y^2 - 10x + 2y - 26 = 0$.

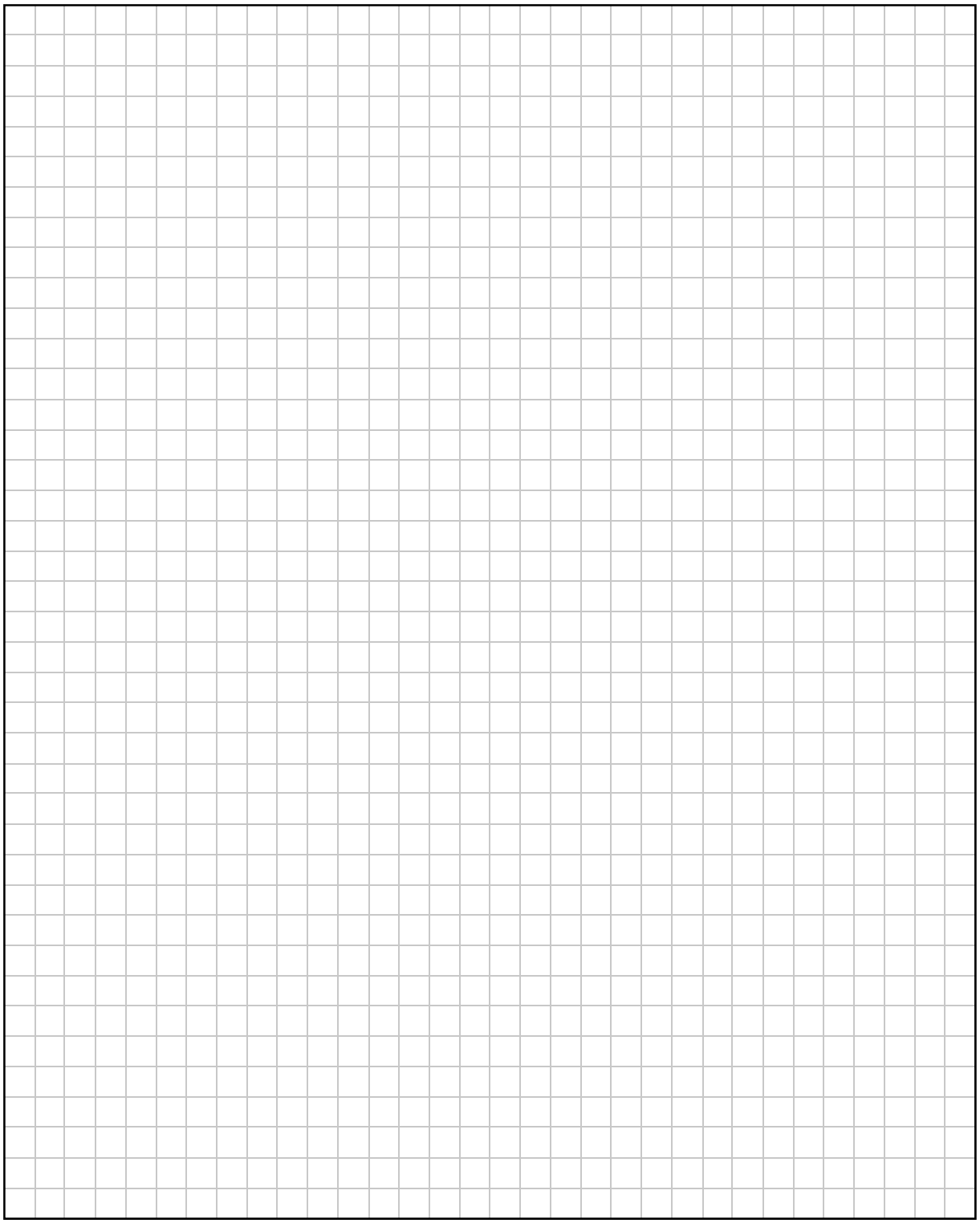
(i) Find the centre **and** radius of s.

Centre $=$ Radius $=$

(ii) The line $2x - 3y + c = 0$ is a tangent to circle s.
Find the two possible values of c.

(b) The lines $x = 2$ and $x = 8$ are tangents to circle k.
The centre of k is on the line $3x + 4y - 7 = 0$.

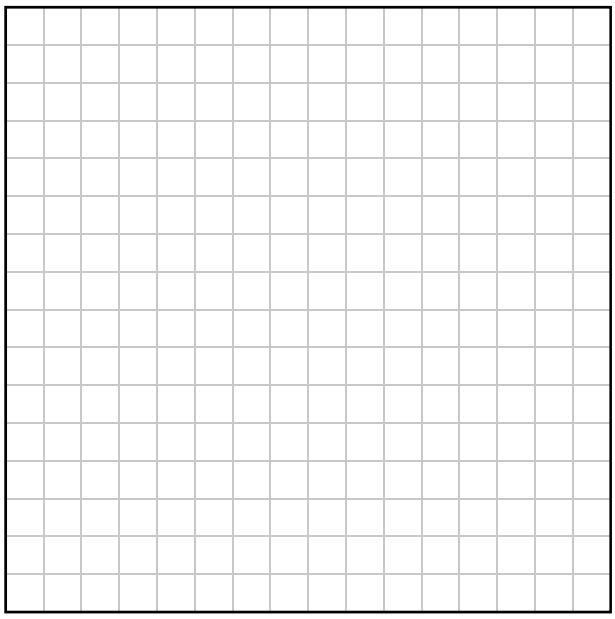
Find the equation of k.

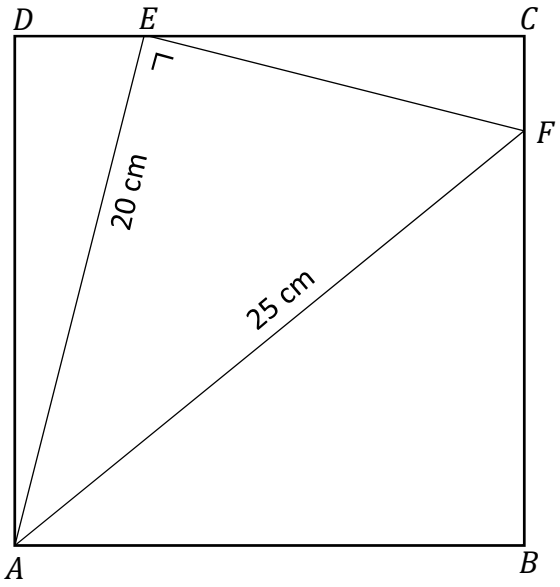


Question 3 **(30 marks)**

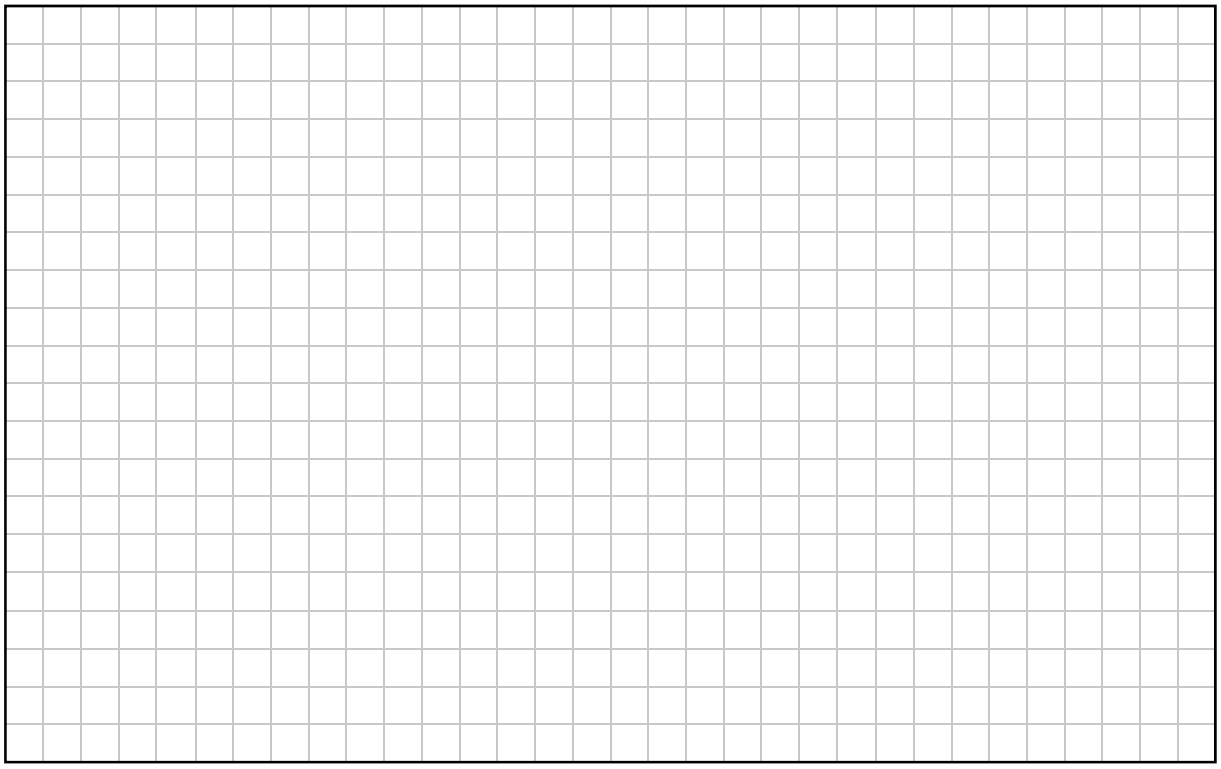
In the diagram below, $ABCD$ is a square. ΔAFE is a right-angled triangle.
The points E and F lie on $[DC]$ and $[BC]$, respectively.
$|AE| = 20$ cm, $|AF| = 25$ cm and $|\angle FEA| = 90°$.

(a) Find $|EF|$.





(b) Prove that ΔAED and ΔEFC are similar (equiangular).
Give a reason for any statement you make use of in your proof.



(c) Given that $|ED| = \frac{1}{4}\ |AD|$, find $|AD|$.

Give your answer in the form $\frac{p}{\sqrt{q}}$ cm, where $p, q \in \mathbb{N}$.

(d) Find the area of ΔABF. Give your answer in the form $\frac{m}{n}$ cm^2, where $m, n\ \in \mathbb{N}$.

Question 4 **(30 marks)**

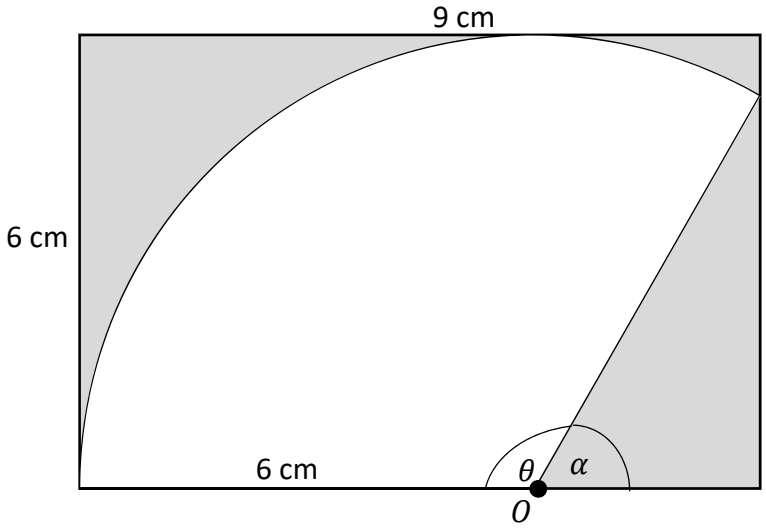
(a) **(i)** Prove that $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

(ii) $\sin A = \frac{2}{\sqrt{5}}$, where $0 < A < \frac{\pi}{2}$.

Without using your calculator, find $\sin 2A$ and $\cos 2A$ in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$.

$\sin 2A =$ $\cos 2A =$

(b) A sector of a circle of centre O and radius 6 cm lies inside a rectangle of length 9 cm and height 6 cm, as shown in the diagram. O lies on one side of the rectangle, as shown.



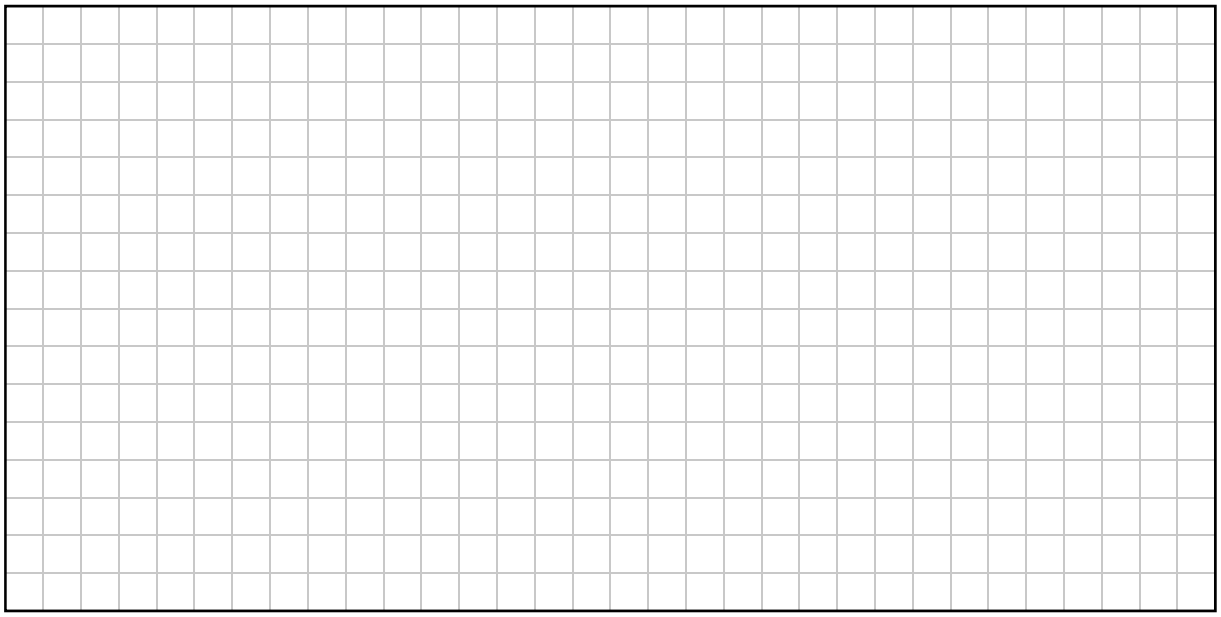
(i) Find the measure of the angle α.

(ii) Find the area of the shaded section in the diagram.
Give your answer in the form $(a - b\pi)$ cm^2, where $a, b \in \mathbb{N}$.

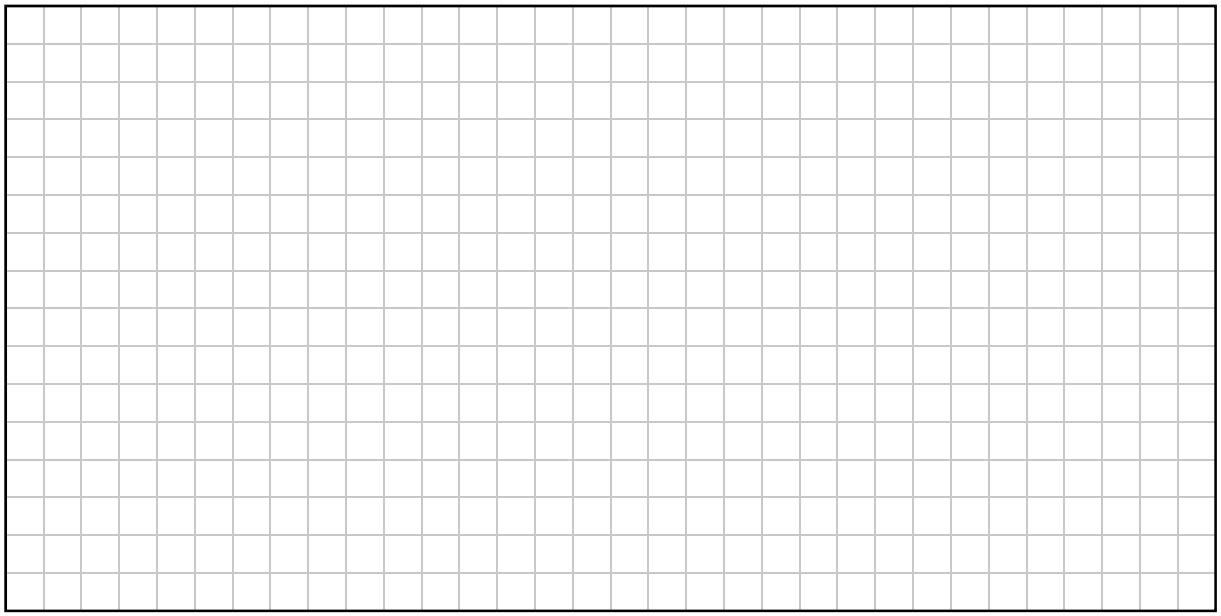
Question 5 **(30 marks)**

(a) In an examination, students are asked to enter the day and month of their birth on the cover of their answer books (for example, 3rd February is entered as 03 02).
You may assume that all months are equally likely to be entered and all days in a given month are equally likely to be entered.

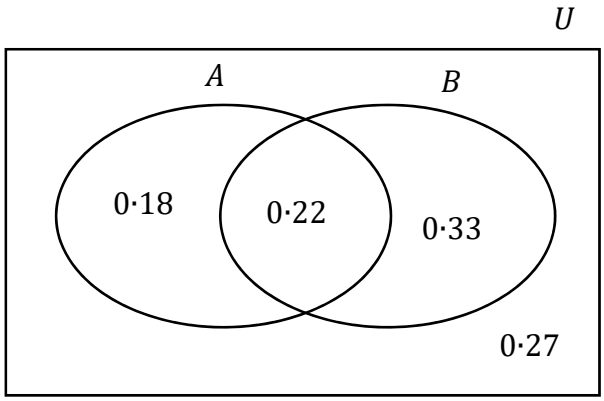
(i) In a particular school, four students enter their data.
What is the probability that all four students entered the same month?
Give your answer correct to 4 decimal places.



(ii) In a particular centre, **five** students were born during the month of April.
What is the probability that at least two of them have entered the same day?
You may assume that all of the 30 days in April are equally likely to be entered.
Give your answer correct to 4 decimal places.



(b) A and B are two events.
In the Venn diagram below, the probability of each event is written in the appropriate region.
For example, from the diagram, the probability that neither A nor B happens is $P(A \cup B)' = 0{\cdot}27$.



Find the value of $P(A'|B)$, where A' is the complement of A.

(c) C and D are independent events such that $P\ (C) = 0{\cdot}5$, and $P(C \cup D) = 0{\cdot}85$.
Find $P(D)$, the probability that D occurs.

Question 6 **(30 marks)**

(a) **(i)** In how many ways can the letters E, D, U, C, A, T, I, O, N be arranged if each letter is used exactly once in each arrangement?

(ii) In how many of these arrangements do all the vowels (A, E, I, O, and U) come together?

(iii) In how many of these arrangements will the letters A, C, T, I, O, N be together and in that order?

(b) **(i)** In how many ways can a jury of 12 people be selected from a panel of 8 men and 8 women?

(ii) Find the probability that one such jury, selected at random, will contain more women than men.

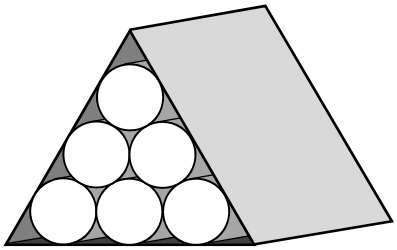
Section B — Contexts and Applications — 150 marks

Answer **any three questions** from this section.

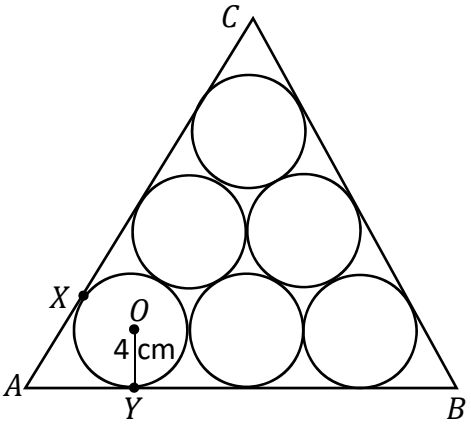
Question 7 **(50 marks)**

The diagrams below show a triangular-based prism containing six identical cylindrical tubes, each of radius 4 cm.
The cylinders all touch the sides of the prism and each other, as shown.
The prism and the cylinders all have the same height.

The cylinder with the centre of its base at O touches $[AC]$ at X and $[AB]$ at Y, where ABC is the base of the prism.



Prism



Base of prism

(a) **(i)** Explain why $|\angle OAY| = 30°$.

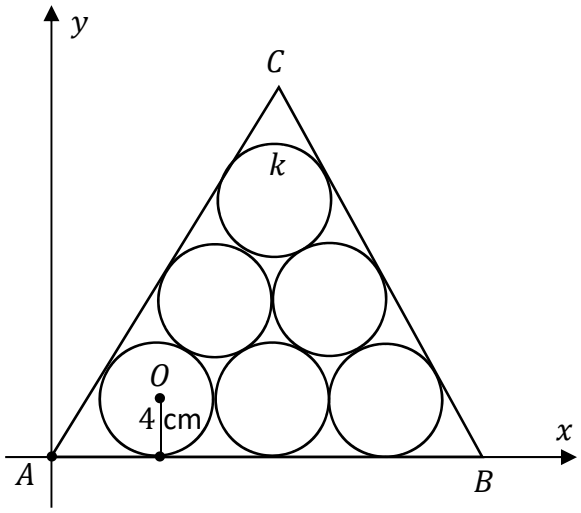
(ii) Find $|AY|$. Give your answer in the form $p\sqrt{q}$ cm, where $p, q \in \mathbb{N}$.

(iii) Find $|AB|$, correct to 1 decimal place.

(iv) The volume of the empty space in the prism is 422·5 cm^3, correct to 1 decimal place.
Find the height of each cylinder.
Give your answer correct to the nearest cm.

This question continues on the next page.

The base of the prism is shown again below, on a co-ordinate plane.
A is the point $(0, 0)$, and B lies on the x-axis.
One of the circles is labelled k.



(v) Write down the equation of each of the lines AB, AC, and BC.

(vi) Find the equation of the circle k.

(b) A solid metal sphere, of radius r, is melted down and recast into a right-circular cylinder.
You may assume that no material was lost in this process.
The resulting cylinder has a height of $3r$.
Find, in terms of r, the radius of the cylinder.

Question 8 **(50 marks)**

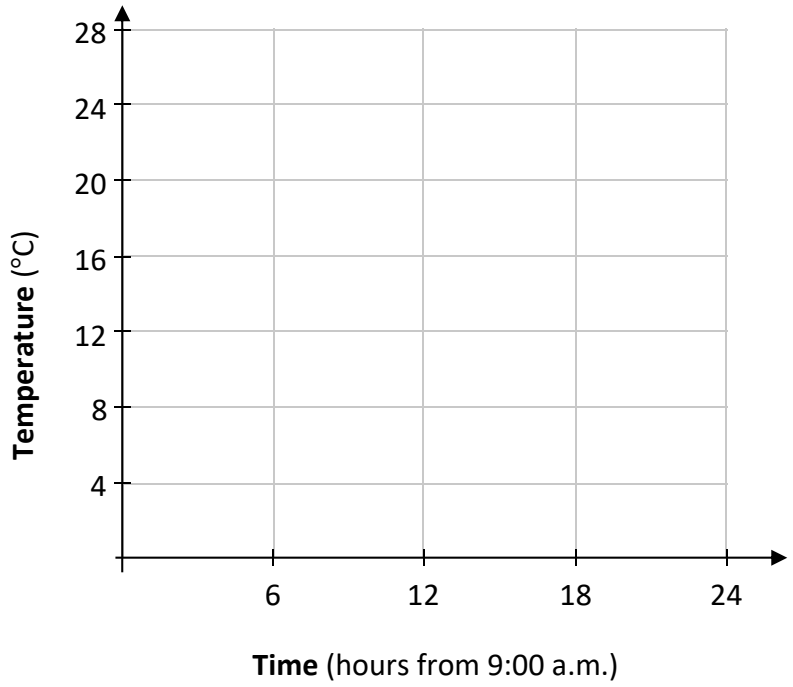
(a) In a particular city, the temperature during a 24-hour period in the month of July, measured in degrees centigrade, could be modelled by the function

$$f(t) = 19 + 6\ \sin\left(\frac{\pi t}{12}\right),$$

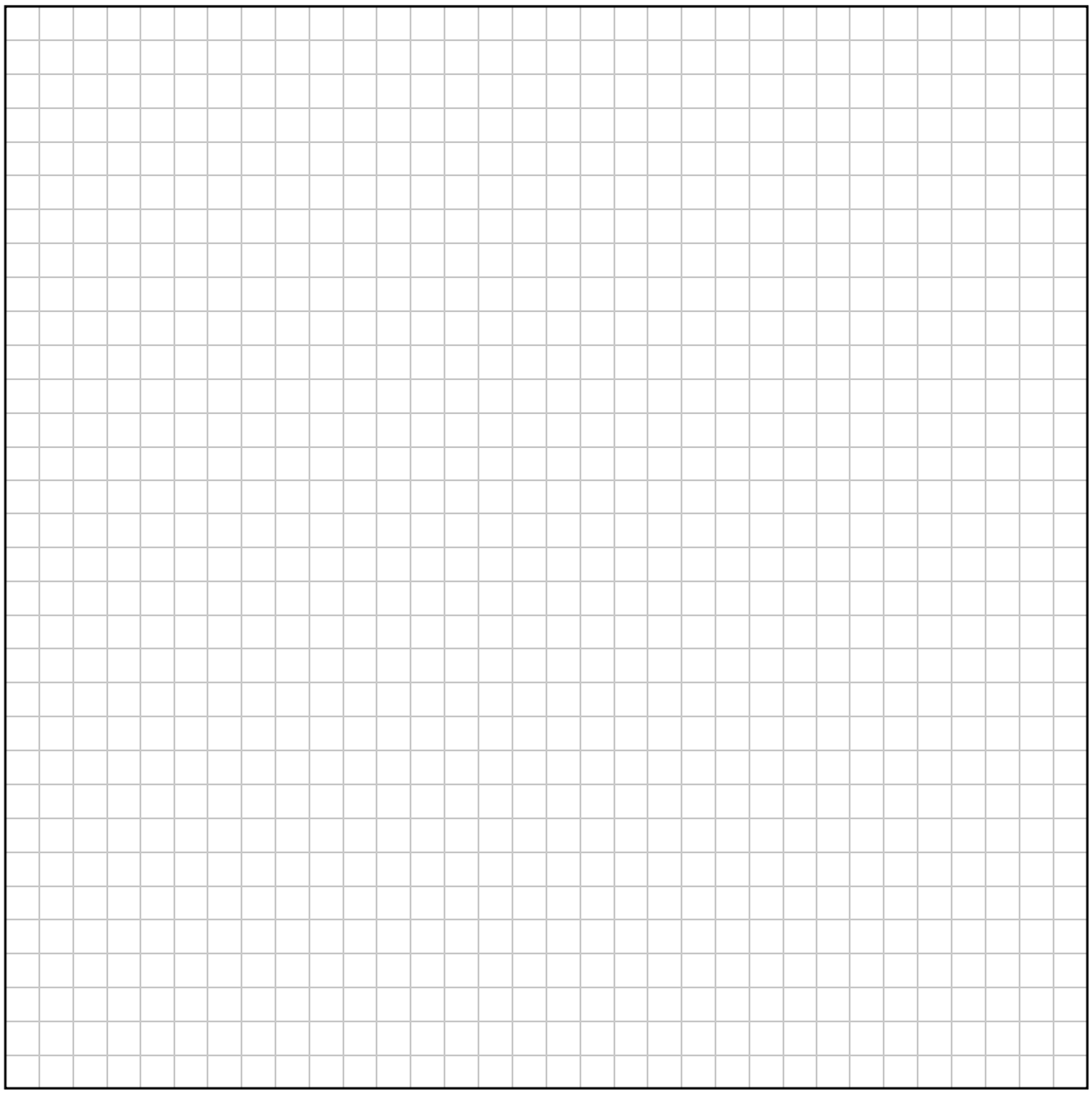
where $t \in \mathbb{R}$ was the time, in hours, after 9:00 a.m., and $\left(\frac{\pi t}{12}\right)$ was measured in radians.

(i) Write down the range of $f(t)$.

(ii) The period of $f(t)$ is 24 hours.
Draw the graph of $f(t)$ on the axes below, in the domain $0 \leq t \leq 24$, $t \in \mathbb{R}$.



(iii) Find the first time during the 24-hour period when the temperature was exactly 21°C. Give your answer correct to the nearest minute.



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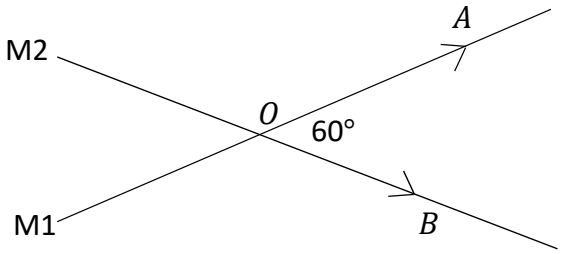
(b) A straight motorway, M1, crosses over another straight motorway, M2, at the point O.
At the point where they cross, the motorways are at an angle of 60° to each other, as shown in the diagram.

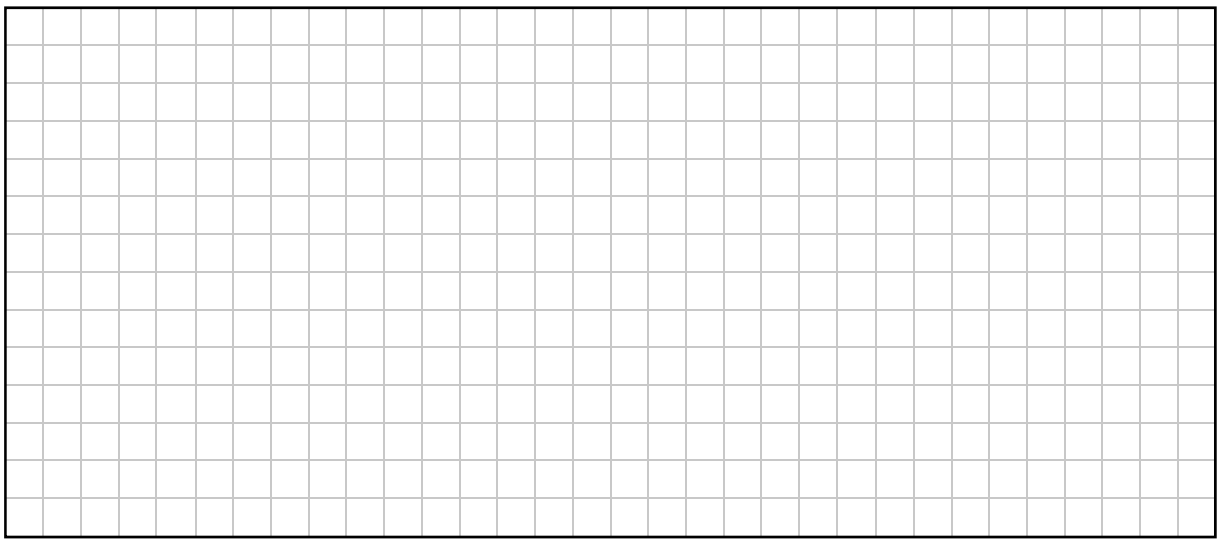


At 9:00 a.m., car A on M1 passes O at a constant speed of 100 km/hr.
Twenty minutes later, car B on M2 passes O at a constant speed of 120 km/hr.
Each car is travelling in the direction shown in the diagram.

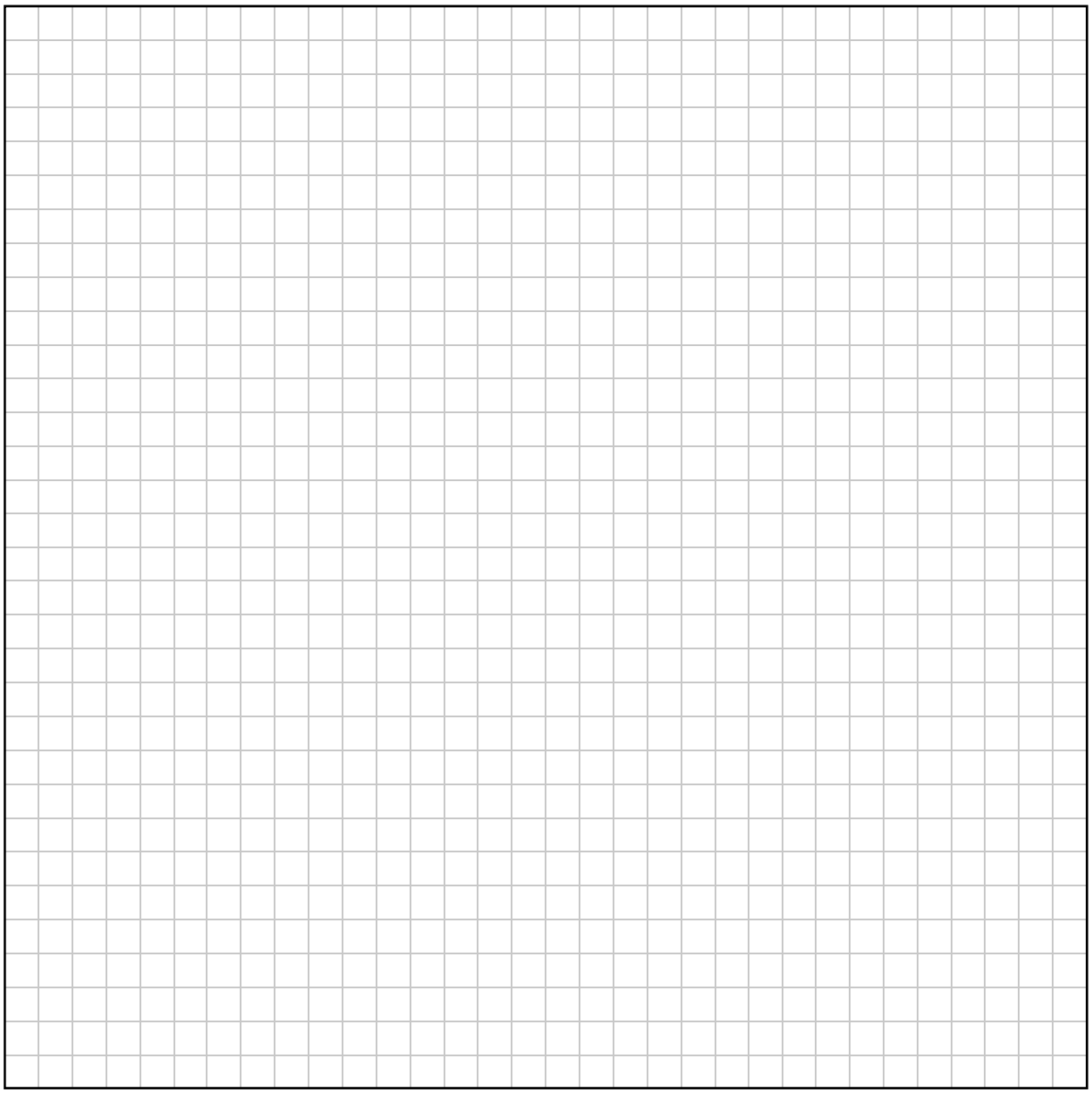
(i) Find the time, after 9:00 a.m., when both cars will be the same distance from O.

(ii) Find how far apart the cars will be at 10:15 a.m.
Give your answer, in km, correct to one decimal place.

There is space for more work on the next page.



(iii) Find the exact time, after 9:00 a.m., when car A and car B will be 52 km apart.



Question 9 **(50 Marks)**

(a) 30 students in a class took a test. The table below gives a summary of their marks.
Use mid-interval values to estimate the mean mark of the students in the test.

Mark (%)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Number of students	4	8	10	6	2

(Note: 20 – 40 means greater than 20 and up to and including 40)

(b) A multiple choice test has 15 questions.
For each question, 5 possible answers are given, only 1 of which is the correct answer.
If a candidate picks the correct answer, they get 10 points.
If a candidate picks an incorrect answer, they lose 2 points.
Candidates must answer all questions.
Candidates can get a positive or negative score on the test.

Find the expected score on the test of a candidate who randomly selects an answer to each of the 15 questions.

(c) The probability that any player on a particular team will score a goal from a penalty is $\frac{2}{3}$.
In a penalty shoot-out, at the end of a game, 5 players from the team each take a penalty.

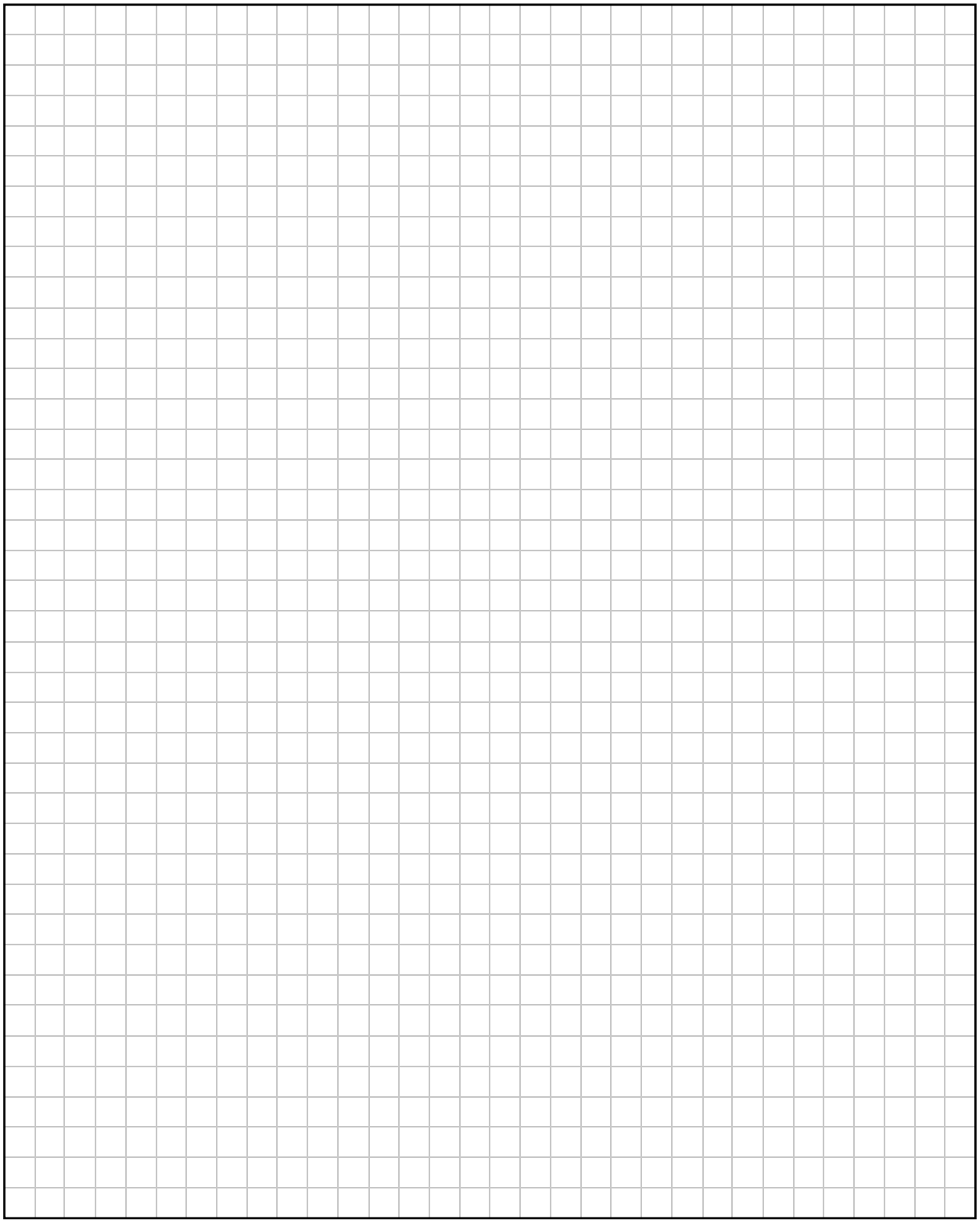
(i) Complete the table below to show the probability of scoring from 0 to 5 goals from these penalties.
Give each answer in the form of $\frac{a}{b}$, where $a, b \in \mathbb{N}$.

Goals scored	0	1	2	3	4	5
Probability $P(x)$	$\frac{1}{243}$		$\frac{40}{243}$		$\frac{80}{243}$	

(ii) If this team were involved in a lot of 5 player penalty shoot-outs, on average, how many goals would you expect this team to score per penalty shoot-out.

This question continues on the next page.

(d) A number of candidates (male and female) completed an examination.
The mean score of the male candidates in the examination was 71.
The mean score of the female candidates in the examination was 73.
The mean score of all the candidates in the examination was 71·8.
Find the proportion of candidates who were male.



Question 10 **(50 marks)**

(a) People are chosen at random from the population in Ireland.
The probability that a person in Ireland wears contact lenses is $\frac{2}{25}$.

(i) If 10 people are chosen at random, find the probability that exactly two of them wear contact lenses. Give your answer correct to 4 decimal places.

(ii) Find the probability that the 7^{th} person chosen is the **second** person who wears contact lenses. Give your answer correct to 4 decimal places.

(iii) Find the minimum number of people who would need to be chosen, in order to be at least 90% certain of including at least one person who wears contact lenses.

This question continues on the next page.

(b) A company produces eye drops. The eye drops are sold in small bottles.
The amount of liquid in the bottles follows a normal distribution with mean 10 ml and standard deviation 0·18 ml.

(i) Find the percentage of bottles that contain less than 9·85 ml of liquid.
Give your answer correct to the nearest percentage.

(ii) It is known that 55% of the bottles contain between 9·85 ml and k ml of liquid, where $k \in \mathbb{R}$ and $k > 9{\cdot}85$.

Find the value of k. Give your answer correct to 2 decimal places.

(c) The machine used by the company to fill the bottles is serviced and adjusted.
The company want to check if the mean amount of liquid in the bottles is still 10 ml.
They take a random sample of 50 bottles and find the mean amount of liquid is 9·96 ml.

Carry out a hypothesis test at the 5% level of significance to see if this shows a change in the mean amount of liquid in the bottles.
State clearly your null hypothesis, your alternative hypothesis, your conclusion, and a reason for your conclusion.

Null Hypothesis:

Alternative Hypothesis:

Calculations:

Conclusion:

Reason for your conclusion:

You may use this page for extra work.
Label any extra work clearly with the question number and part.

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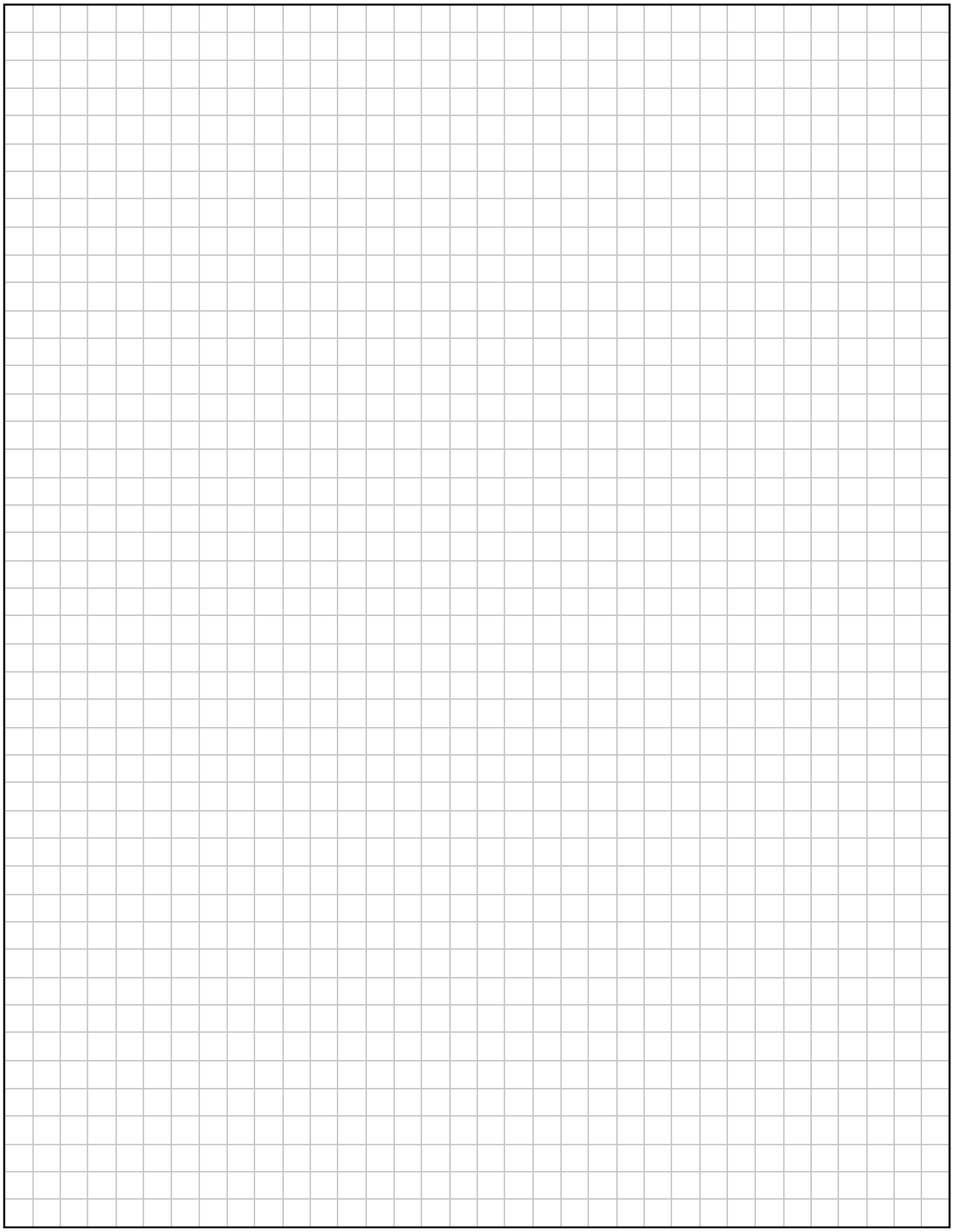
Label any extra work clearly with the question number and part.



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Leaving Certificate – Higher Level

Mathematics - Paper 2

2 hours 30 minutes