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Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination

Mathematics

Paper 1

Ordinary Level

2 hours 30 minutes

300 marks

Examination Number

Day and Month of Birth

For example, 3rd February is entered as 0302

Centre Stamp

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- any **five** questions from Section A – Concepts and Skills
- any **three** questions from Section B – Contexts and Applications

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Diagrams are not generally drawn to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

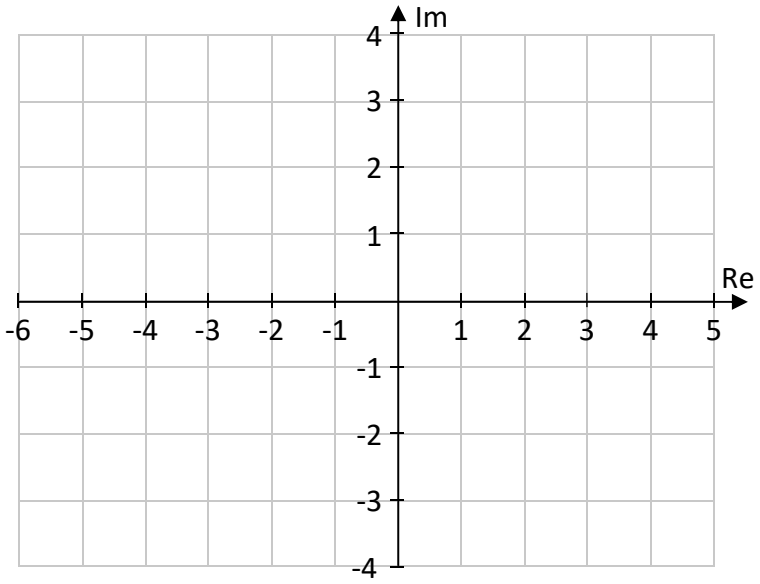
Answer **any five** questions from this section.

Question 1 **(30 marks)**

Note: In this question, $i^2 = -1$.

(a) Let $z_1 = 3 - 2i$ and $z_2 = -2 + 3i$.

(i) Plot and label z_1, z_2, and $\overline{z_2}$ on an Argand Diagram below, where $\overline{z_2}$ is the complex conjugate of z_2.



(ii) Find $|z_1 - z_2|$. Give your answer in surd form.

(b) Write $3(2-i)+i(5+2i)$ in the form $x+yi$, where $x, y \in \mathbb{R}$.

(c) The real numbers p and k are such that $2-3pi-9-ki=4k+1-i$.
Find the value of p and the value of k.

$p =$ $k =$

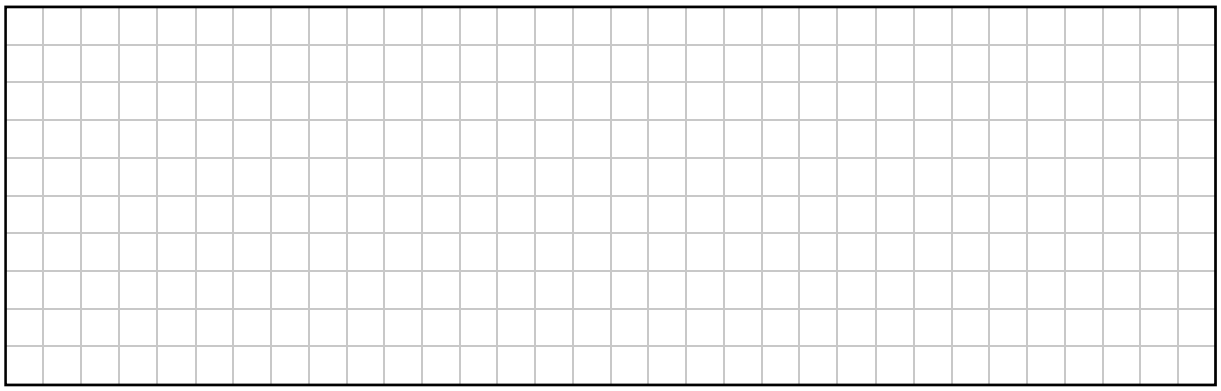
Question 2 **(30 marks)**

(a) Solve the inequality $\frac{2x-5}{3} \geq \frac{x-4}{2}$, where $x \in \mathbb{R}$, **and** show the solution set on a number line.

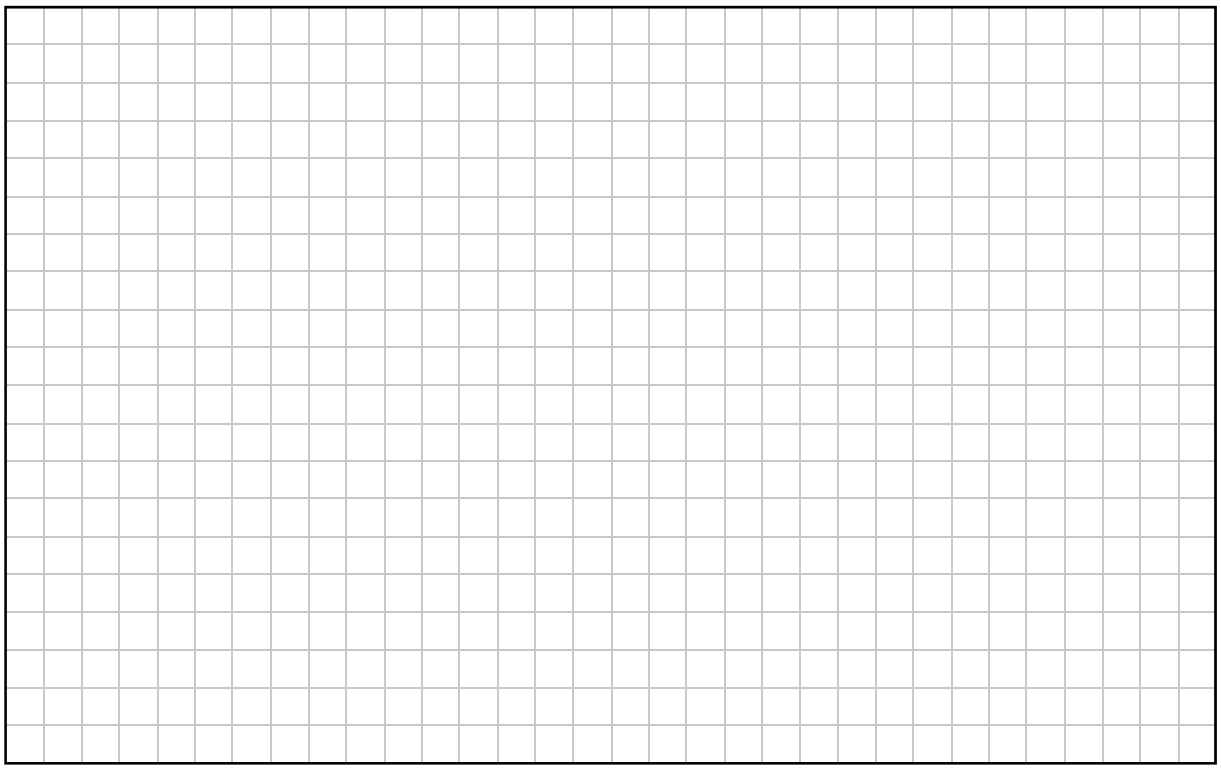
(b) Write q in terms of p and t when $3(2p + q) = t$.

(c) The function f is as follows for $x \in \mathbb{R}$: $f(x) = 3x^2 - 8x - 35$.
The function g is as follows for $x \in \mathbb{R}$: $g(x) = 2x - 7$.

(i) Find $f(6)$.



(ii) Find $f(g(2))$.



Question 3 **(30 Marks)**

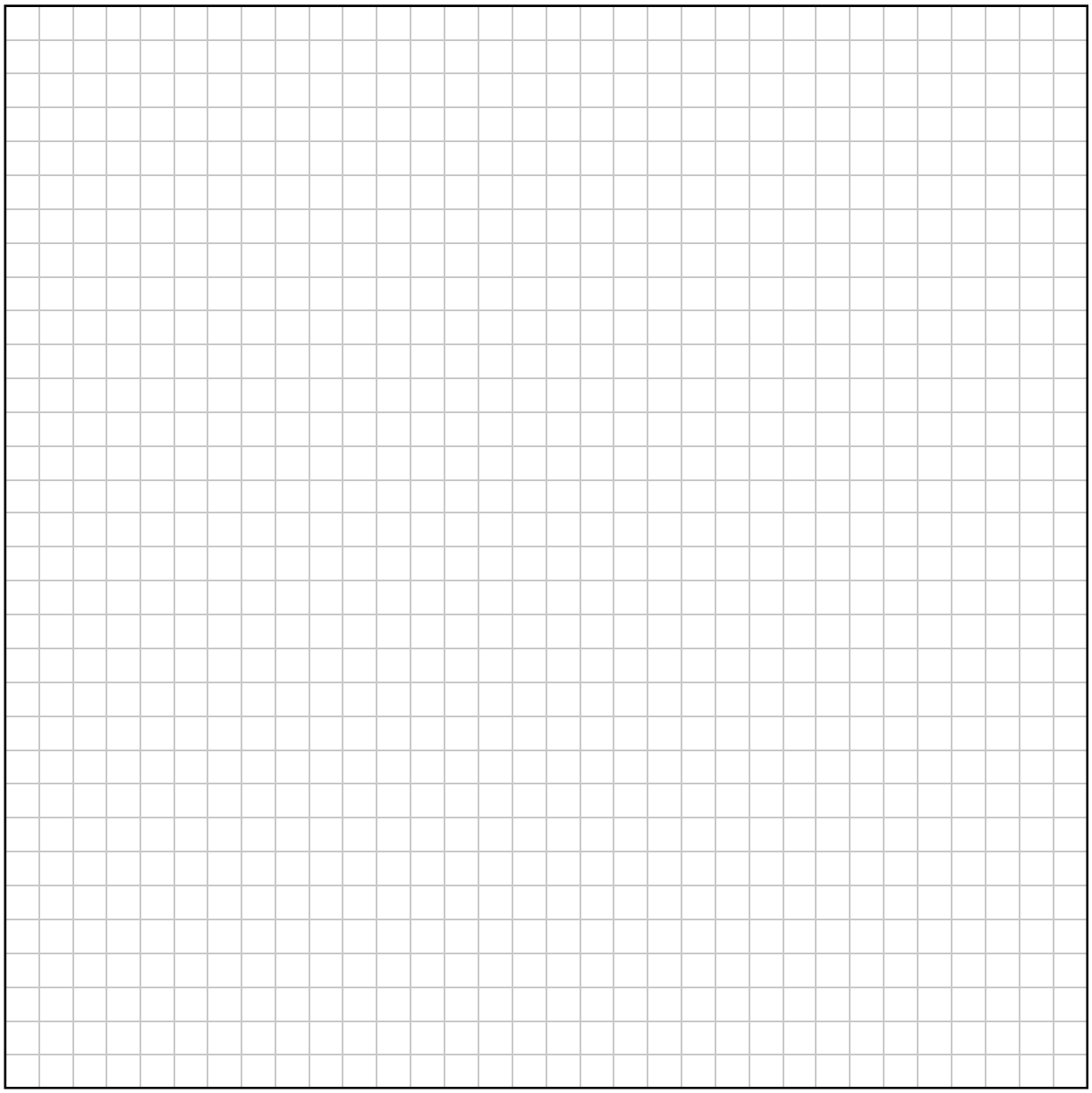
(a) Given that $5x - 4y = 1$, find the value of y when $x = -7$.

(b) Find the value of x for which $2^x = \sqrt{32}$, where $x \in \mathbb{Q}$.

(c) Solve the simultaneous equations:

$$x - y = 2$$

$$x^2 + y^2 = 10$$



Question 4 **(30 Marks)**

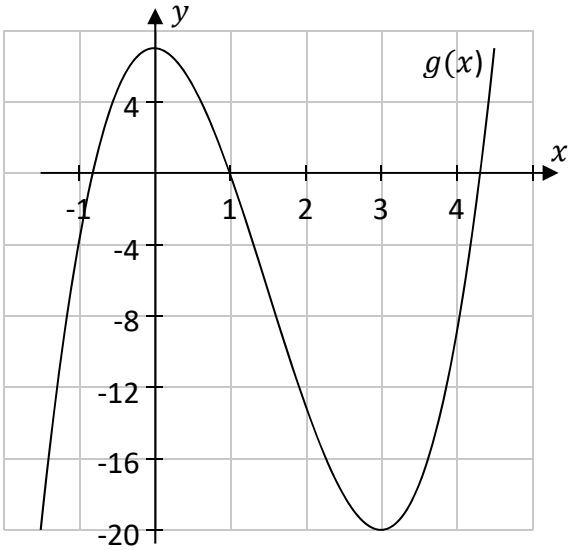
(a) Solve the equation $x^2 - 6x + 3 = 0$.
Give each solution correct to 1 decimal place.

(b) The function h is as follows for $x \in \mathbb{R}$:

$$h(x) = \frac{6x^2 - 23x + 20}{2x - 5}$$

Express $h(x)$ in the form $ax + b$, where $a, b \in \mathbb{Z}$.

(c) The graph of a cubic function $g(x)$ is shown below.



Use the graph to estimate each of the following:

(i) the 3 roots of $g(x)$.

Note: In the case of the next two parts, show your work clearly on the graph.

(ii) $g(3{\cdot}5)$.

(iii) The values of x for which $g(x) = -8$.

Question 5 **(30 Marks)**

(a) Term n of an arithmetic sequence is given by $T_n = 4n + 1$, for $n \in \mathbb{N}$.

(i) Write down the first three terms of the sequence.

(ii) Term k of the sequence is 101. Find the value of k.

(b) The following identity is true for all $n \in \mathbb{N}$:

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2},$$

(i) Use the expression $\frac{n(n+1)}{2}$ to find the sum of the natural numbers from 1 to 12 inclusive.

(ii) The sequence of numbers $1, 2, 3, 4, \cdots, n$ add to 325.

Solve the equation

$$\frac{n(n+1)}{2} = 325,$$

to find the value of n.

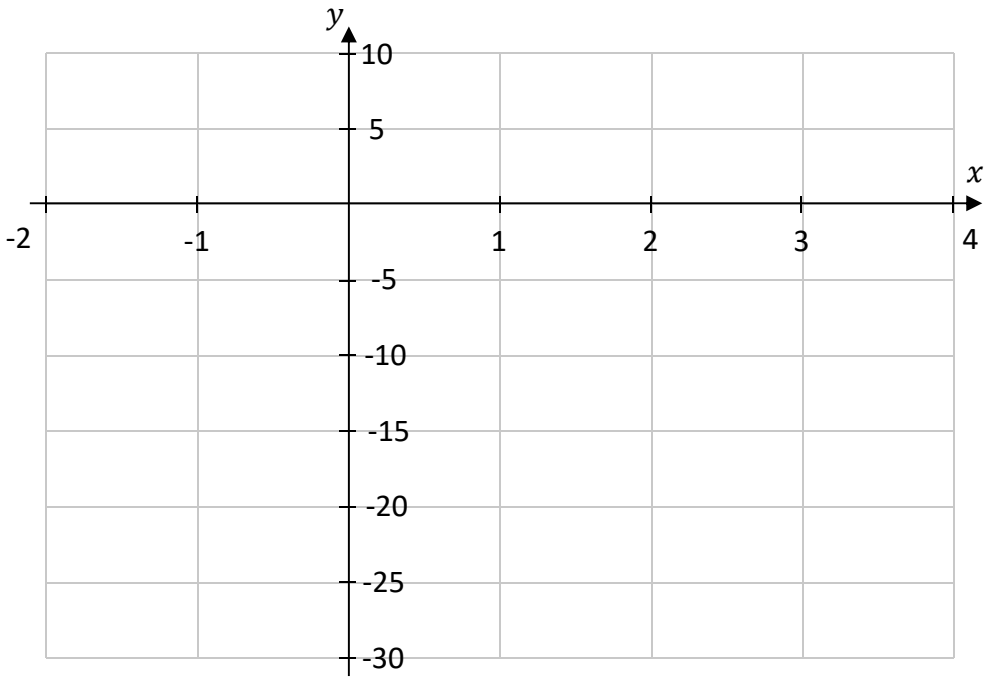
Question 6 **(30 Marks)**

Let $f(x) = x^3 - 3x^2 - 9x$, where $x \in \mathbb{R}$.

(a) Complete the table below to show the value of $f(x)$ for each of the given values of x.

x	-2	-1	0	1	2	3	4
$f(x)$							-20

(b) Draw the graph of $f(x)$ in the domain $-2 \leq x \leq 4$ where $x \in \mathbb{R}$, on the grid below.



(c) **(i)** Find $f'(x)$, the derivative of $f(x)$.

(ii) Use your derivative to find the co-ordinates of the local maximum point **and** the local minimum point of $f(x)$.

Local maximum point (,) Local minimum point (,)

Section B — Contexts and Applications — 150 marks

Answer **any three** questions from this section.

Question 7 **(50 marks)**

A 5 km run takes place every Saturday morning over the same course, starting at **9:30 a.m.**

(a) The average time taken to finish the run is 34 minutes 30 seconds.
Find the average time it takes to complete 1 km of the run.
Give your answer in minutes and seconds.

(b) **(i)** One Saturday morning Aoife ran as follows:

First km:	5 minutes 10 seconds
Second km:	5 minutes 45 seconds
Third km:	6 minutes 25 seconds
Fourth and fifth km:	constant speed.

She finished the race at 10:05 a.m.

How long did it take her to run the last km?

(ii) Find Aoife's average speed over the 5 km run.
Give your answer, **in km/hr**, correct to 1 decimal place.

(c) One Saturday Brian arrived late and started his run 3 minutes later than everyone else (at 9:33 a.m.)
His average speed, while he was running, was 15 km/hr.
Shane, who started on time (9:30 a.m.), ran at an average speed of 12·5 km/hr.
Which of the two finished the run first **and** by how many minutes?

First to finish:

Time Difference:

This question continues on the next page.

(d) Shane is training for the run. One of his training sessions is as follows, where $n \in \mathbb{N}$:

Warm up	Sprints	Warm down
10 minutes	n sprints for 10 seconds each Jog for 60 seconds between each pair of sprints	9 minutes

It takes Shane exactly 32 minutes to complete this session.
Work out the value of n, the number of 10-second sprints he does during this session.

Question 8 **(50 marks)**

An oil lorry delivered heating oil to John's home.
Before the delivery the meter reading in the lorry showed that it contained 10 360 litres of oil.
After the delivery the meter reading was 9160 litres.

(a) **(i)** One litre of oil costs €0·6168.
Find the cost of the heating oil that was delivered to John's home.

(ii) The cost in **part (a)(i)** does **not** include VAT.
Find the total cost of the heating oil after VAT at 13·5% is included.

(b) Six months later the same amount of oil, including VAT at the same rate, cost €1157·70.
What was the price **per litre** of this oil **excluding** VAT?
Give your answer correct to the nearest cent.

This question continues on the next page.

(c) John discovered a leak in his oil tank.
He contacted two companies and asked for quotes to repair the tank.
Each company included a call out charge and a rate per hour, or part of an hour, for the repair work.
They emailed the following quotes for the repair work:

Company A: €100 call out charge plus €60 per hour (or part thereof)
Company B: €80 call out charge plus €70 per hour (or part thereof)

(i) If the repair will take 4 hours 15 minutes to complete, how much would each company charge?

Company A: Company B:

(ii) Write a formula to show the total amount that each company charges for n hours of repair work, where $n \in \mathbb{N}$.

Formula for Company A:

Formula for Company B:

(iii) After how many hours would the cost be the same for both companies?

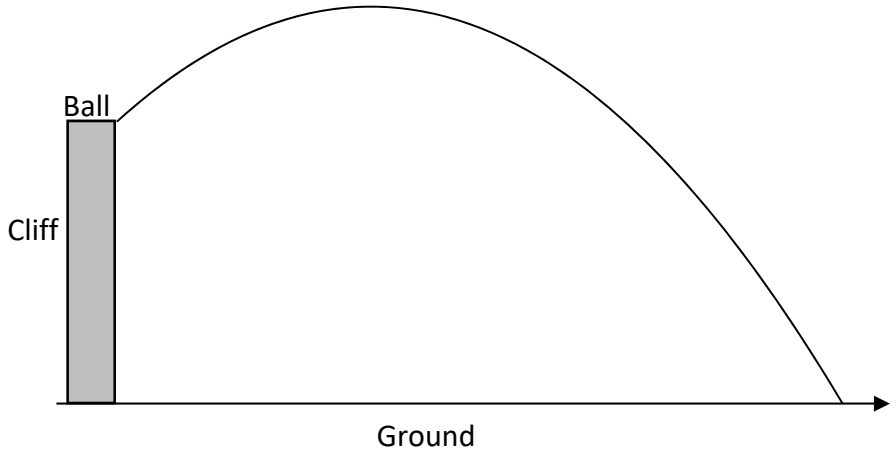
(d) John's oil tank is in the shape of a cylinder.
It is 2 m long and can hold 1500 litres of oil.
Find the radius of the oil tank.
Give your answer correct to the nearest cm.
(**Note**: 1 Litre $=$ 1000 cm^3.)

Question 9 **(50 marks)**

(a) A golf ball was placed on top of a vertical cliff above some level ground.
Florence, standing on the edge of the cliff hit the ball with a golf club, off the cliff and into the air.
While it was travelling through the air, the height of the ball above the ground was given by the function:

$$h(t) = 40 + 10t - 5t^2,$$

where h is the height in metres and t is the time in seconds after the ball was hit.



(i) Find the height of the cliff, in metres.

(ii) What was the height of the ball above the ground after 1·5 seconds?

(iii) Find the maximum height of the ball above the ground.

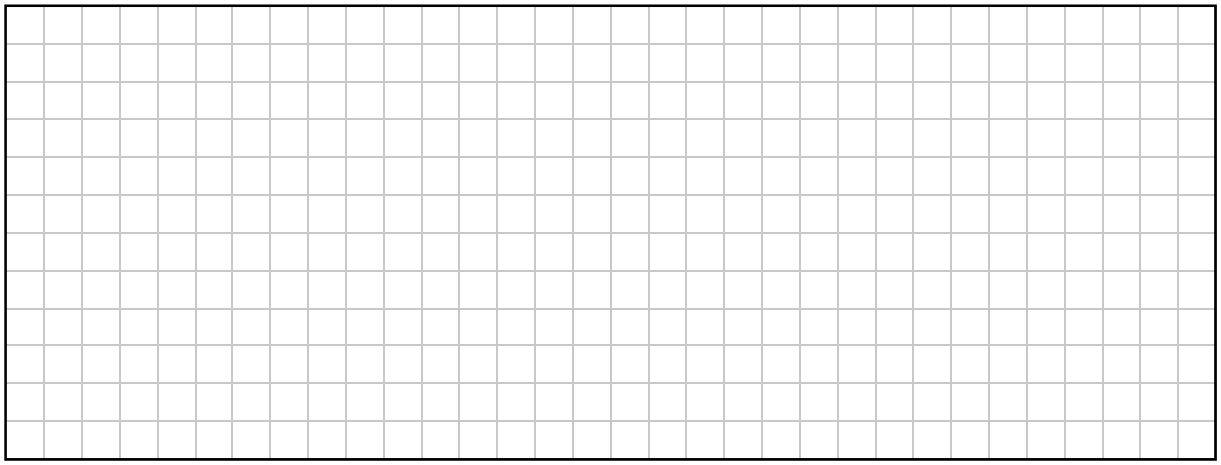
(iv) How long was the ball in the air before it hit the ground?

(v) The ball landed 240 m from the foot of the cliff.
Find the distance between the ball and the top edge of the cliff when it landed.
Give your answer, in metres, correct to 2 decimal places.

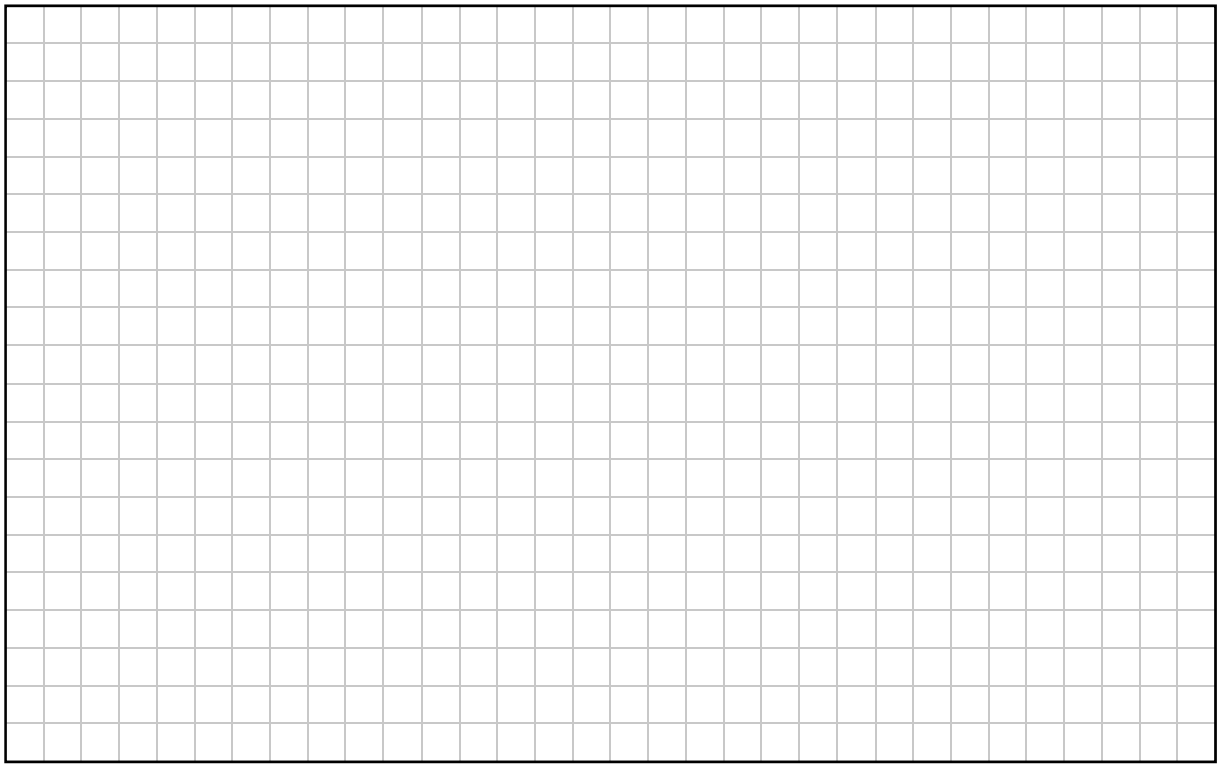
This question continues on the next page.

(b) **(i)** Florence bought a golf club online from a company in Britain.
The cost was £138·75 sterling.
The exchange rate was €1 = £0·8338.

How much did the golf club cost in euro?
Give your answer correct to the nearest cent.



(ii) Florence also bought a golf bag at her local golf club during a sale.
The original cost was €125, including VAT at 23%.
She got a discount of 15% in the sale.
Work out how much **VAT** Florence paid.
Give your answer correct to the nearest cent.



Question 10 **(50 Marks)**

(a) Mary got a new job where she was earning €950 per week. When she started the job, Mary had to pay emergency tax as she did not have a tax credit certificate.
Emergency tax meant that all of her income was taxed at 40% and she had no tax credits.

(i) How much tax did she pay each week while she was on emergency tax?

(ii) When Mary got her tax credit certificate, she started to pay tax at the correct rate.
Now, each week she pays tax at the standard rate of 20% on the first €678·85 of her income and 40% on the remainder.
She also had tax credits of €65·38 each week.

How much does Mary now pay in tax each week?

(iii) Mary was on emergency tax for a total of six weeks.
At the end of that time she got a refund of the tax **that she had overpaid** while she was on the emergency tax rate.

Find the total amount of the refund she got for these six weeks.

(iv) Mary gets a pay increase of € x per week.
She continues to pay tax at the standard rate of 20% on the first €678·85 of her income and 40% on the remainder.
She still had tax credits of €65·38 each week.
She now pays €218·85 in tax each week.
Find the value of x.

This question continues on the next page.

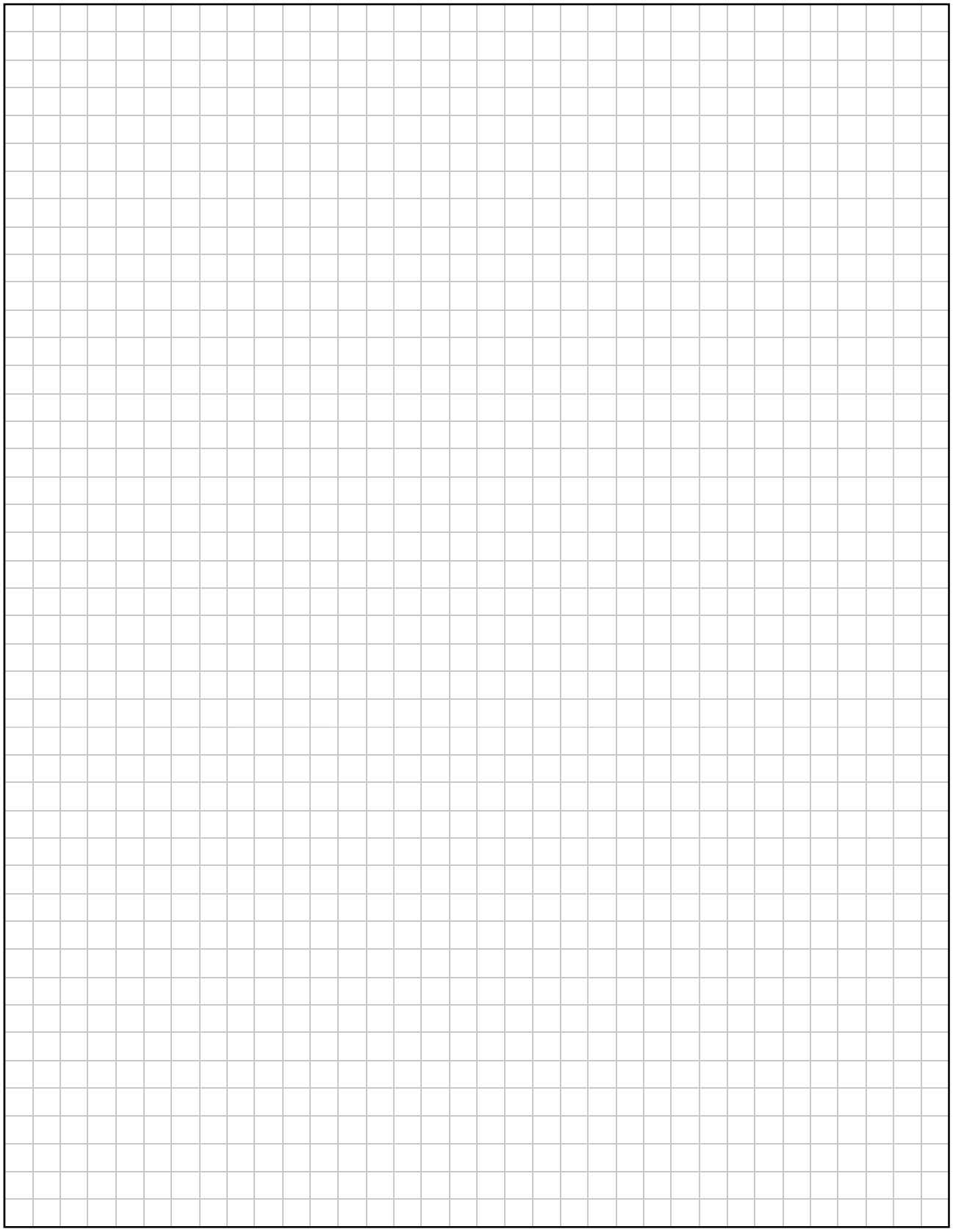
(b) Mary has €15 000 in a savings account and would like to invest the money for a maximum of 3 years. An investment broker is offering two investment options, Option 1 and Option 2, as shown in the table below.

Option 1	Option 2
Invest €15 000 at an annual compound rate of $r\%$ and receive €16 153·36 at the end of 3 years	Invest €5000 at the beginning of each year for 3 years at an annual compound rate of 4·2%

(i) Find the value of the annual compound interest rate, r, in Option 1.

(ii) Mary chooses Option 2.
She will invest €5000 at the beginning of year 1, another €5000 at the beginning of year 2, and another €5000 at the beginning of year 3.
Find how much money Mary will receive at the end of the 3 year period.
Give your answer correct to the nearest cent.

Page for extra work.
Label any extra work clearly with the question number and part.



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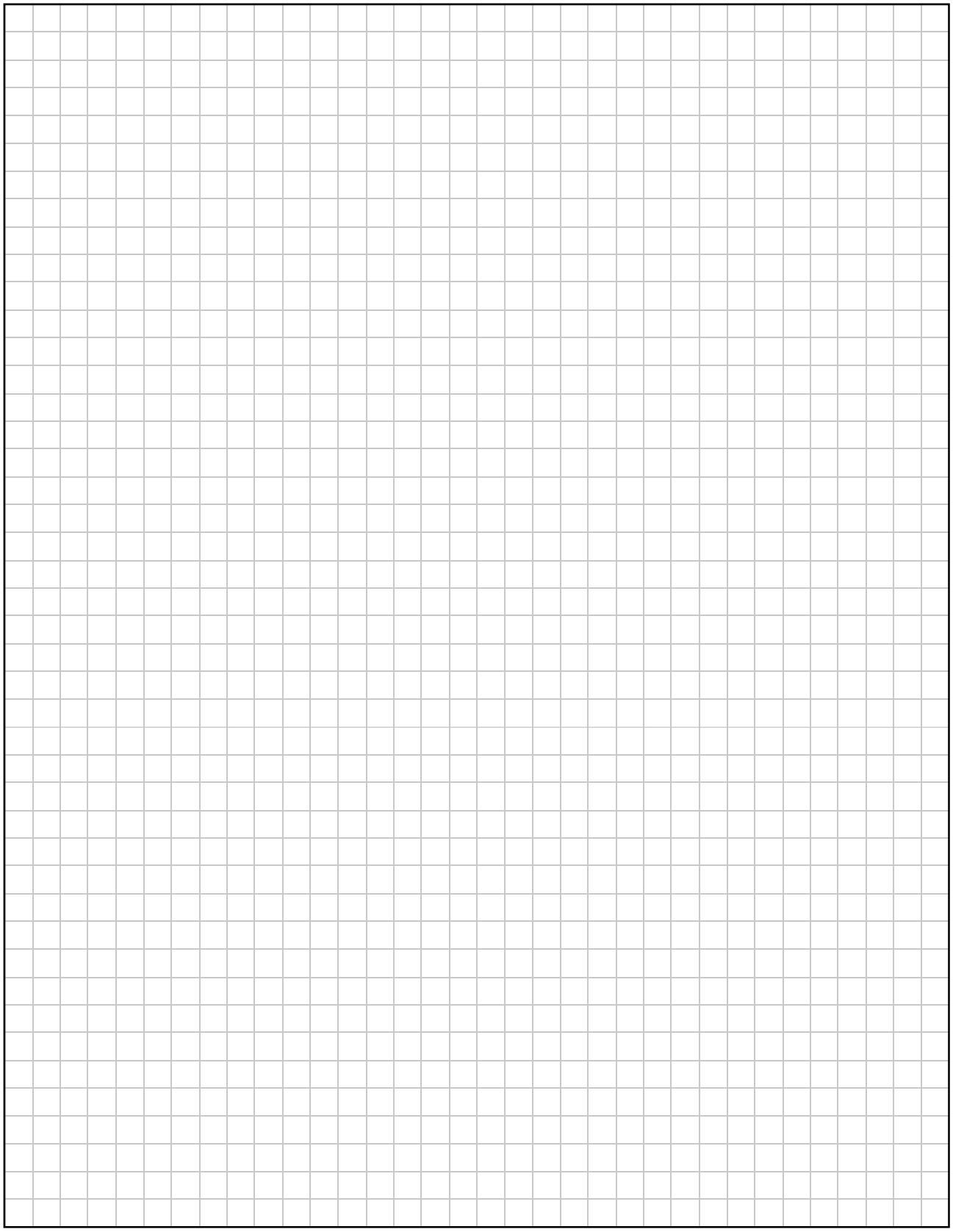

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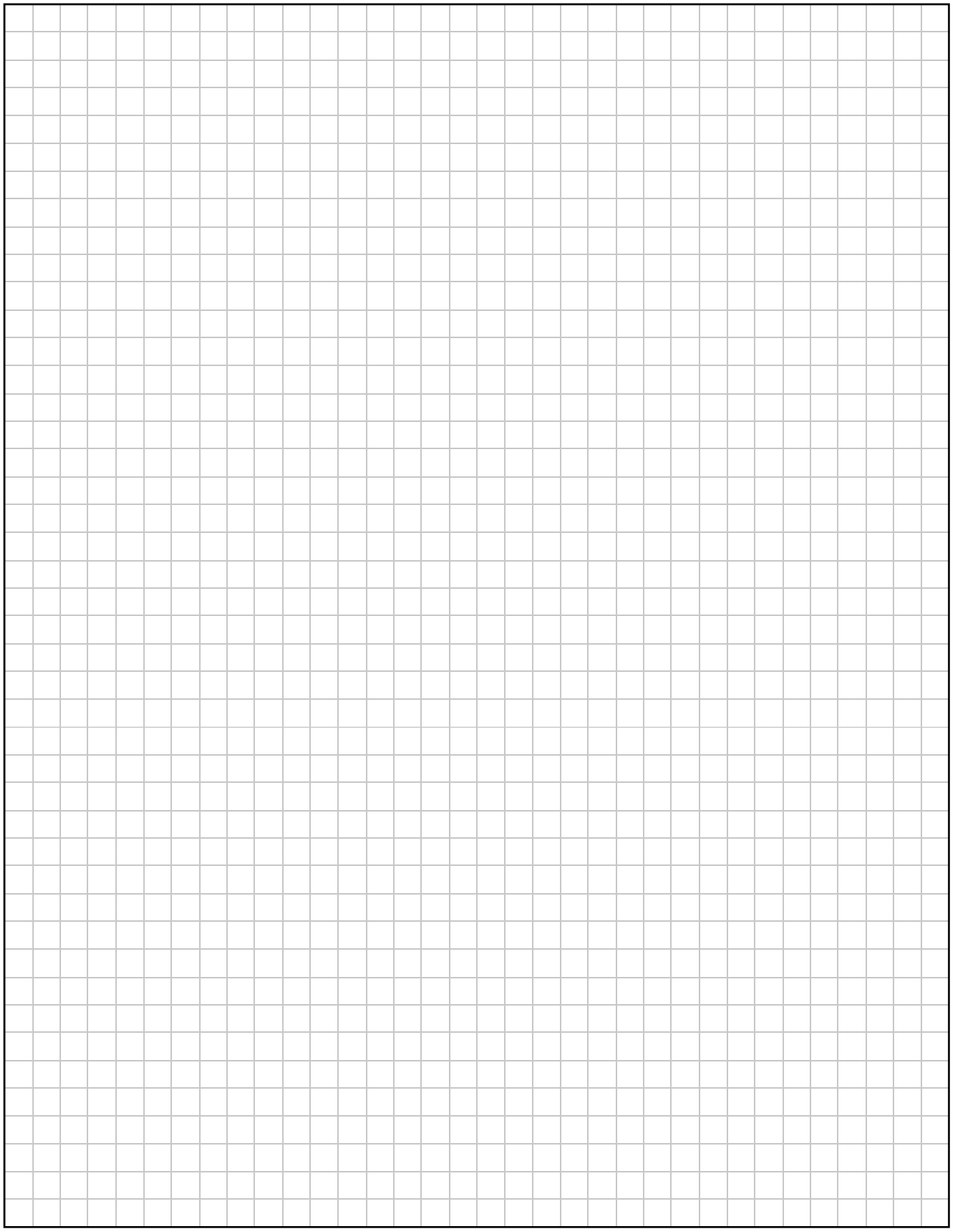

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2 hours 30 minutes