Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2023

Mathematics

Paper 1

Ordinary Level

Friday 9 June   Afternoon 2:00 - 4:30

300 marks

Examination Number

Day and Month of Birth

For example, 3rd February is entered as 0302

Centre Stamp

Do not write on this page

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:
- any **five** questions from Section A   –   Concepts and Skills
- any **three** questions from Section B   –   Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet.
If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **any five** questions from this section.

Question 1 (30 marks)

The prices of houses in a particular area have been increasing over a number of years.

(a) A house is valued at €240 000 at the start of 2019.

By the start of 2020, the price of the house has increased by 8%.
By the start of 2021, the price of the house has increased by a further 9%, based on the 2020 price.

Work out the price of the house at the start of 2020, **and** the price at the start of 2021.
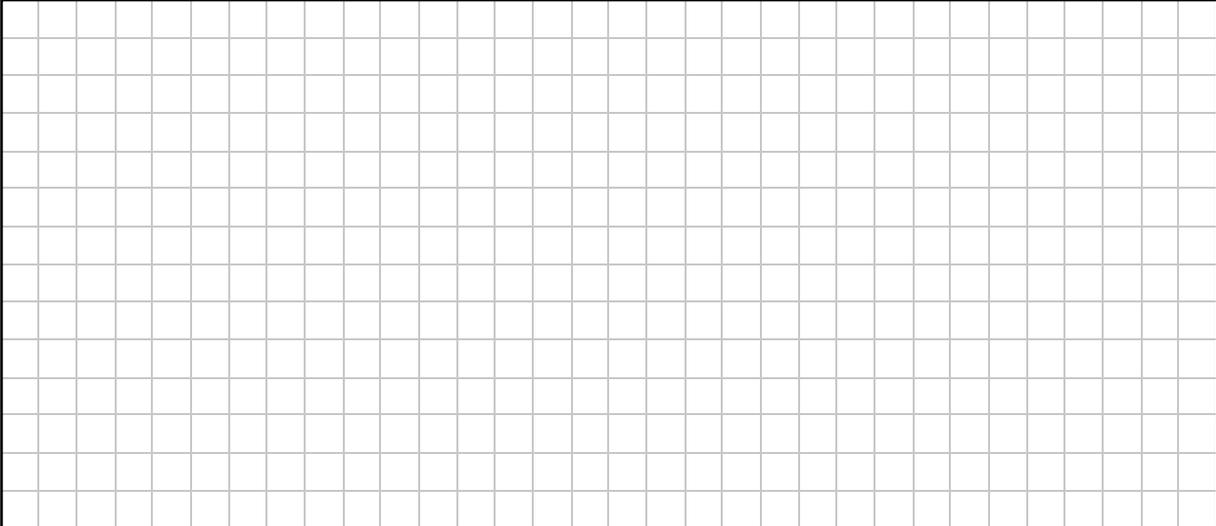
Price at start of 2020 = _____   Price at start of 2021 = _____

(b) Another house was valued at €460 000.
One year later, the value of this house had gone up to €472 000.

Work out the **percentage** increase in the value of this house over the year.
Give your answer correct to 1 decimal place.
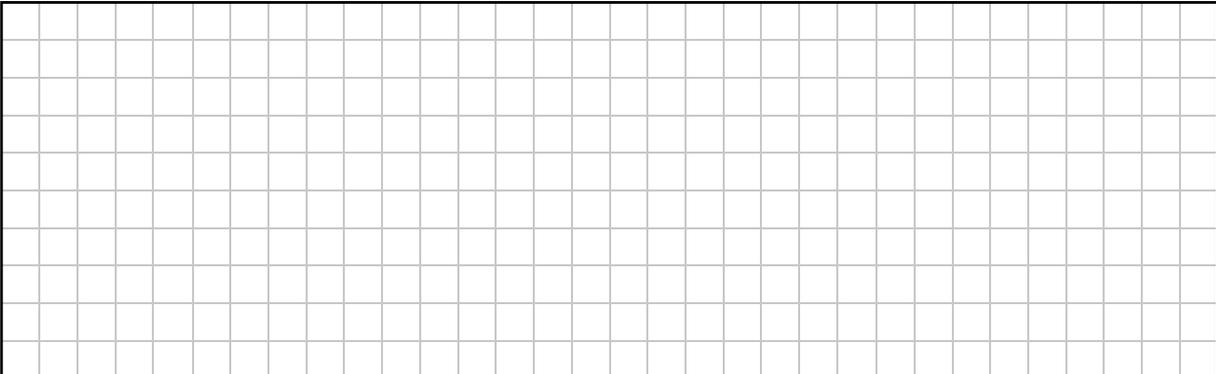
**(c)** A third house was valued at €265 000.

Over the following 4 years, the value of the house increased by $r\%$ each year compared to the previous year, where $r \in \mathbb{R}$.

After 4 years, the house was valued at €370 000.

Work out the value of r, by setting $F = 370\,000$, $P = 265\,000$, and $t = 4$ in this equation:

$$F = P\left(1 + \frac{r}{100}\right)^t$$

Give your answer correct to 1 decimal place.
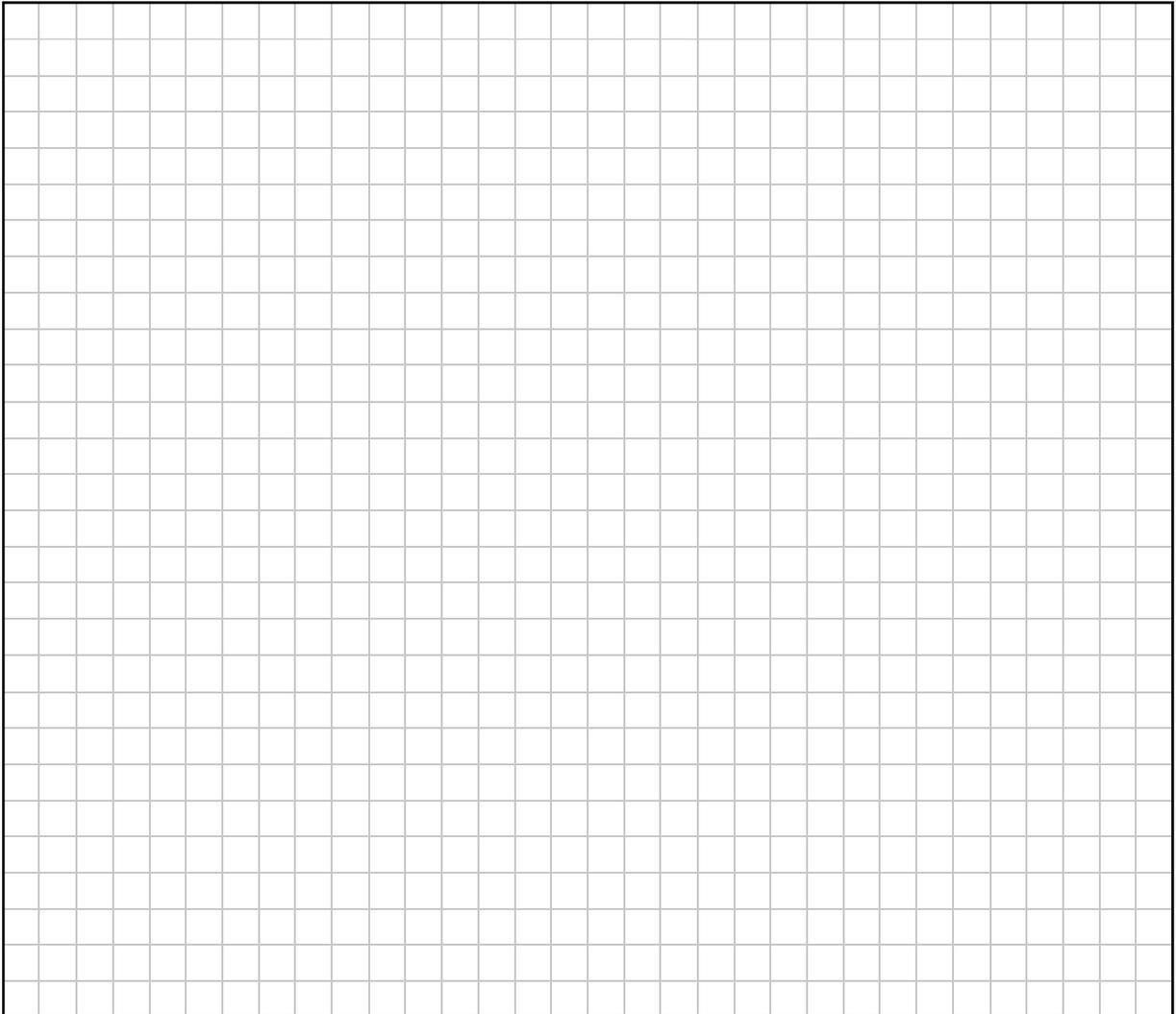
Question 2 (30 marks)

In this question, $i^2 = -1$.

**(a)** The complex number z_1 is shown on the Argand diagram below.
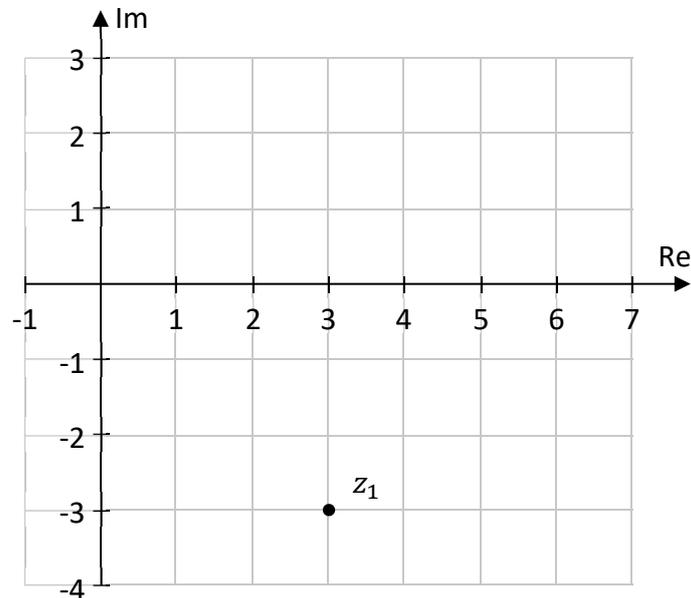
(i) **Plot** and **label** $\overline{z_1}$, the complex conjugate of z_1 , on the diagram above.

z_2 and z_3 are two other complex numbers.

$z_2 = 5 + i$ and $z_3 = 6 - 2i$.

**(ii)** **Plot** and **label** z_2 and z_3 on the diagram above.

(iii) Write $4z_2 - 5z_3$ in the form $a + bi$, where $a, b \in \mathbb{Z}$.
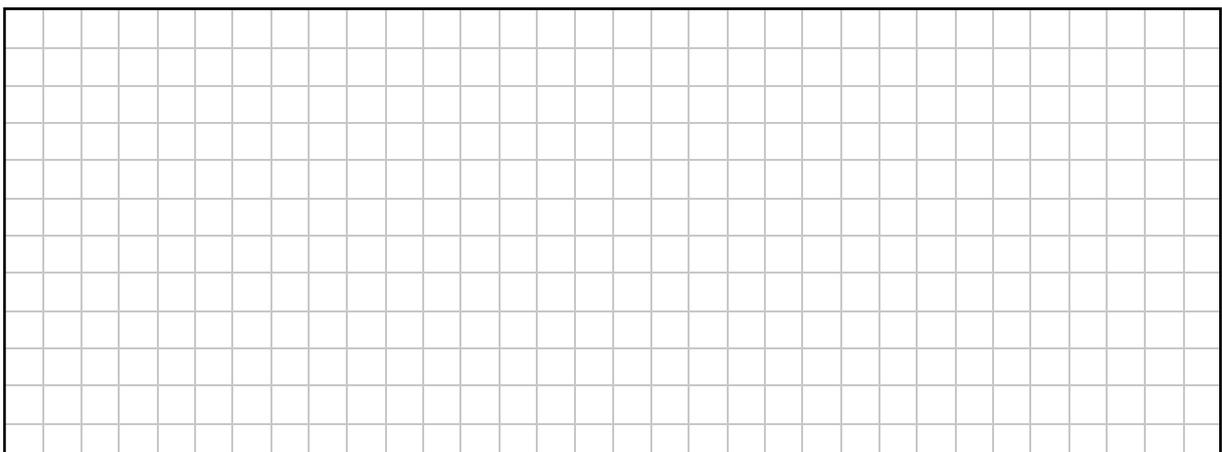
(iv) Work out the value of $|z_3|$. Give your answer in surd form.
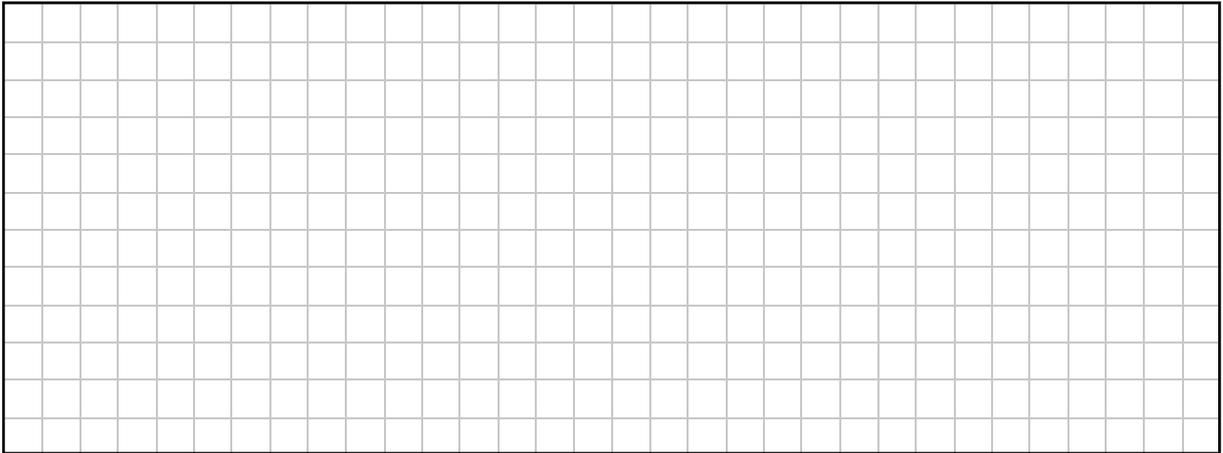
(b) The complex numbers z_4, z_5, z_6, and z_7 are shown on the Argand diagram below. The diagram is to scale.
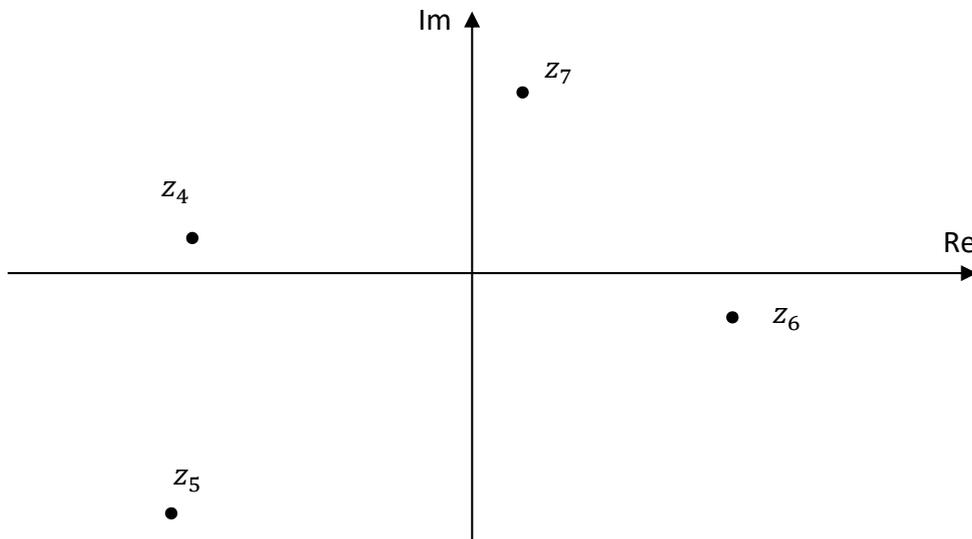
Tick one box to indicate which of z_4, z_5, z_6, or z_7 has the largest **modulus**. Give a reason for your answer.

Number with the largest modulus: z_4 z_5 z_6 z_7
(tick **one** box only)   ☐   ☐   ☐   ☐

Reason:

Question 3 **(30 marks)**

(a) Solve the following equation in $x \in \mathbb{R}$:

$$3(2x + 4) - 5 = 3$$
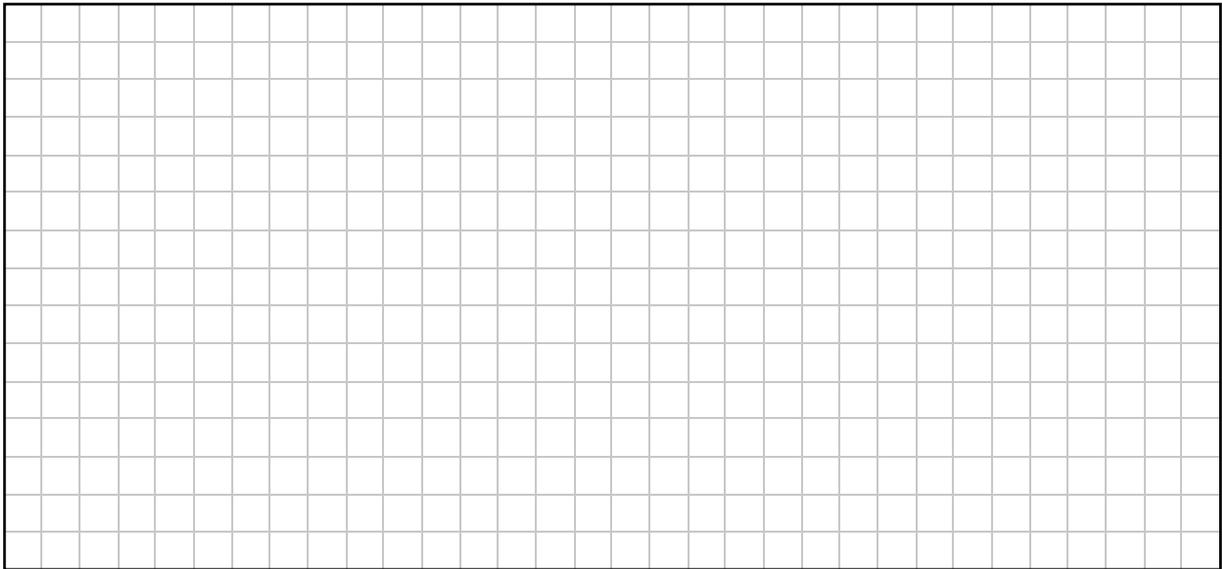
(b) Graph each of the following inequalities on the given number line.
Note that x comes from a **different set of numbers** in each case.

(i) $x \leq 2{\cdot}8$, where $x \in \mathbb{R}$.

```
  ┼──┼──┼──┼──┼──┼──┼──┼──┼──┼
 -4  -3  -2  -1  0   1   2   3   4
```

(ii) $x \leq 2{\cdot}8$, where $x \in \mathbb{Z}$.

```
  ┼──┼──┼──┼──┼──┼──┼──┼──┼──┼
 -4  -3  -2  -1  0   1   2   3   4
```

(iii) $x \leq 2{\cdot}8$, where $x \in \mathbb{N}$.

```
  ┼──┼──┼──┼──┼──┼──┼──┼──┼──┼
 -4  -3  -2  -1  0   1   2   3   4
```
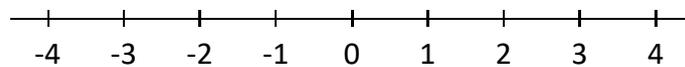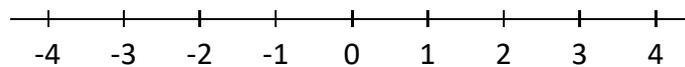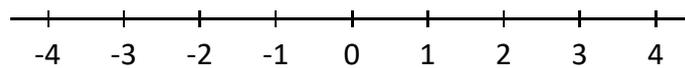
(c) Use algebra to solve the following simultaneous equations:

$$x - y + 1 = 0$$
$$2x + 4y = 19$$

Question 4 (30 marks)

(a) The graph of the function g is shown on the co-ordinate diagram below, for $-3 \leq x \leq 2$, $x \in \mathbb{R}$. Use the graph to answer **parts (a)(i)** to **(a)(iv)**.

(i) Estimate the value of $g(1\cdot5)$.
Show your work on the graph.

$g(1\cdot5):$ ☐

(ii) Estimate the value of x for which $g(x) = -6$.
Show your work on the graph.

$x:$ ☐

(iii) Tick **one** box to show how many real roots the function $g(x)$ has in this domain.
Give a reason for your answer.

Number of real roots:	0	1	2	3
(tick **one** box only)	☐	☐	☐	☐

Reason:

(iv) From the graph, estimate the co-ordinates of the local maximum point and the local minimum point of $g(x)$, as accurately as possible.

Local maximum point: (,)

Local minimum point: (,)

(b) A different function, $h(x)$, has the following equation:

$$h(x) = x^3 + 2x^2 - x - 8$$

(i) Find $h'(x)$, the derivative of $h(x)$.

(ii) Find the equation of the tangent to $h(x)$ at the point $(2, 6)$.

Question 5 **(30 marks)**

(a) A company is repairing a railway line. As part of this work, it is laying new railway tracks.

(i) It costs €12 000 to lay 240 metres of railway track.
Work out how much would it cost to lay 320 metres of railway track, at the same rate.
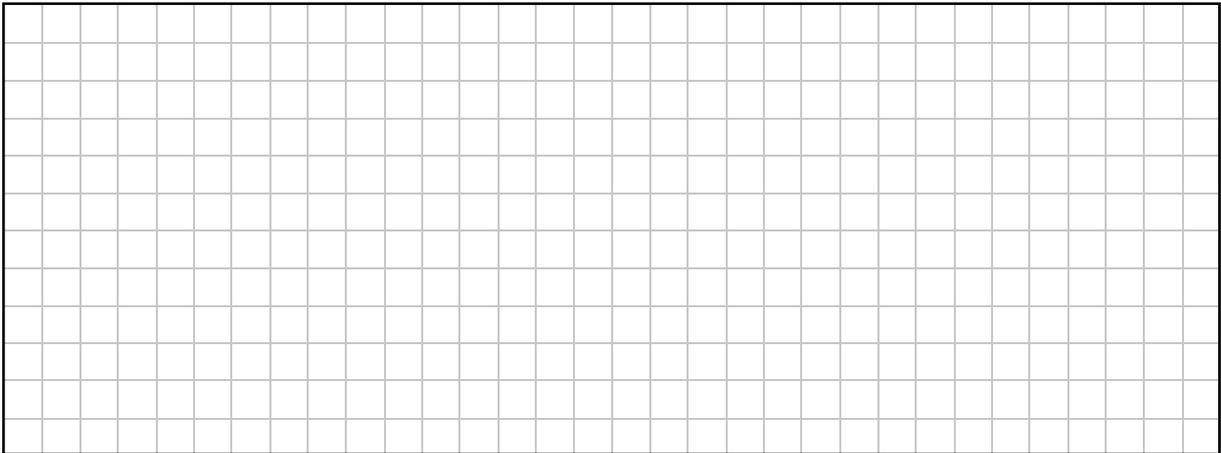
(ii) It would take 6 people 8 days to replace the tracks on a different part of the railway line. Work out how long it would take 4 people to replace these tracks, each working at the same speed.
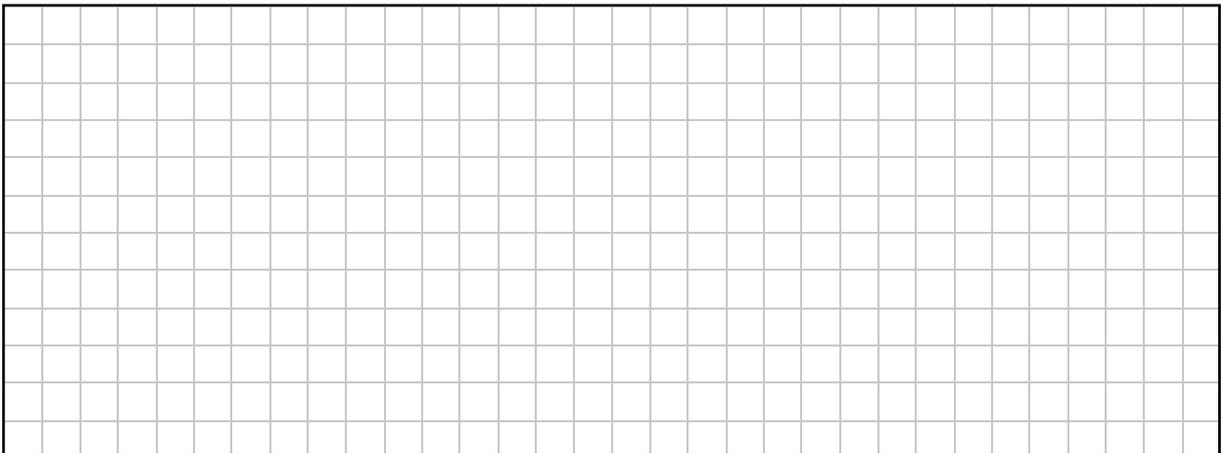
(b) Two towns, **A** and **B**, are 120 km apart.
A train travels from **A** to **B** at an average speed of 180 km/hour.

(i) Work out the time it takes the train to get from **A** to **B**.
Give your answer in hours, as a fraction.

(ii) On the way back from **B** to **A**, the train travels at an average speed of 220 km/hour.
Work out the average speed of the train for the entire journey, while it is travelling from **A** to **B** and back again. Give your answer in km/hour.

Hint: first work out the total time taken for the entire journey.

Question 6 (30 marks)

(a) $g(x)$ is the following function in $x \in \mathbb{R}$:

$$g(x) = x^2 + 8x - 6$$

(i) Find the value of $g(-5)$.

(ii) Use calculus to find the value of x which gives the minimum value of $g(x)$.

(b) Expand and simplify $(2x - 4)^2 - 6$.

(c) Solve the following equation in x:

$$9x^2 + 6x - 5 = 0$$

Give each answer correct to 2 decimal places.
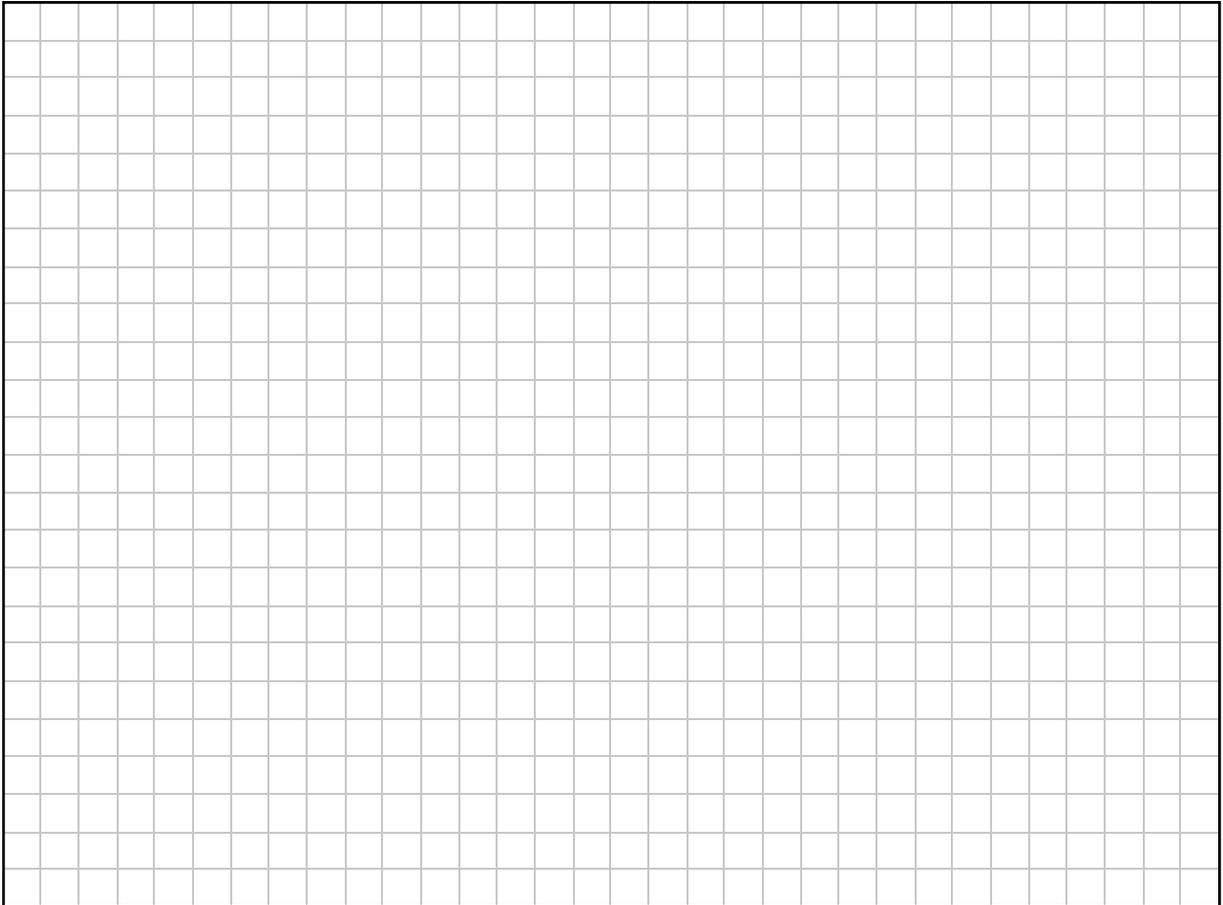
Answer **any three** questions from this section.

Question 7 **(50 marks)**

(a) When it rains on land, some of the rain soaks into the land, and the rest runs off the land. The runoff curve number, C, is a number used when estimating the amount of rain that runs off a particular area of land. C is given by:

$$C = \frac{1000}{S + 10}$$

where S is a measure of the maximum amount of rain that can soak into the soil.

(i) Find the value of C when $S = 15$.

(ii) When S **increases**, does C increase or decrease?

Justify your answer, using the equation $C = \frac{1000}{S+10}$.

Answer (increases **or** decreases):

Justification:

(iii) Rearrange the equation to write S in terms of C.
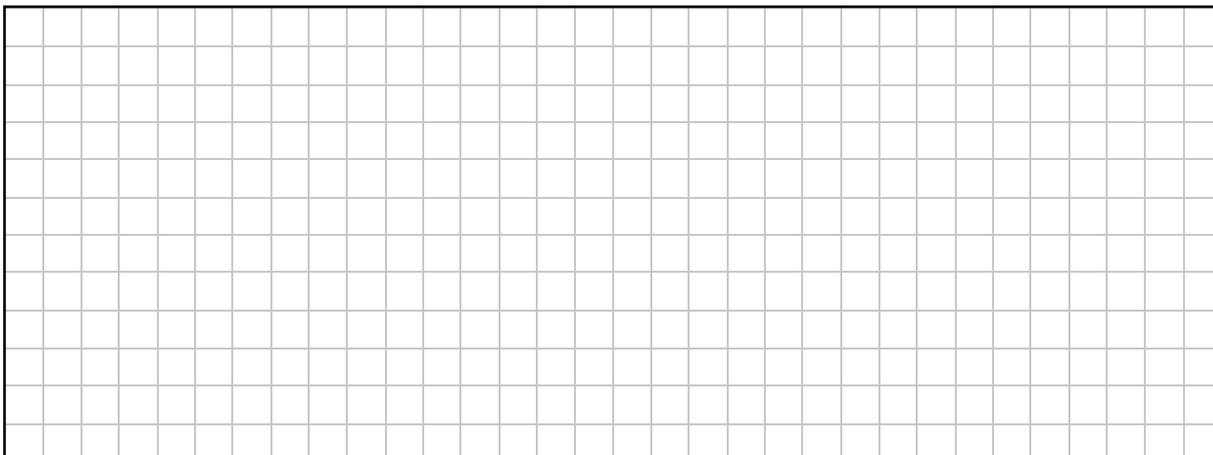
(iv) The values for C go from 30 to 100.
Use your answer from **part (a)(iii)**, or otherwise, to find the range of values of S.
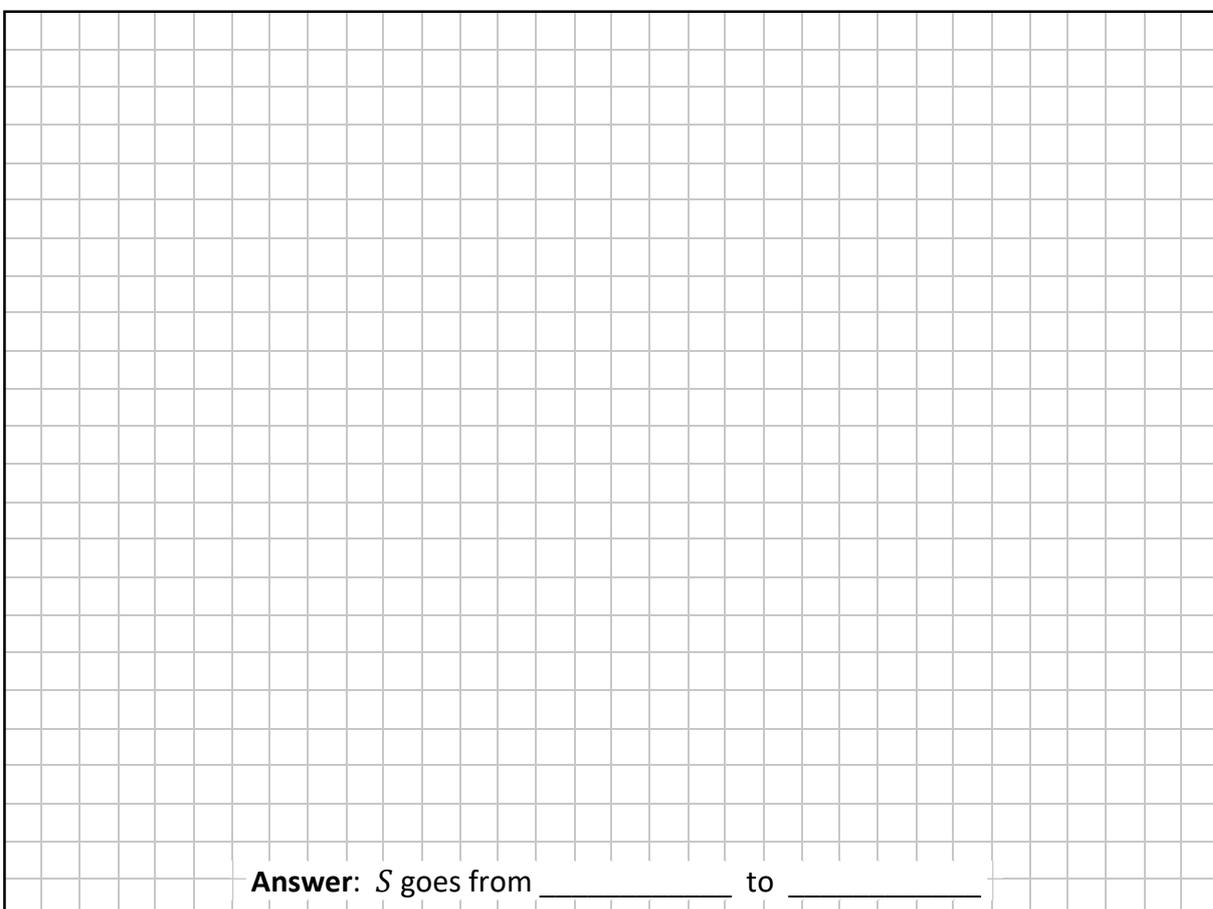
Answer: S goes from _____ to _____

This question continues on the next page.

(b) Over the course of one day (24 hours), the probability that it is raining in Waterville, $P(t)$, could be modelled by the following function:

$$P(t) = 0.3 + 0.02\,t.$$

Here, $P(t)$ is the probability that it is raining t hours after the start of the day, with $0 \leq t \leq 24$, $t \in \mathbb{R}$.

(i) Write down the probability that it is raining at the start and the end of the day, that is, find the value of $P(0)$ and $P(24)$.

(ii) Find the value of $P'(t)$, the derivative of t, **and** explain what the value of $P'(t)$ means in this context.

$P'(t)$:

Meaning of $P'(t)$:

(iii) At a certain time, the probability that it is raining, according to this model, is 0.35. Work out the probability that it is raining exactly 1 hour later, according to this model.

Question 8 **(50 marks)**

A rectangular garden (50 m by 30 m) contains an irregularly shaped pond of length 18 m, as shown in the diagram below (not so scale).

(a) Find the area of the whole garden (that is, the total area of the rectangle).

(b) The width of the pond is measured at intervals of 3 metres, starting at the right-hand edge of the pond. These measurements are given on the diagram, correct to the nearest metre.

Use the **Trapezoidal Rule** (and the measurements in the diagram above) to work out an estimate for the area of the pond.

This question continues on the next page.

(c) Suggest a way by which the Trapezoidal Rule could be used (with different measurements) to get a potentially more accurate estimate of the area of the pond.
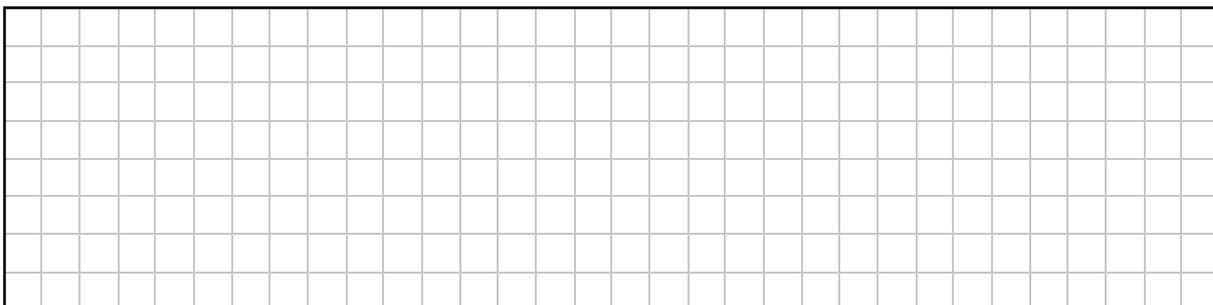
(d) Apart from the pond, the rest of the rectangular garden is covered in grass.
The actual area of the pond is 172 m².

All of the grass in the garden needs to be resown.
The cost is €2·55 per square metre that needs to be resown.
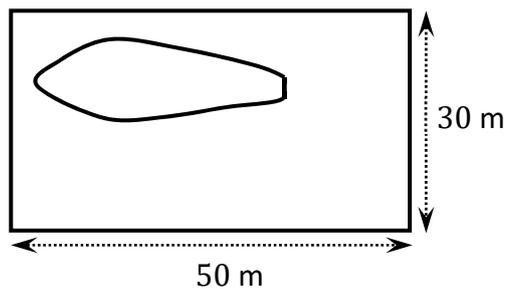
Work out the cost of resowing the grass.
Give your answer correct to the nearest **ten euro**.

30 m

50 m
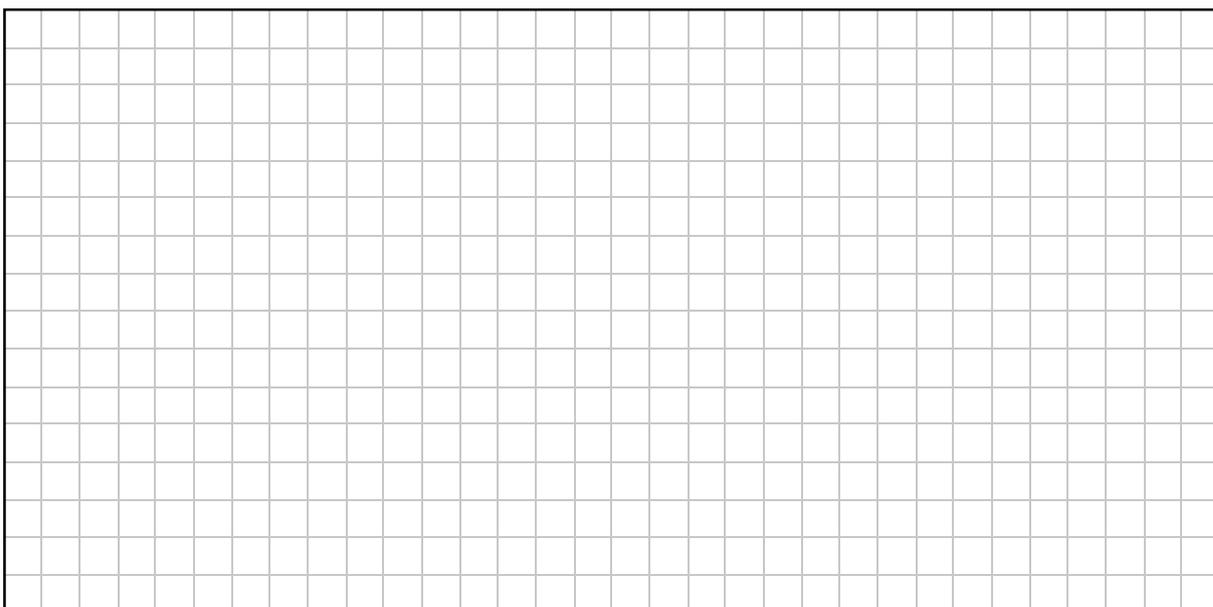
(e) Some materials were imported from England by the gardener for resowing the grass.
The cost of the materials was £840. This included an Import Duty of 16·9%.
Work out the cost of the materials **before** the Import Duty was added.
Give your answer correct to the nearest penny (that is, to 2 decimal places).

(f) The gardener paid the £840 in euro.
The exchange rate was €1 = £0·8547.
He also paid a fee of 2·5% of the £840 as a commission.

Work out how much he paid in total, in euro, correct to the nearest cent.

Question 9 (50 marks)

(a) The diagram below shows the actual daily number of new cases of a disease, from day $t = 0$ to day $t = 25$. It also shows the number of cases according to a mathematical model over this period of time.
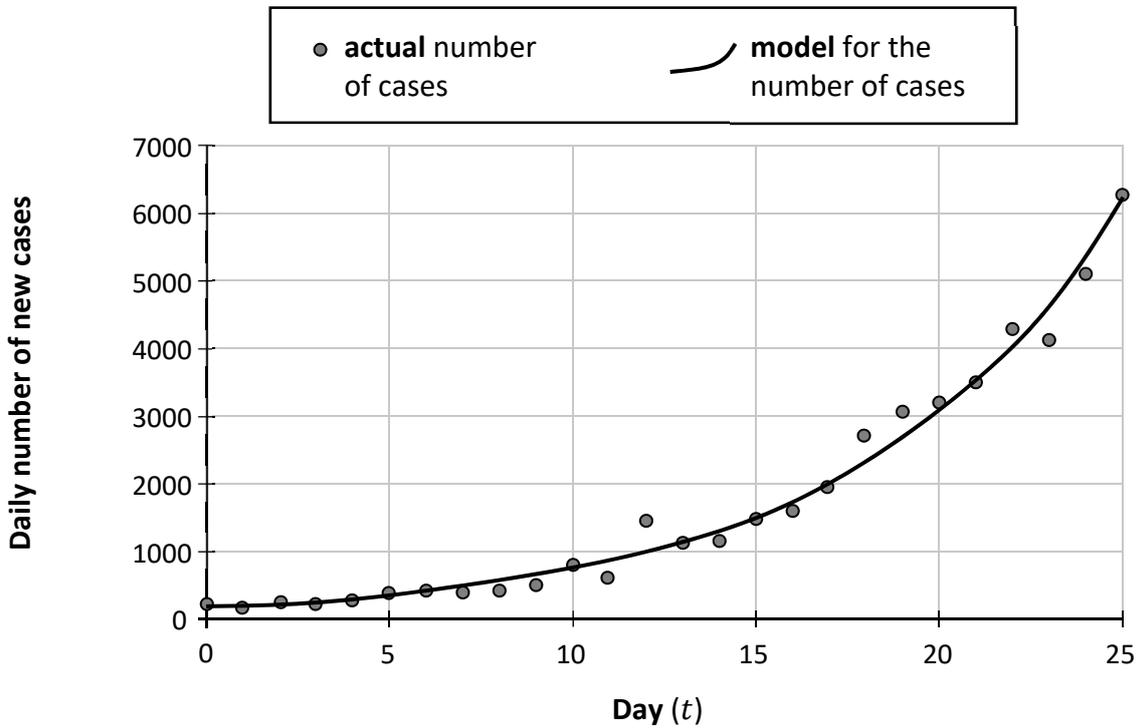
(i) From the diagram, estimate the number of **actual** new cases there were on day $t = 12$, using the relevant **dot**.

Answer for day $t = 12$: _____

According to the **model** (the curve), there were 1000 new cases on day $t = 12$.

(ii) Work out the percentage error in the model's value for day $t = 12$.
Use your answer to **part (a)(i)** above as the actual value for day $t = 12$.
Give your answer correct to the nearest percent.
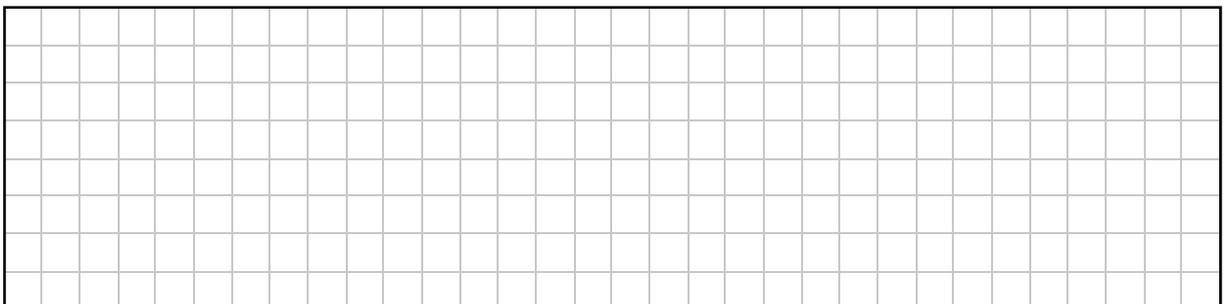
(iii) Show that it takes approximately 5 days for the daily number of new cases to **double**, according to the **model** (that is, according to the curve in the diagram).
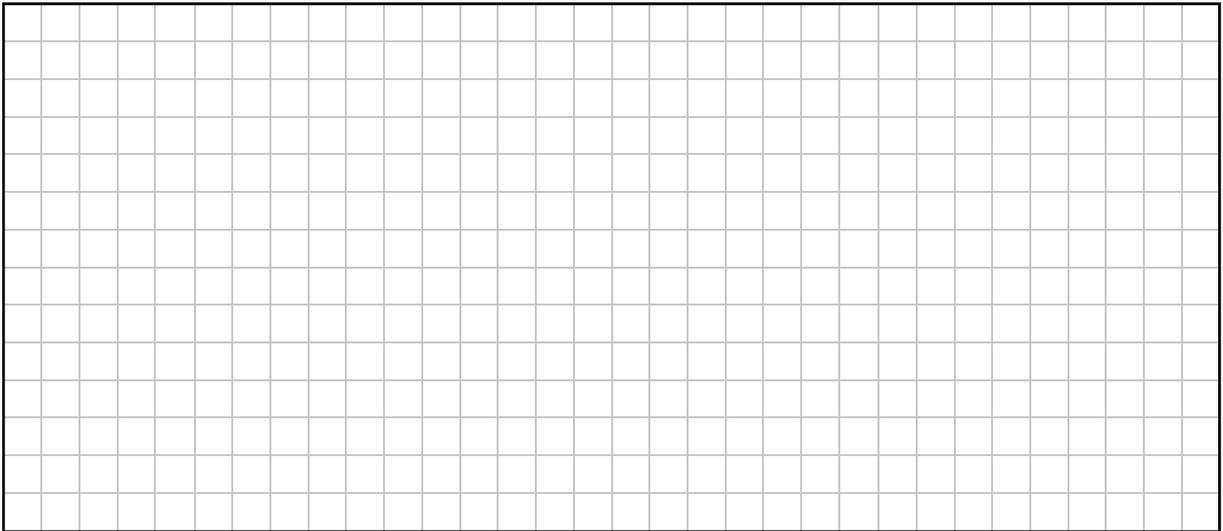
(iv) Hence, estimate on what day (value of t) the number of new cases will first be at least 100 000, according to the **model**. Show your working out.
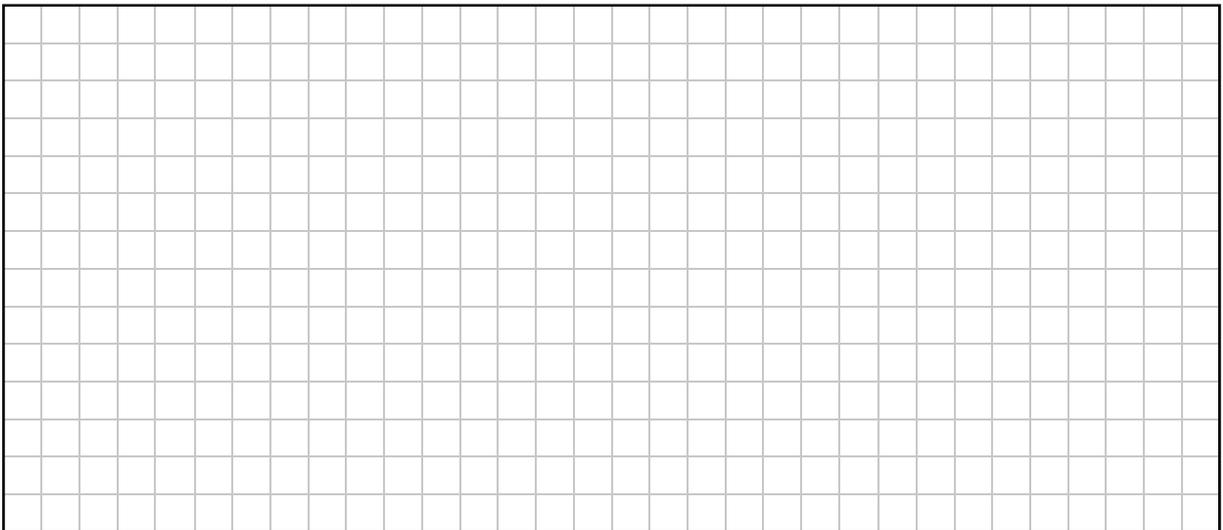
This question continues on the next page.

(b) The daily number of new cases of a different disease can be modelled using the function:

$$D(t) = 2100 \times 1{\cdot}18^t$$

where D is the daily number of new cases and t is the time in days from $t = 0$ to $t = 14$.

(i) Fill in the table below to show the values of $D(t)$ for the given values of t.
Give each value correct to the nearest whole number.

t	0	1	2	3	4	5	6
$D(t)$			2924				5669
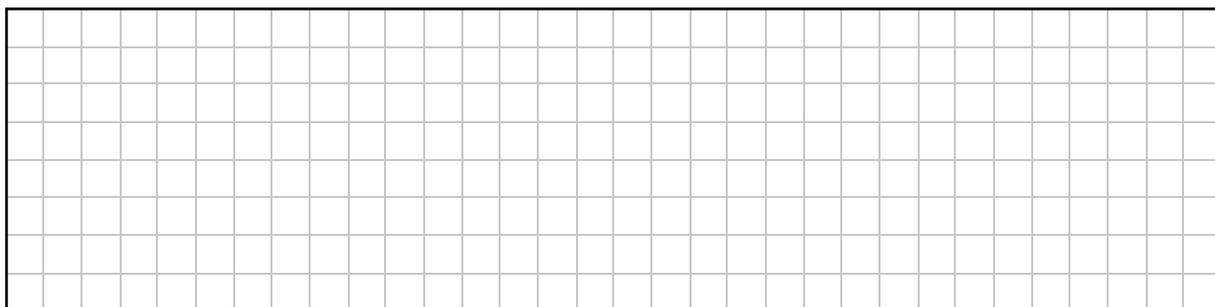
(ii) Draw the graph of $y = D(t)$ on the axes below, for $0 \le t \le 6$ and $t \in \mathbb{R}$.
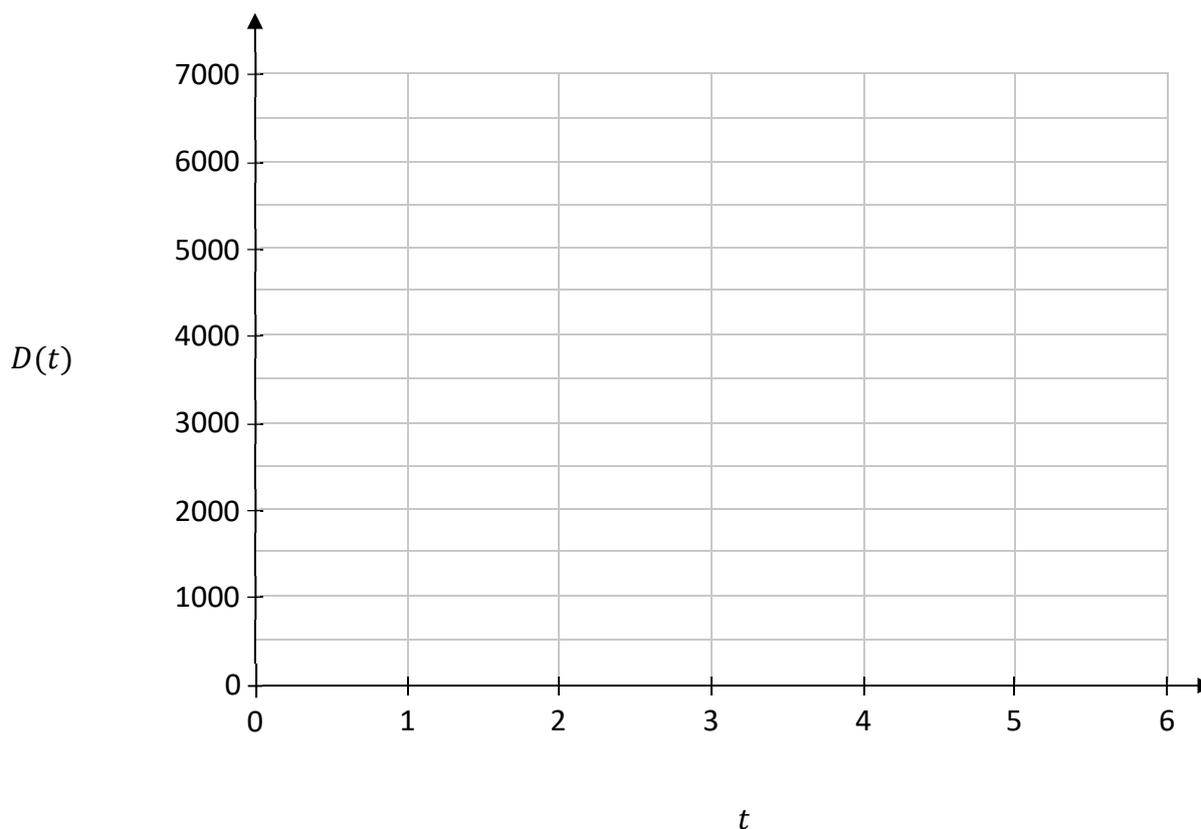
(iii) Measures are put in place to slow down the spread of the disease.
From $t = 14$ days onwards, the number of new daily cases starts to decrease.

For $t \geq 14$, the daily number of new cases on day t can be modelled by:

$$K(t) = a \times 0{\cdot}83^t$$

On $t = 14$ days, there are 21 309 cases, according to this model.
Use this to work out the value of a.

Give your answer in the form $b \times 10^n$, where $1 \leq b < 10$, $n \in \mathbb{N}$, and b is correct to 2 decimal places.
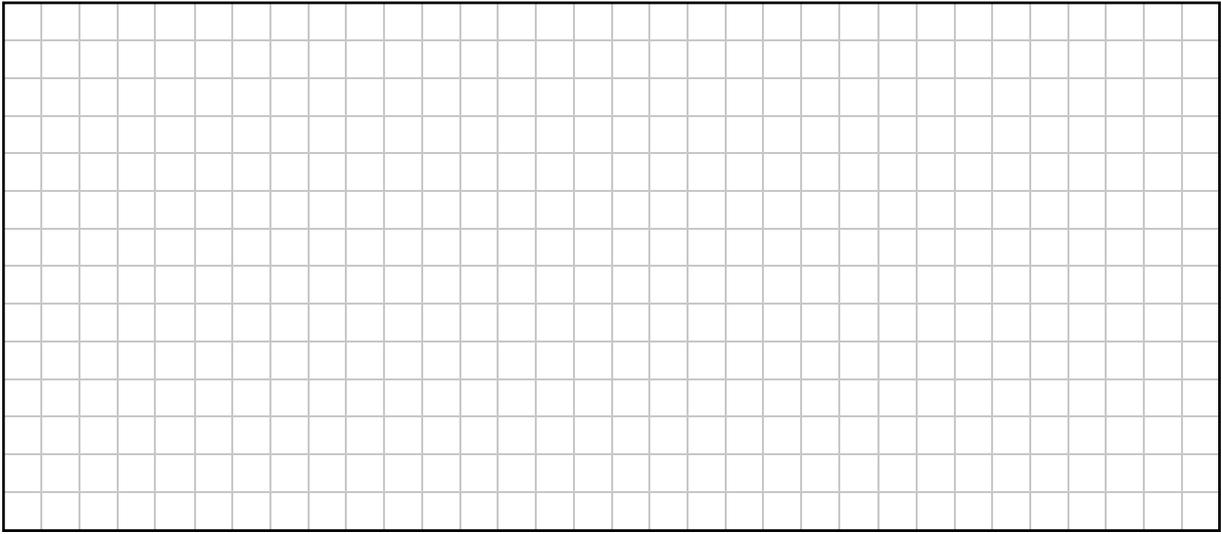
Question 10 (50 marks)

(a) **(i)** Find the **volume** of a cylinder with a diameter of 18 mm and a height of 5 mm.
Give your answer correct to the nearest mm^3.
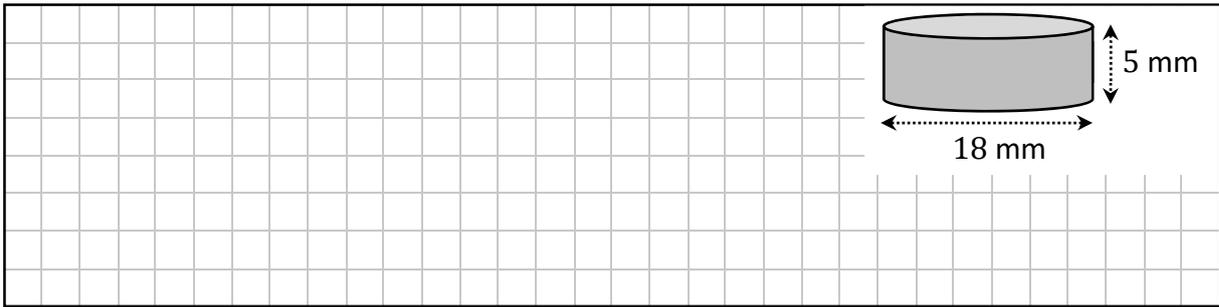
(ii) A metal ring could be made by removing a cylinder of diameter 14 mm and height 5 mm from the cylinder in **part (a)(i)**.

Work out the volume of metal in this ring, correct to the nearest mm^3.
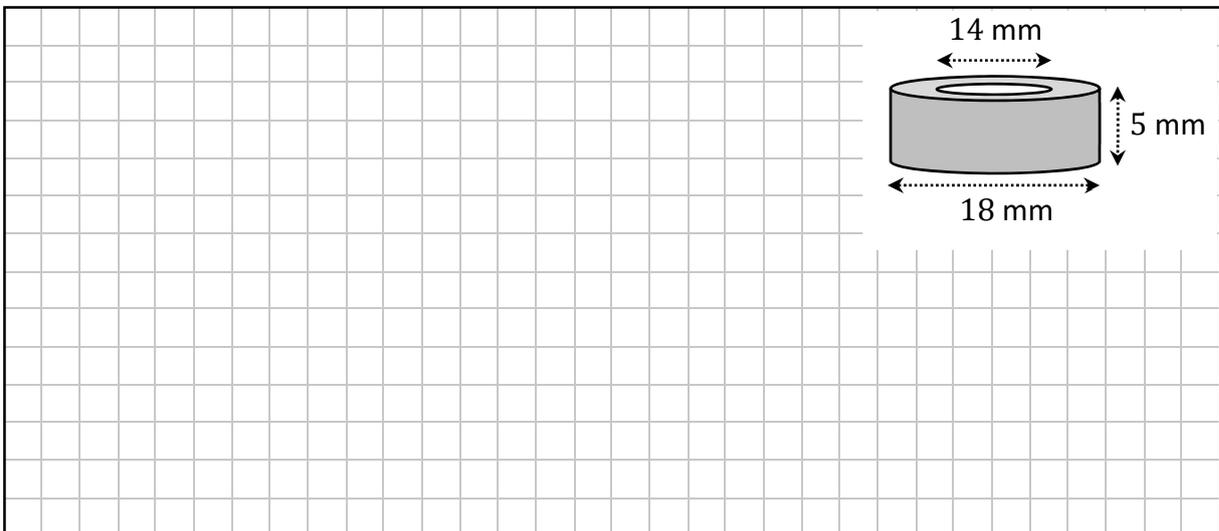
(b) Many gold rings are made of a mixture of pure gold and other metals.

(i) One particular ring weighs 8 grams.
It contains 3 grams of pure gold.

Work out the percentage of the weight of this ring that is pure gold.
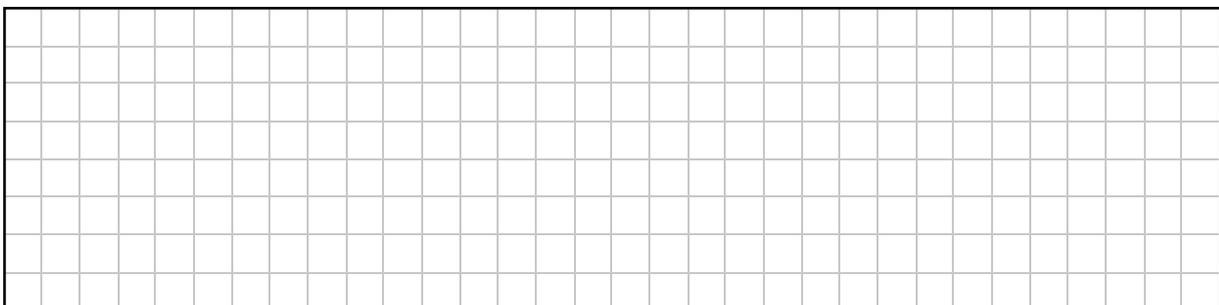
(ii) Pink gold is made up of pure gold, copper, and silver.
The ratio of the weight of pure gold to copper to silver is $15 : 4 : 1$, respectively.

Work out the number of grams of pure gold in a pink gold ring that weighs 25 grams.

(iii) Rose gold is made up of 75% pure gold, 22·25% copper, and 2·75% silver.
Express the ratio of pure gold : copper : silver in rose gold in the ratio $a : b : c$,
in its simplest form, where $a, b, c \in \mathbb{N}$.

This question continues on the next page.

(c) The cost of insuring a piece of jewellery depends on its value.
The table below shows the rates charged per year by one insurance company, where $r \in \mathbb{R}, r > 0$.

Value	Rate per year
First €1000	1% of the value
Next €9000	2% of the excess over €1000, plus the cost of the insuring the first €1000
Amount in excess of €10 000	r % of the excess over €10 000, plus the cost of insuring the first €10 000

(i) Work out the cost per year of insuring a ring valued at €1800.

(ii) The cost of insuring a particular necklace is €142·80 per year.
Work out the value of this necklace.

(iii) A brooch is valued at €17 500.
The cost per year of insuring the brooch is €435.

Work out r, the rate of the insurance charged on the amount of the value of the brooch which is in excess of €10 000. Give your answer as a percentage, correct to 1 decimal place.
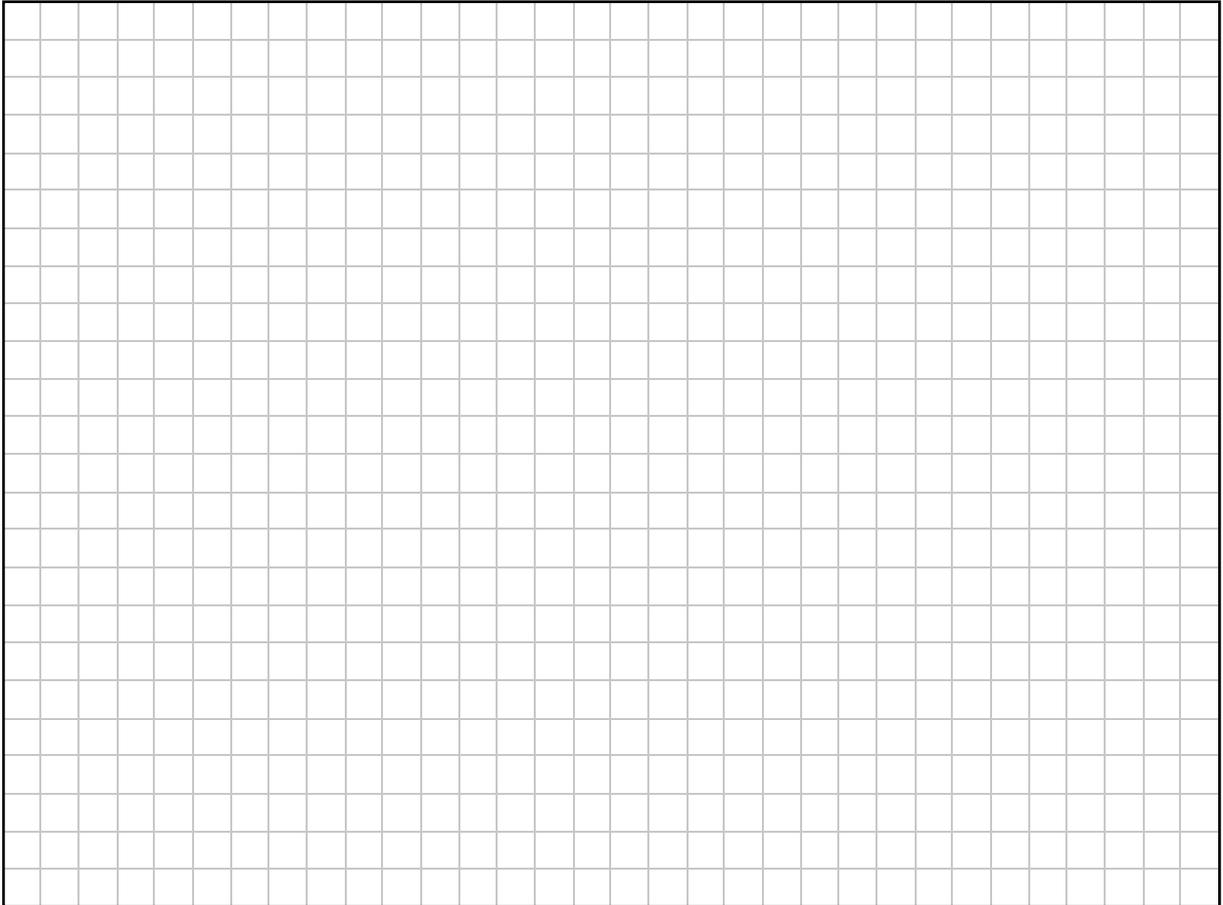
Page for extra work.
Label any extra work clearly with the question number and part.
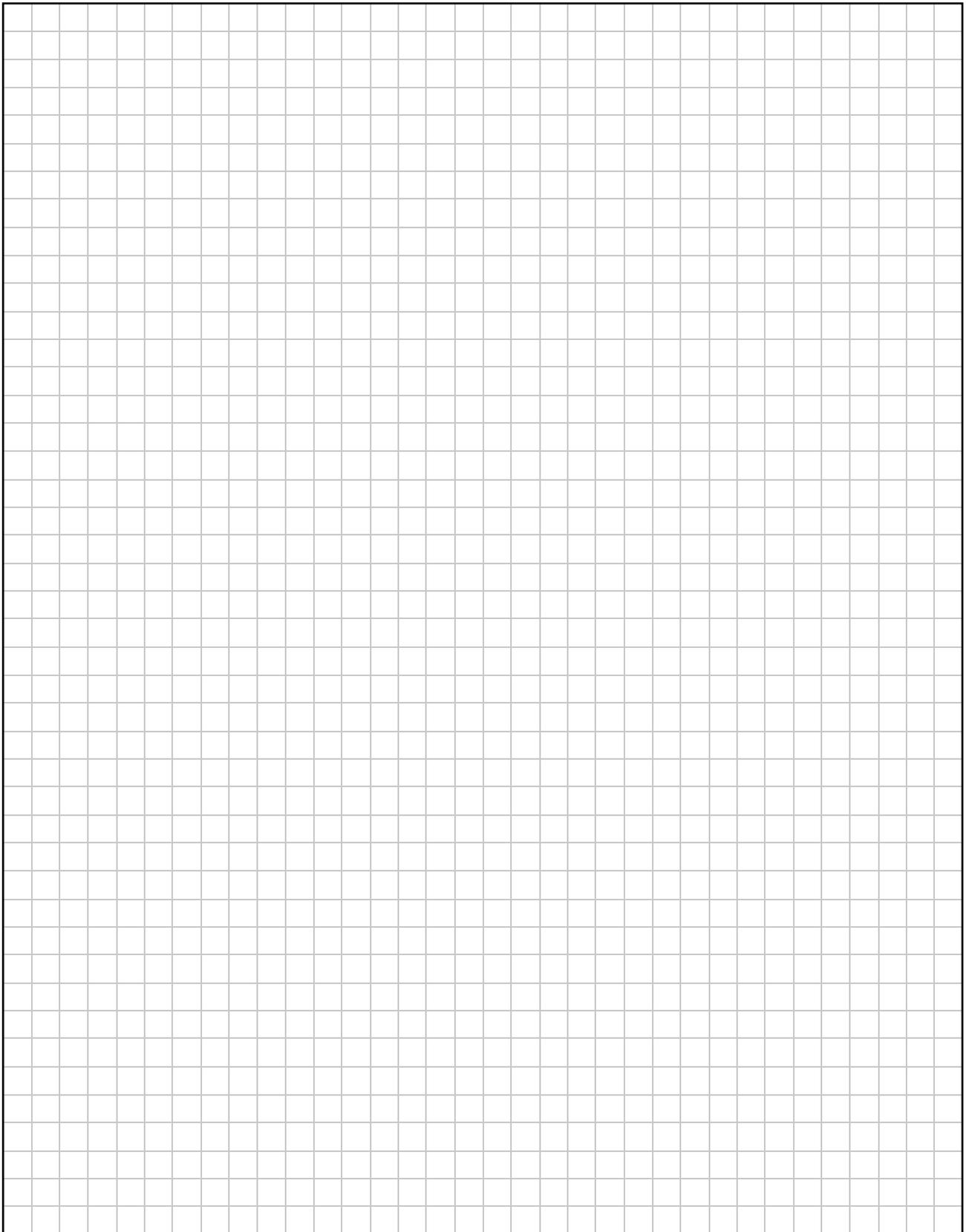
Page for extra work.
Label any extra work clearly with the question number and part.
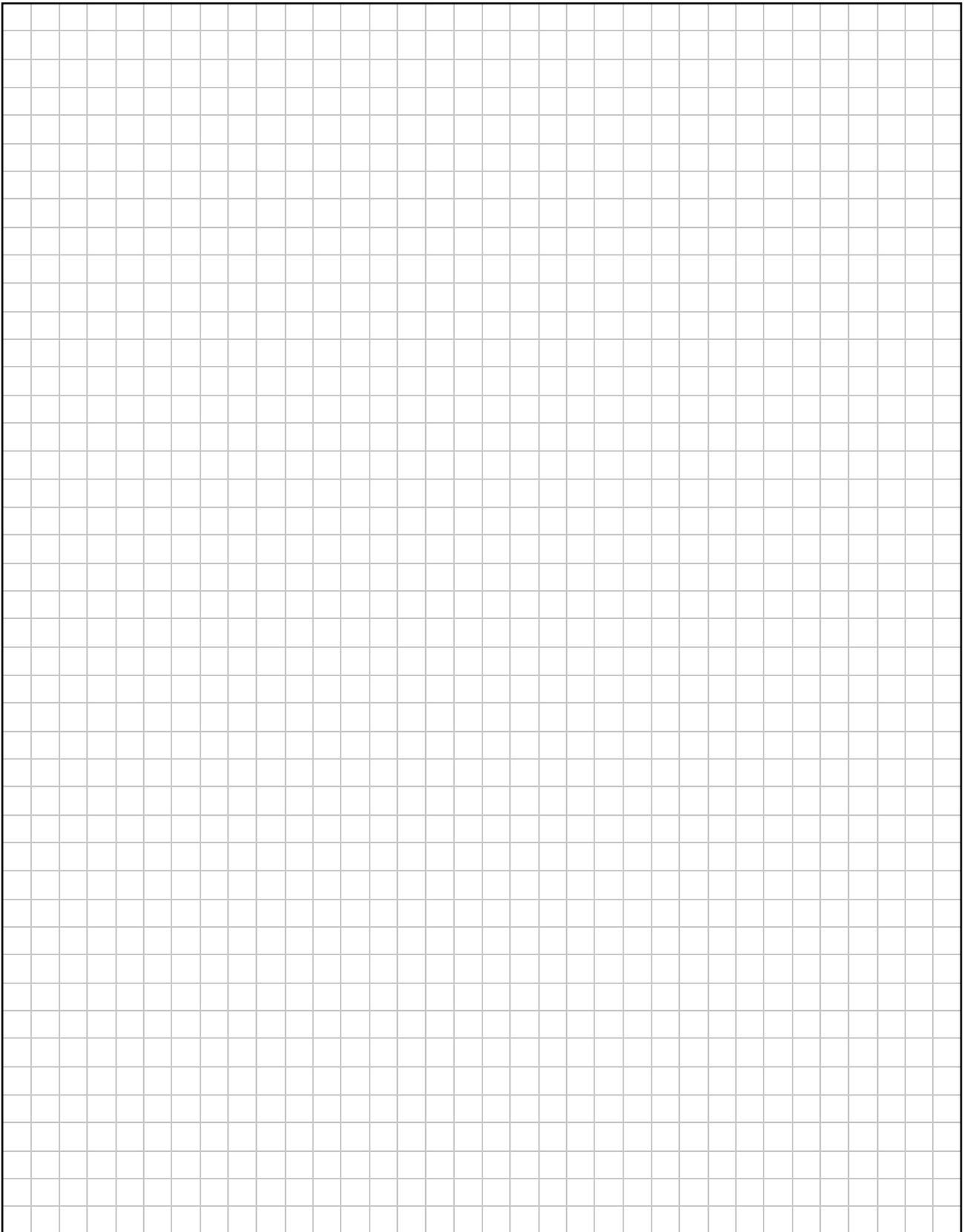
Do not write on this page

Leaving Certificate – Ordinary Level

Mathematics Paper 1

Friday 9 June

Afternoon 2:00 - 4:30