Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2023

Mathematics

Paper 2

Ordinary Level

Monday 12 June   Morning 9:30 - 12:00

300 marks

Examination Number

Day and Month of Birth

For example, 3rd February
is entered as 0302

Centre Stamp

Do not write on this page

Instructions

There are **two** sections in this examination paper.

| Section A | Concepts and Skills | 150 marks | 6 questions |
| Section B | Contexts and Applications | 150 marks | 4 questions |

Answer questions as follows:
- any **five** questions from Section A – Concepts and Skills
- any **three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet.
If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

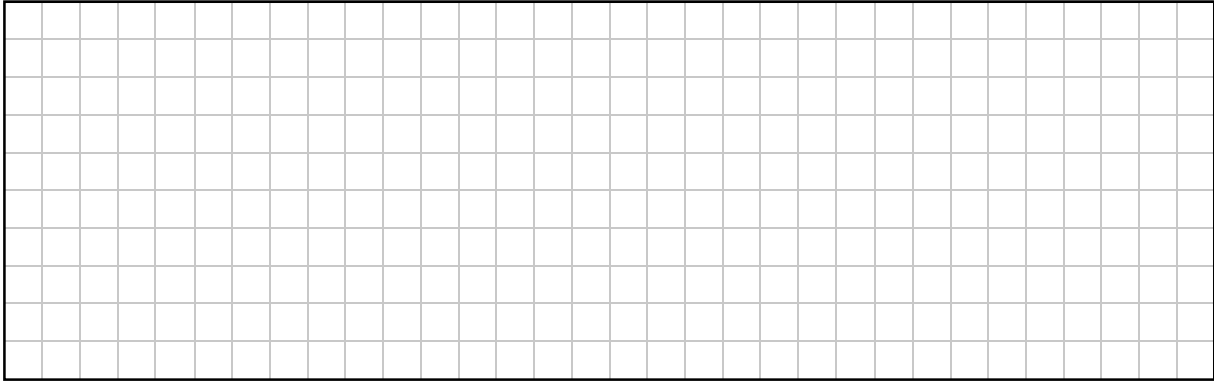You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:
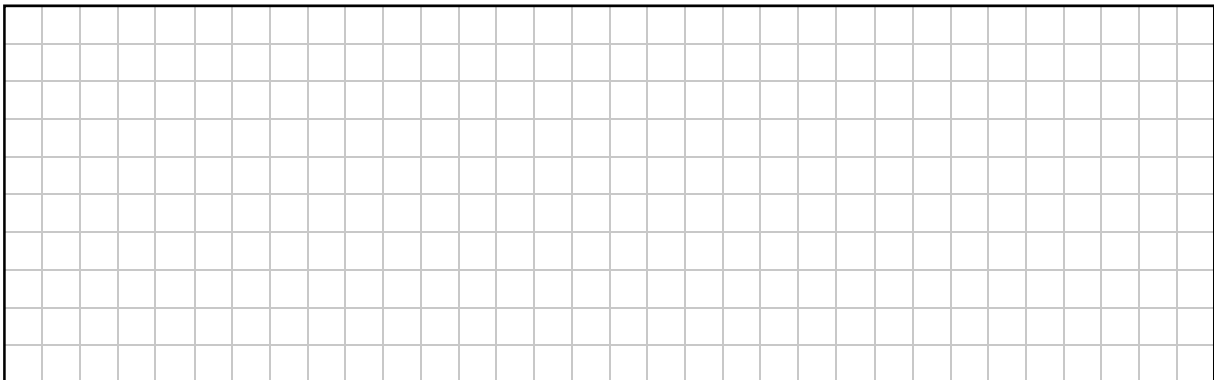
Answer **any five** questions from this section.

Question 1   **(30 marks)**

(a) $A(4, 2)$ and $B(1, 8)$ are two points in the co-ordinate plane.
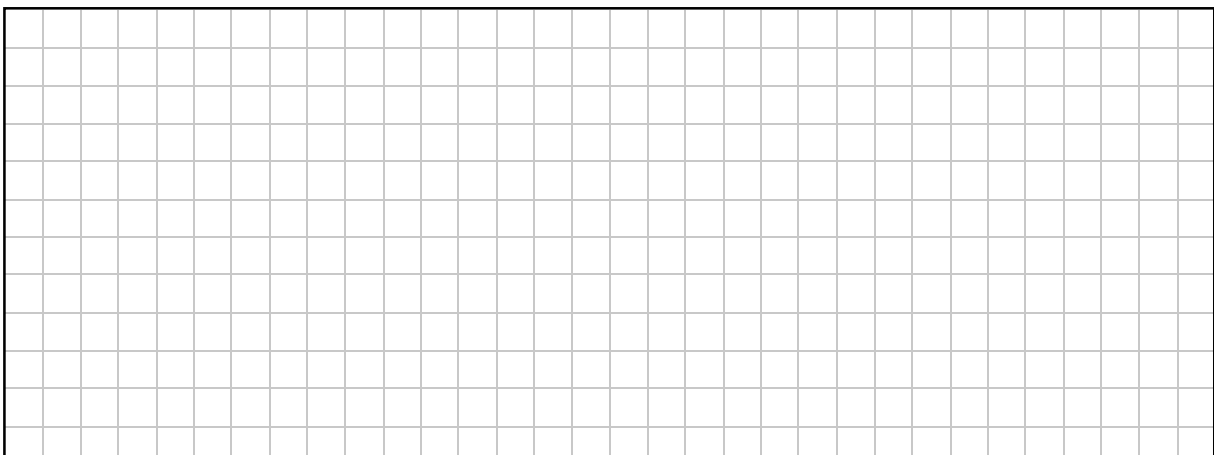
     (i)   Work out the slope of the line AB.

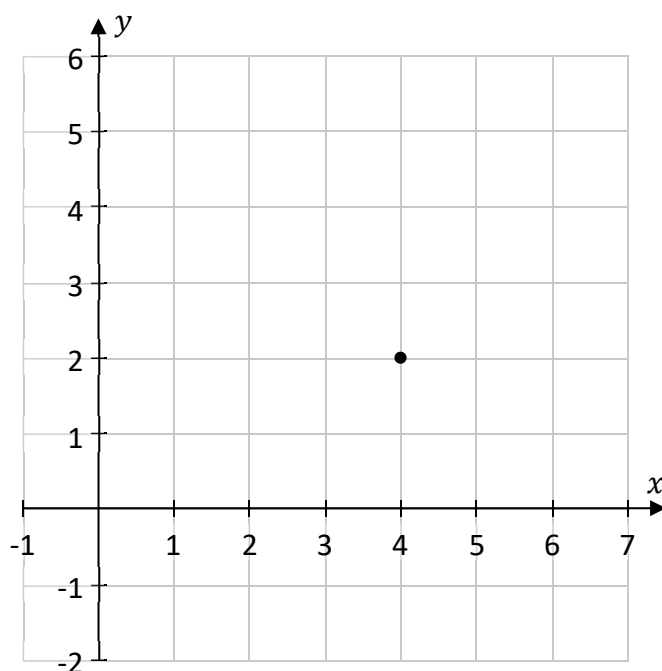   **(ii)**   Work out the distance $|AB|$.

(b)   Find the equation of the line through the point $(-2, 7)$ with a slope of $\dfrac{1}{3}$.

     Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

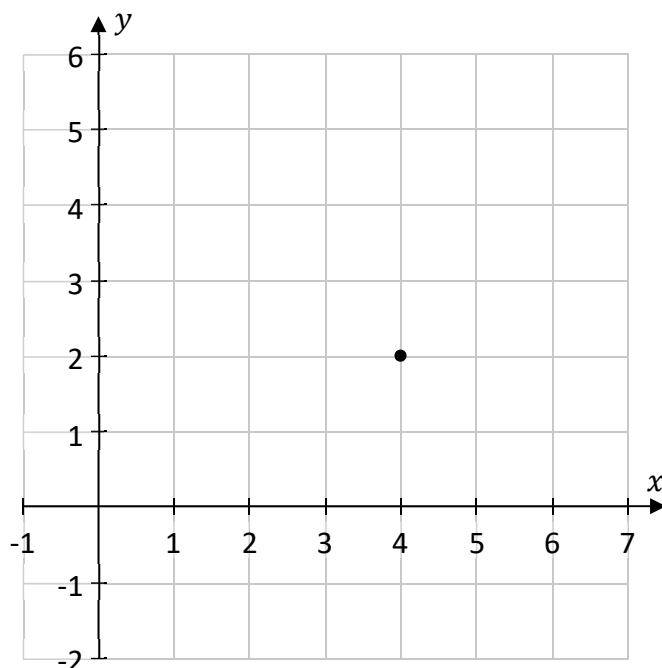**(c)** The point $(4, 2)$ is shown in each of the two co-ordinate diagrams below.

(i) Draw the line segment through the point $(4, 2)$ with a slope of 2 on the co-ordinate diagram below, with the range $-2 \leq y \leq 6$, $y \in \mathbb{R}$.

(ii) Draw the line segment through the point $(4, 2)$ with a slope of $-\dfrac{2}{3}$ on the co-ordinate diagram below, on the domain $-1 \leq x \leq 7$, $x \in \mathbb{R}$.

Question 2 (30 marks)

(a) s is the circle $x^2 + y^2 = 25$.

    (i) Write down the centre and radius of the circle s.

        Centre $= ($, $)$ Radius $= $ _____

    (ii) Show that the point $(3, -4)$ is on the circle s.

    (iii) Write down the co-ordinates of **two** other points that are also on the circle s.

(b) Find the points of intersection between a different circle, k, and the line l, where:

$$l: \quad 5x - y - 13 = 0$$

$$k: \quad x^2 + y^2 = 13$$
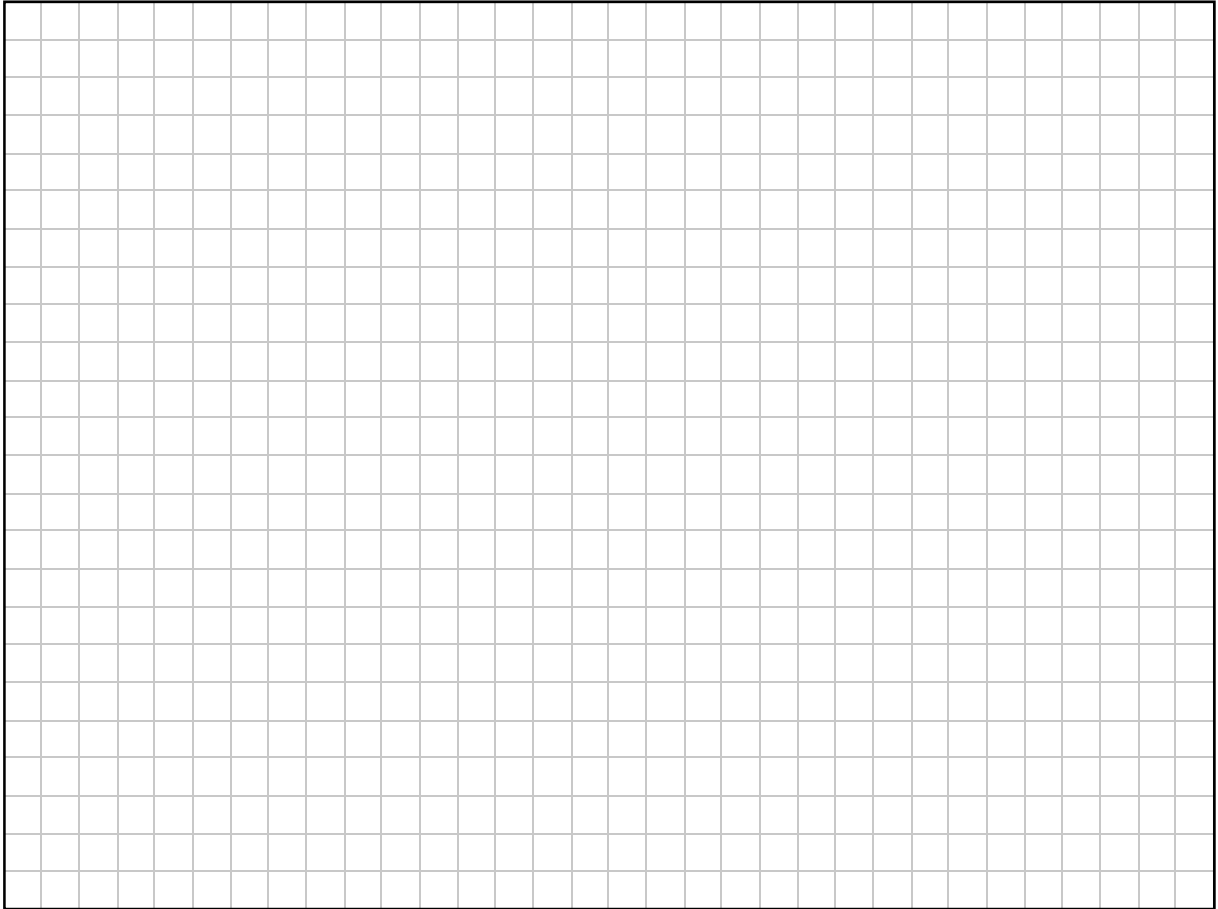
Question 3 (30 marks)

(a) (i) In how many ways can the letters in the word CAMOGIE be arranged?

(ii) How many of the arrangements in **part (a)(i)** start with M **and** end with a vowel (A, O, I, or E)?

(iii) How many arrangements of 3 different letters can be made from the letters in the word CAMOGIE?

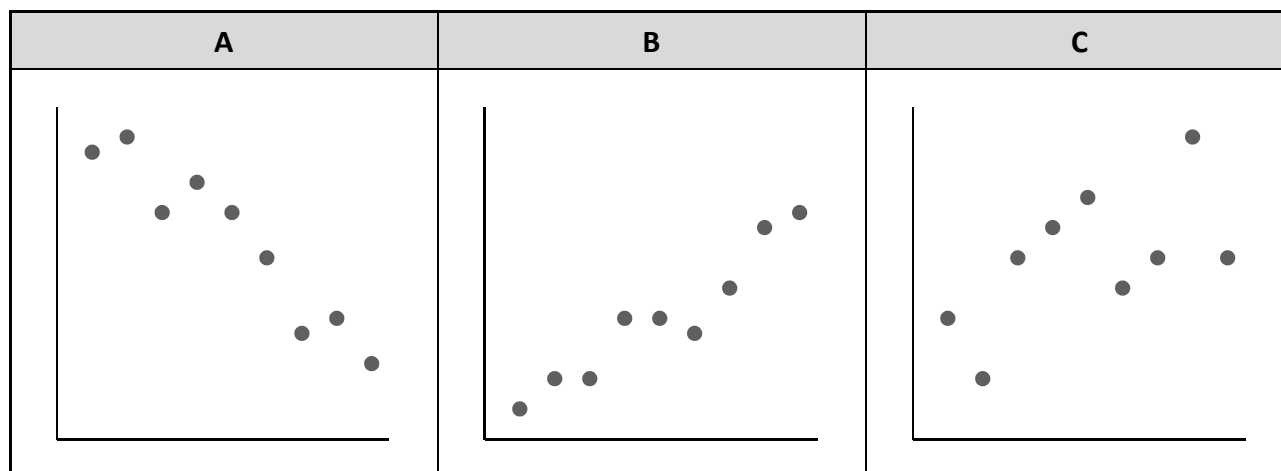**(b)** Three scatterplots are shown below. They are labelled **A**, **B**, and **C**.
Each scatterplot has the same axes and scales.

The correlation coefficient for each scatterplot was calculated.
The results were 0.95, 0.6, and -0.95.

A	B	C

(i) Write each of the letters **A**, **B**, and **C** in the correct place in the table below to show which scatterplot corresponds to each correlation coefficient.
Use each letter only once.

Correlation coefficient	0·95	0·6	−0·95
Scatterplot (**A**, **B**, or **C**)			

(ii) For scatterplot **B**, explain why the value you picked is correct.

Question 4 (30 marks)

Rohan plays basketball.

(a) During a match, Rohan takes a number of free throws.
He scores 70% of his free throws, on average.

Assume that scoring his free throws are independent events.

(i) Find the probability that he scores his first 3 free throws.
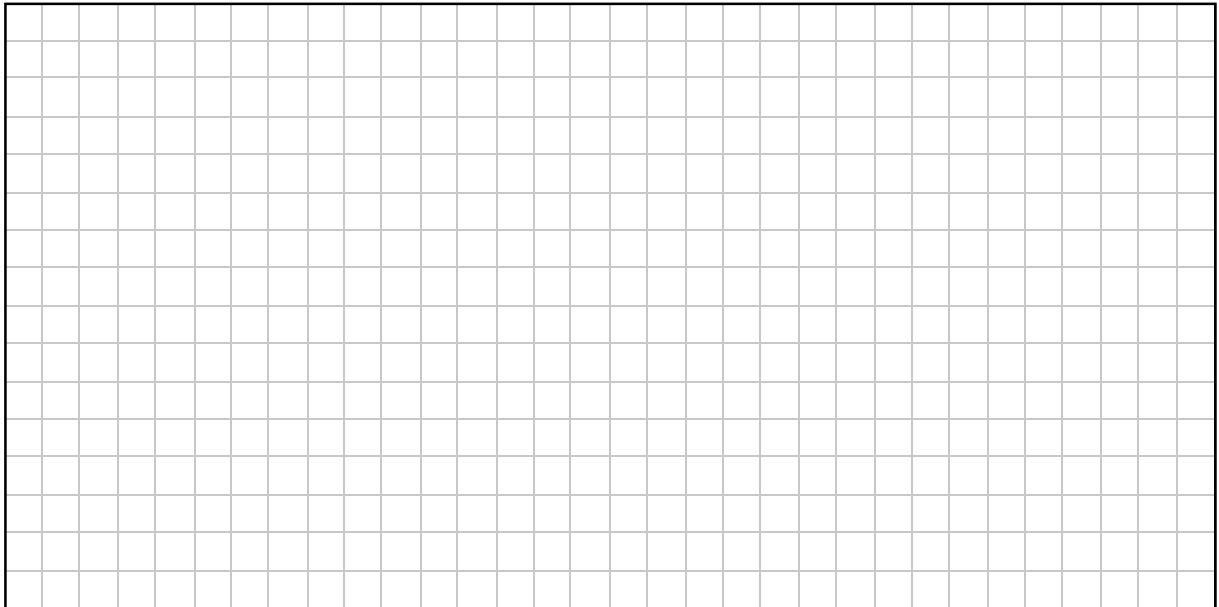
(ii) Find the probability that he scores exactly 2 of his first 3 free throws.

(iii) Give one reason why scoring his free throws might **not** be independent events.

(b) The table below shows the number of games in which Rohan scored each of the given number of points. For example, Rohan scored 12 points in 3 games.

Number of points	0	6	8	10	12	16
Number of games	4	5	4	2	3	1

Work out Rohan's mean number of points per game, based on the values in the table. Give your answer correct to 2 decimal places.

Question 5 **(30 marks)**

The diagram below (not to scale) shows a square with sides of length 12 cm.
The circle c is inside the square, and touches the four sides of the square.
The circle k is outside the square, and goes through the four corners of the square.

(a) Find the length of the radius of the circle c.

(b) Work out the area of the circle c. Give your answer correct to 2 decimal places.

(c) Work out the percentage of the area of the square that is taken up by the circle c, correct to the nearest percent.
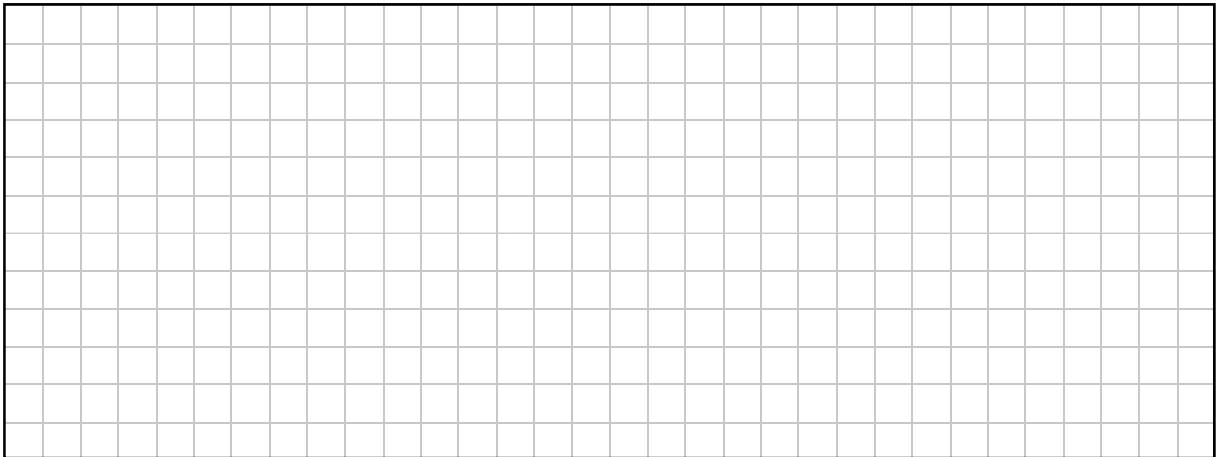
(d) Show that the radius of the circle k is $8 \cdot 49$ cm, correct to 2 decimal places.
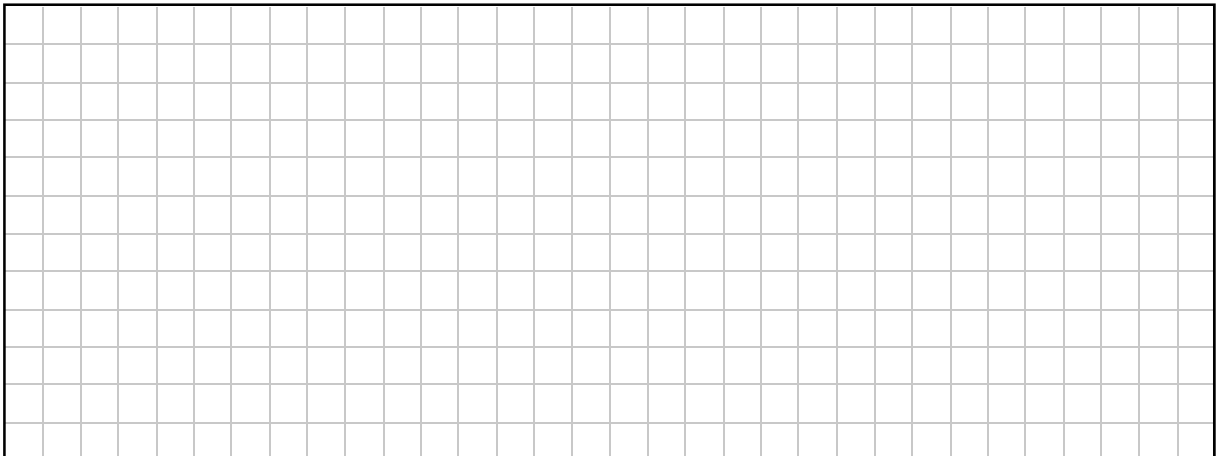
(e) Work out the circumference of the circle k, correct to 2 decimal places.
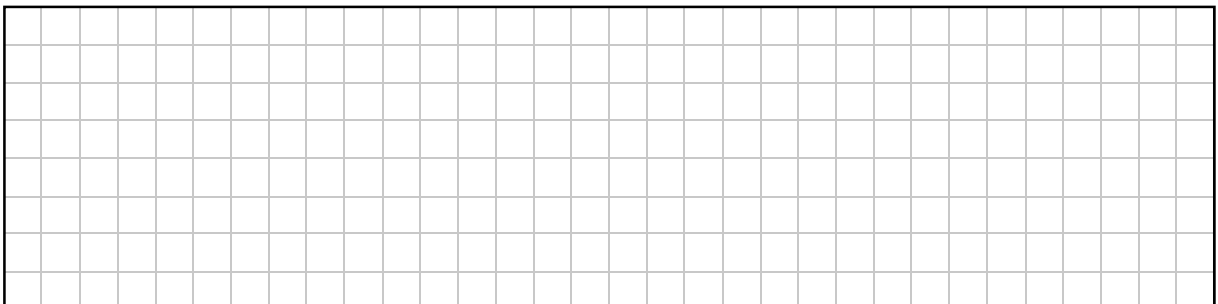
Question 6 **(30 marks)**

(a) The diagram below shows the parallelogram $ABCD$ (not to scale).
$|AB| = 8$ cm, $|BC| = 6$ cm, and $|\angle ABC| = 130°$, as shown.

Construct the parallelogram $ABCD$ in the space below.
The point A is given, as is part of the line AB.

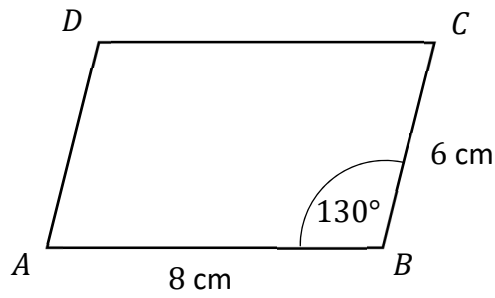**(b)** The diagram below shows another parallelogram (not to scale).
The sizes of three of the angles are given, where $x, y \in \mathbb{R}$.

Use the information in the diagram to work out the value of x and the value of y.
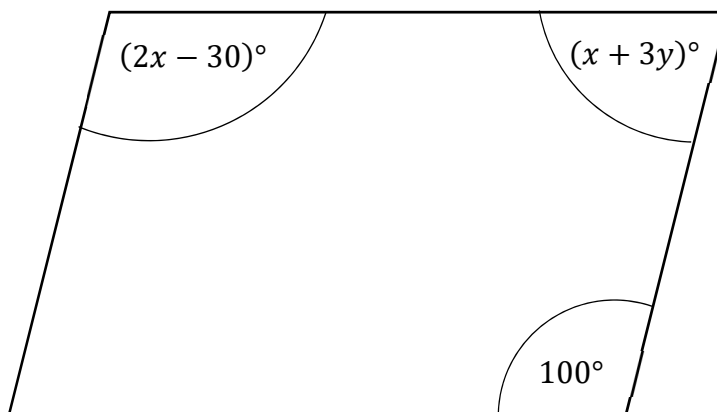
$x =$ _____ $\qquad\qquad y =$ _____

(c) State whether the following statement is true or false. Justify your answer.

Statement:　　　"**Every square is a parallelogram**."

True or false?　　　　　　　　　True　　　　　False
(Tick **one** box only)　　　　　　☐　　　　　☐

Justification:

Answer **any three** questions from this section.

Question 7   **(50 marks)**

Olga is a cyclist. She is measuring how long and how steep different roads are.

(a)   The diagram below shows a road $[AB]$. AC is horizontal and BC is vertical.
        The diagram is not to scale.

The **gradient** of the road $[AB]$ is:
$$\frac{|BC|}{|AC|}$$

written as a **percentage**. It is a measure of how steep the road is.

In parts **(a)(i)** to **(a)(iv)**, there are four different values for $|AB|$ and $|AC|$, and four different gradients.

(i)   Find the gradient of the road $[AB]$ when $|BC| = 24$ m and $|AC| = 135$ m.
        Give your answer correct to the nearest percent.

(ii)   Find $|AC|$ when $|BC| = 16$ m and the gradient of the road $[AB]$ is 12%.
        Give your answer correct to the nearest metre.

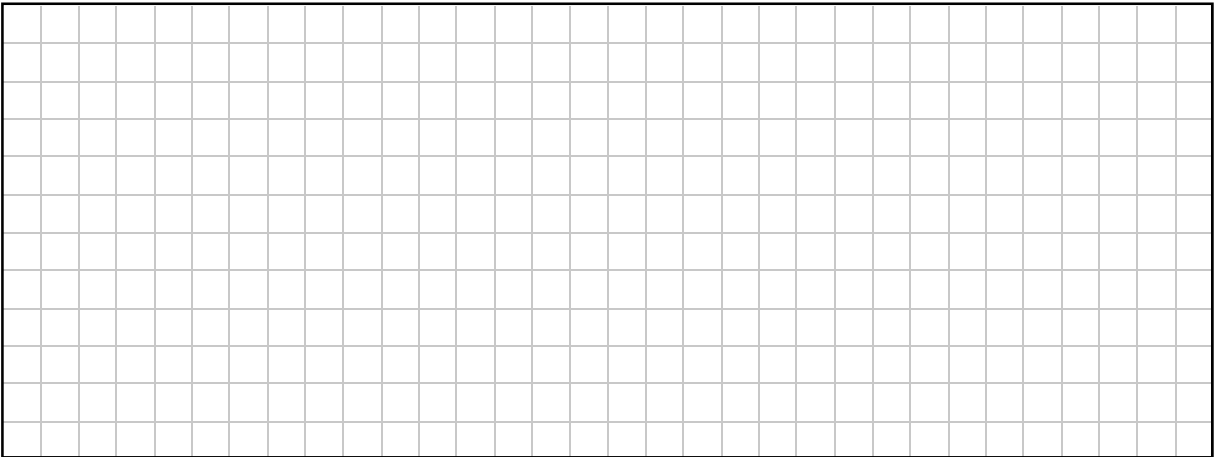**(iii)** Use the Theorem of Pythagoras to find $|AC|$ when $|BC| = 27$ m and $|AB| = 105$ m.
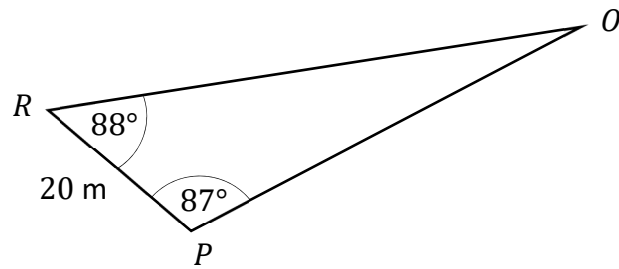Give your answer correct to the nearest whole number.
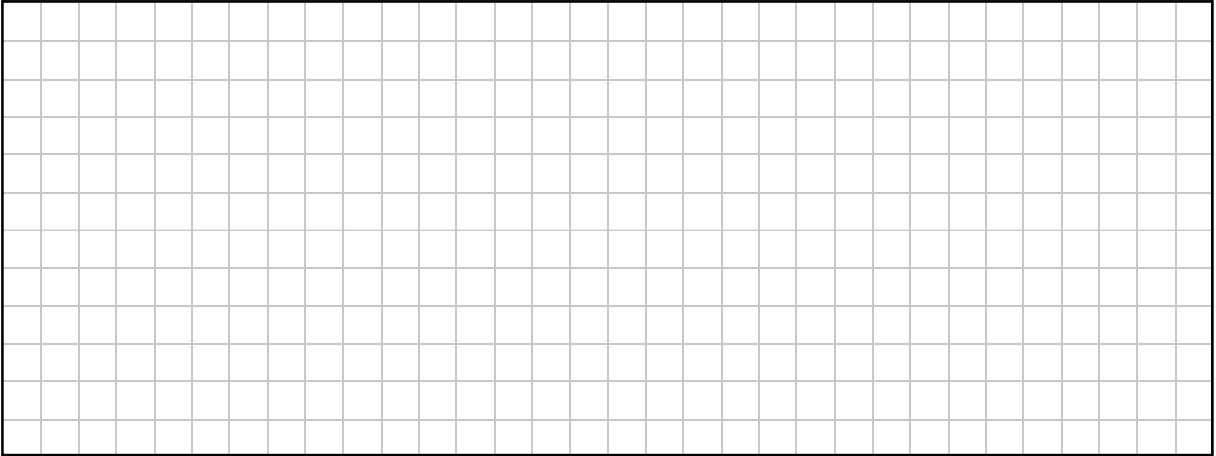
(iv) Find the size of the angle $\angle BAC$ when the gradient of the road $[AB]$ is 9%.
Give your answer correct to the nearest degree.

*This question continues on the next page.*

(b) Olga wants to measure the distance between two points, R and O.
She marks a point P on the ground, so that the distance from R to P is 20 m.
She measures the angles $\angle OPR$ and $\angle ORP$.
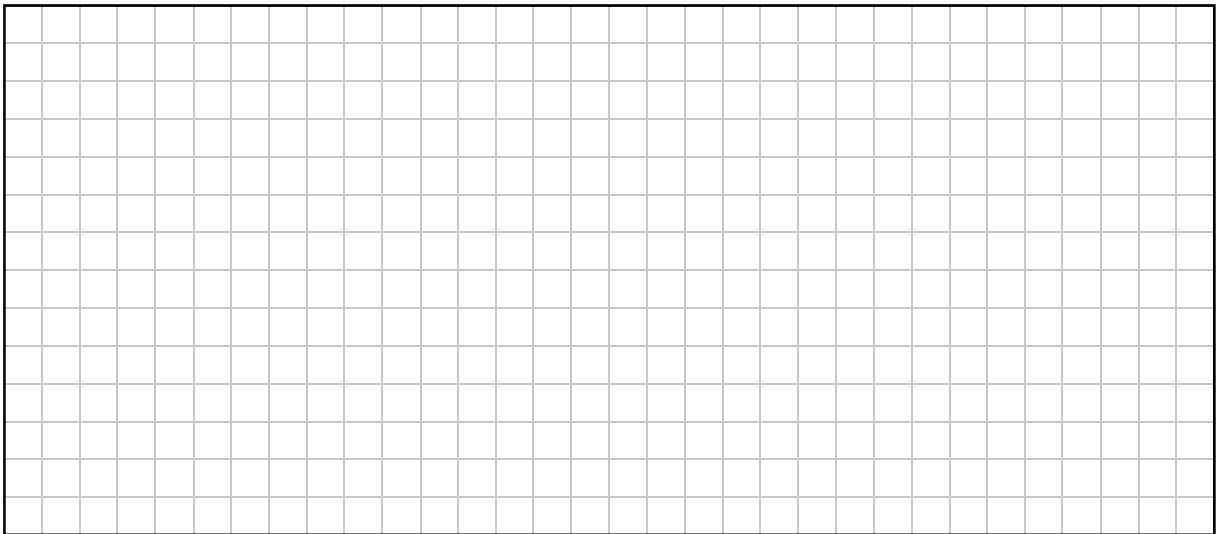All of these are shown in the diagram below (not to scale).

Work out the distance $|OR|$. Give your answer correct to the nearest metre.
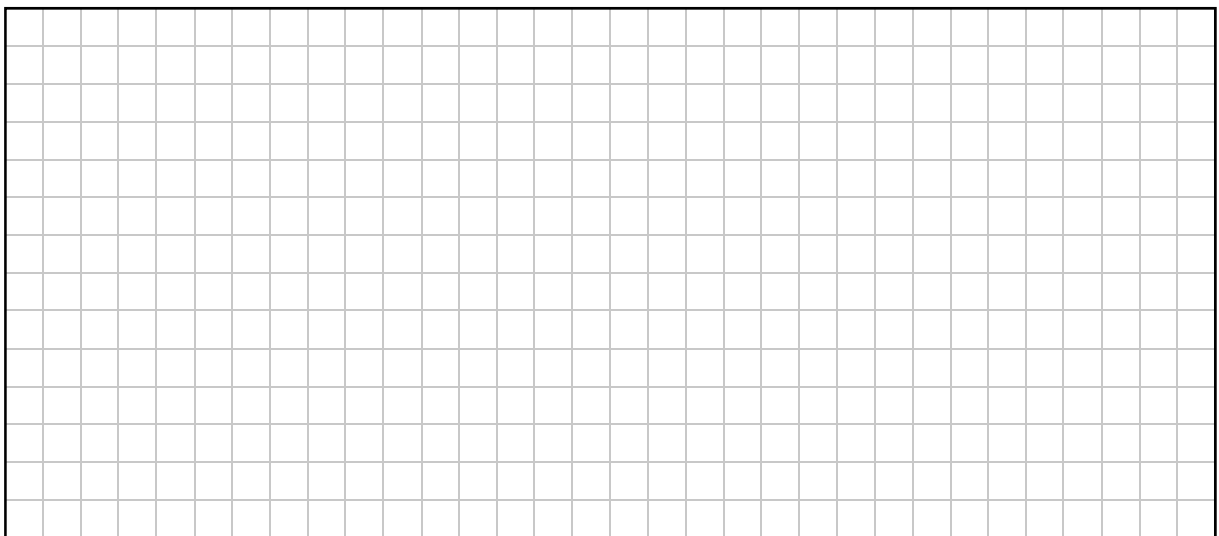
(c) Olga has already worked out the distances between three points, labelled X, Y, and Z in the diagram on the right (not to scale).
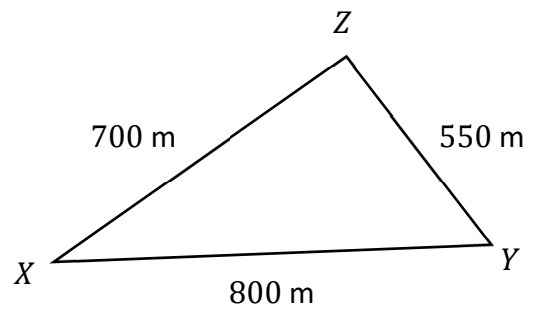
Work out the size of the angle at X in the triangle XYZ.
Give your answer in degrees, correct to 1 decimal place.

Question 8 **(50 marks)**

A family decides to buy a small paddling pool for their back garden.
The pool is in the shape of a cylinder.
It has an internal diameter of 2·4 m and an internal depth of 0·9 m.
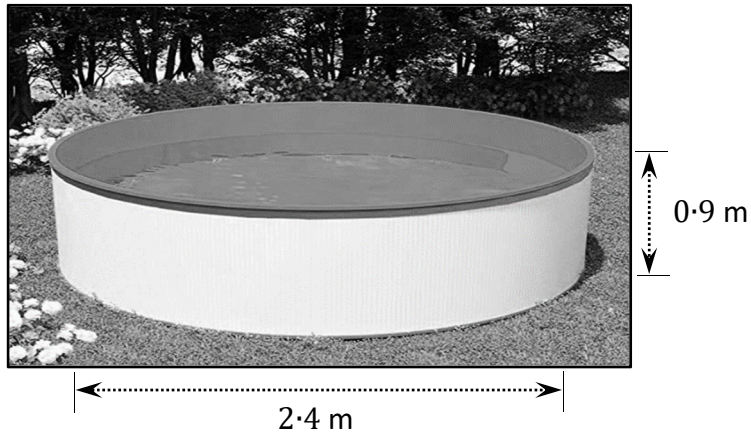
(a) The instruction manual recommends that the pool should be filled to a maximum depth of 0·75 m.

Work out the volume of water in the pool when it is filled to this maximum depth.
Give your answer in m³, correct to 2 decimal places.
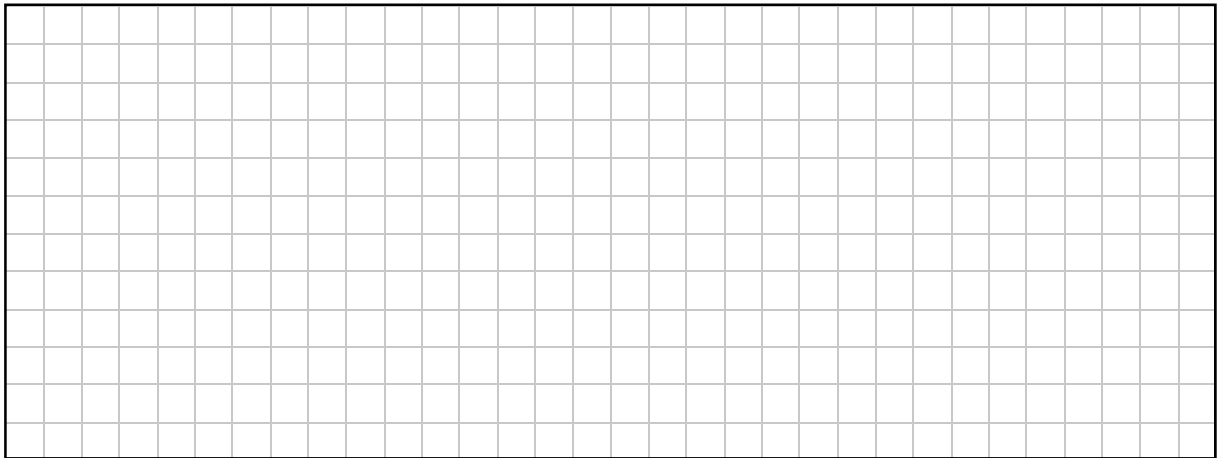
Source of image: www.vidaxl.ie

(b) The family buys 3 pool seats. Each pool seat is a cube, with sides of length 0·5 m.

(i) Show that the **total** volume of the 3 pool seats is 0·375 m³.
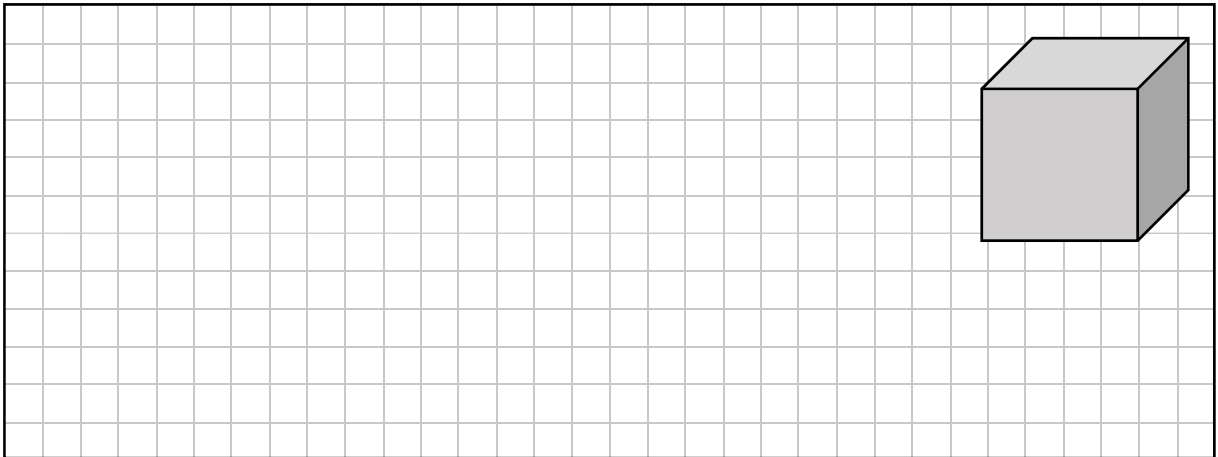
(ii) The pool is filled with water to a height of 0·6 m.
The 3 pool seats are put into the pool, so that each seat is fully covered in water.

Find the amount by which the water level in the pool rises.
Give your answer in metres, correct to 2 decimal places.
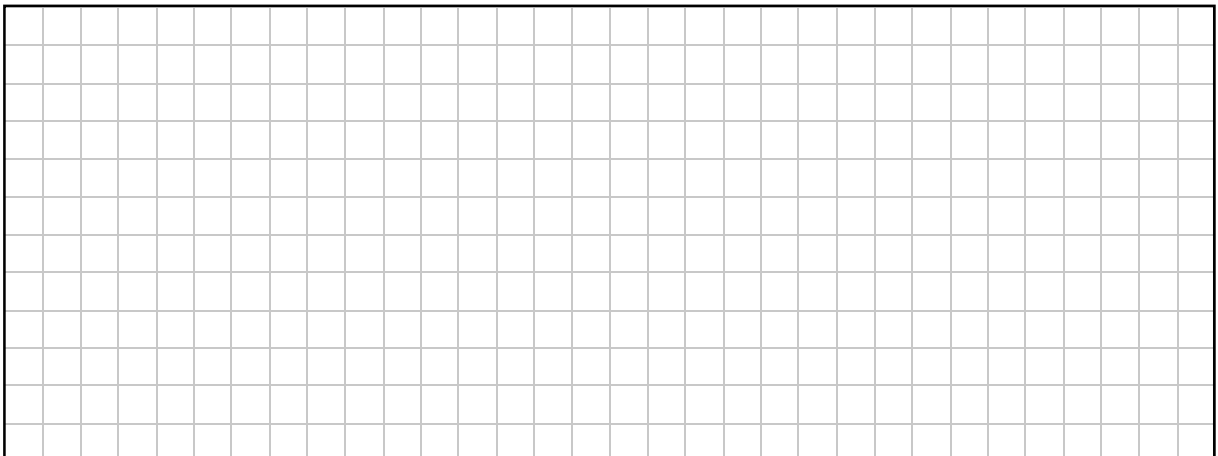
This question continues on the next page.

(c) The family also buys a cover for the pool, to help keep the water clean.
It is in the shape of a right circular cone, with a radius of 1·3 m and a vertical height of 0·8 m.

(i) Show that the slant height of the cone, marked l in the diagram above, is 1·53 m, correct to 2 decimal places.

(ii) Work out the curved surface area of the conical cover.
Give your answer correct to 2 decimal places.

(iii) The diagram below shows the net of the conical cover (not to scale).
It is a sector of a circle of radius x.

Write down the value of x **and** work out the size of the angle A, correct to the nearest degree.

$x =$

Size of angle $A =$

Question 9 **(50 marks)**

(a) A group of students decided to investigate the average amount of time that students in Ireland spend reading each day.

One Monday morning, they asked the first 200 students who came into their school how long, on average, they spend reading each day. Their results are shown in the table below.

Minutes spent reading per day	$0 - 30$	$30 - 60$	$60 - 90$	$90 - 120$	$120 - 150$	$150 - 180$
Number of students	16	35	62	59	18	10

Note: $30 - 60$ means "30 minutes or more, but less than 60 minutes", and so on.

(i) Work out the **maximum** number of students in this sample who, on average, could have read for more than 100 minutes per day.

(ii) Estimate the median time spent reading by these 200 students, as accurately as possible. Justify your answer.

(iii) Explain why the method of choosing the students for this survey could limit the validity of the conclusions.

(iv) Draw a graph or chart to represent the data in the table.
Show clearly any relevant calculations. Label your graph or chart clearly.

This question continues on the next page.

(b) A random sample of 500 students in Cork took a statistics test.
61 of these students were given a rating of *Excellent*.

(i) Work out the percentage of students in this sample who were given a rating of *Excellent*.

(ii) Show that the margin of error for this sample is 4.5%, correct to 1 decimal place.

(iii) In general, 10% of students get a rating of *Excellent* on this statistics test.

Use your answers to parts **(b)(i)** and **(b)(ii)** to test the claim that the percentage of students in Cork who would get a rating of *Excellent* is different to the population percentage, at the 5% level of significance.

Show relevant calculations, state your conclusion, and give a reason for your conclusion.

Calculations:

Conclusion:

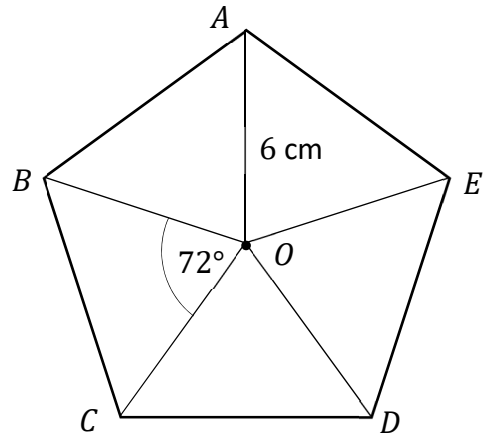Reason for the conclusion:

Question 10 **(50 marks)**

A toy company makes spinners in the shape of a regular pentagon, as shown in the diagram on the right (not to scale).

Each of the five sides of the pentagon $ABCDE$ is the same length.

The pentagon is divided into five congruent isosceles triangles. Each triangle contains the point O.

$|\angle BOC| = 72°$ and $|AO| = 6$ cm.

(a) Work out $|\angle CBA|$, the size of the angle CBA.

(b) Work out the **area** of the triangle AOB, and hence work out the **area** of the pentagon $ABCDE$. Give each answer correct to 1 decimal place.
(You do **not** need to use your answer from **part (a)**.)

Area of triangle $AOB =$ _____ cm² 　　 Area of pentagon = _____ cm²

This question continues on the next page.

(c) The triangles in the spinner are numbered 1 to 5, as shown. The spinner is spun twice, and each number landed on is recorded.
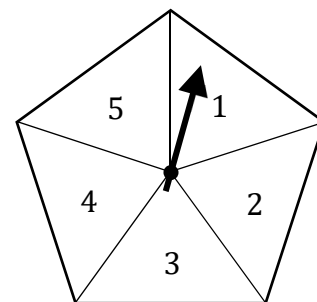
(i) Write down a sample space to show **all** of the 25 possible outcomes when the spinner is spun twice.

It may be useful to draw a two-way table or to list the outcomes in a systematic way.
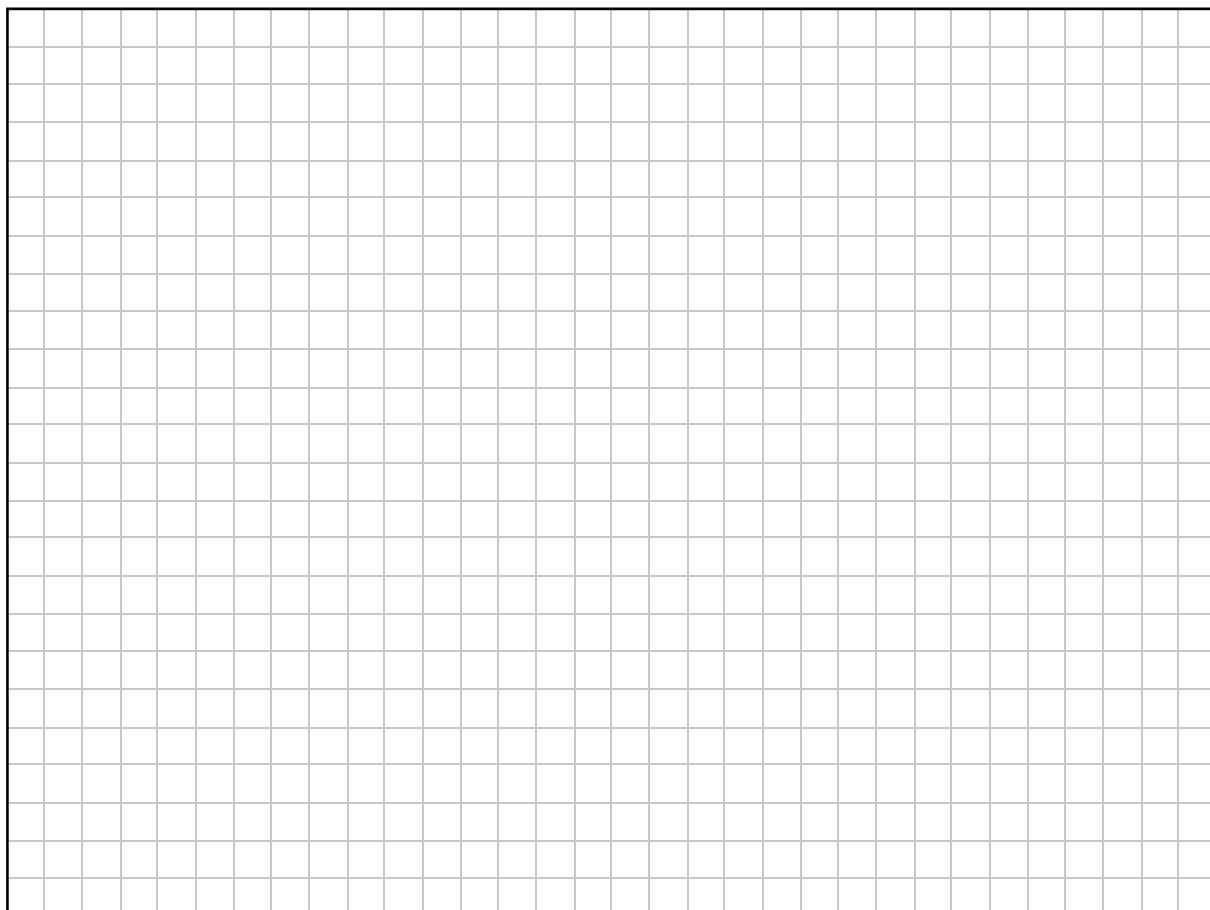
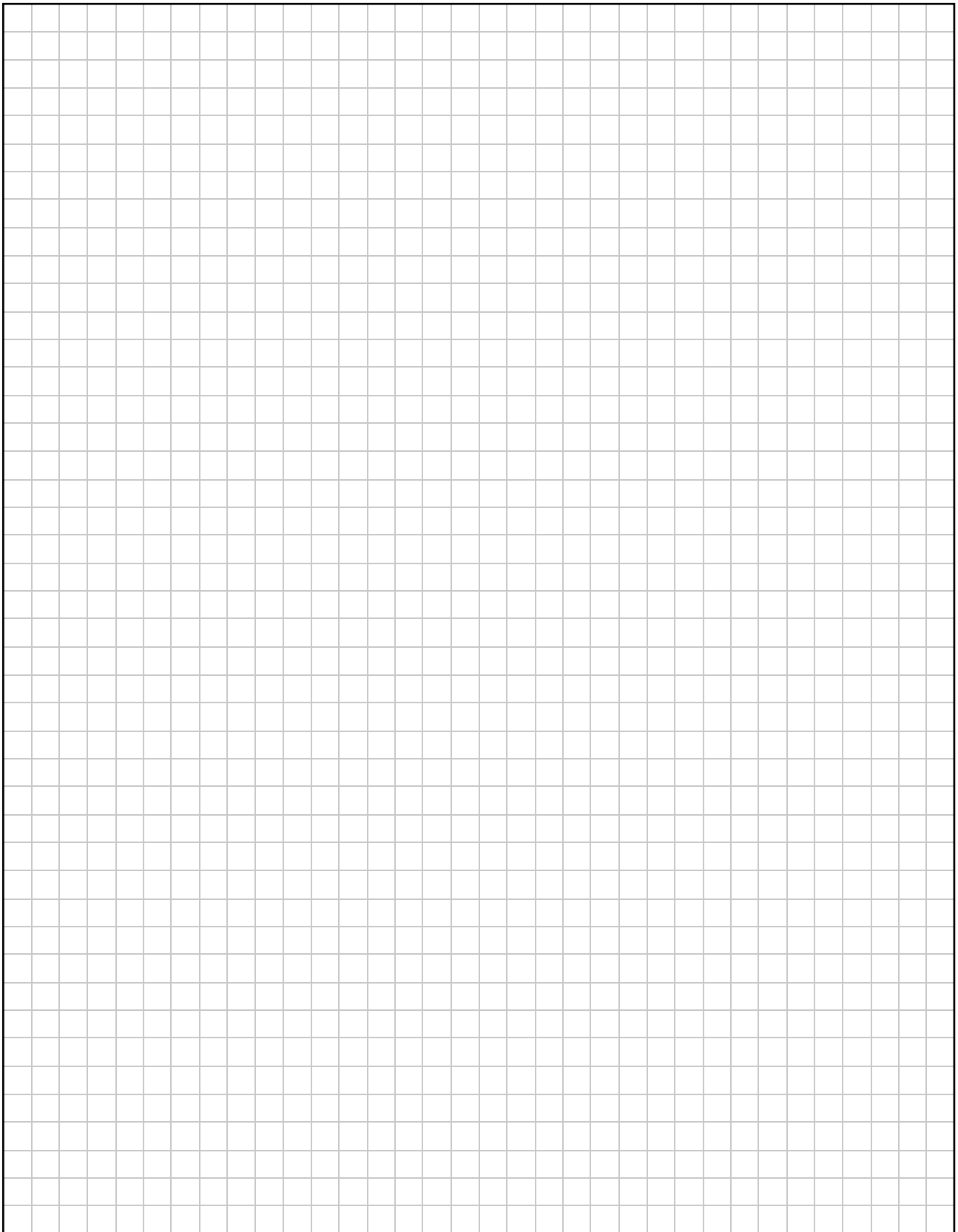The spinner is spun twice, and the two numbers are added.

(ii) What are the **maximum** and **minimum** possible totals when two numbers from the spinner are added?

Maximum = ☐   Minimum = ☐

(iii) Each time the spinner is spun, each number (1 to 5) is equally likely. Work out the probability of getting a total of either 4 or 5, when the two numbers are added.

(d) In a different spinner, there are 4 different outcomes: **N**, **S**, **E**, or **W**.

In a game, Jena **pays** €1 to spin this spinner. The table below shows the probability of each outcome, as well as the money Jena will get, depending on the outcome.

Outcome	N	S	E	W
Probability	0·3	0·2	0·4	0·1
Money Jena gets	€1	€0·50	€0	€5

Is this game **fair**? Justify your answer.
Work out the **expected value** of this game as part of your solution.
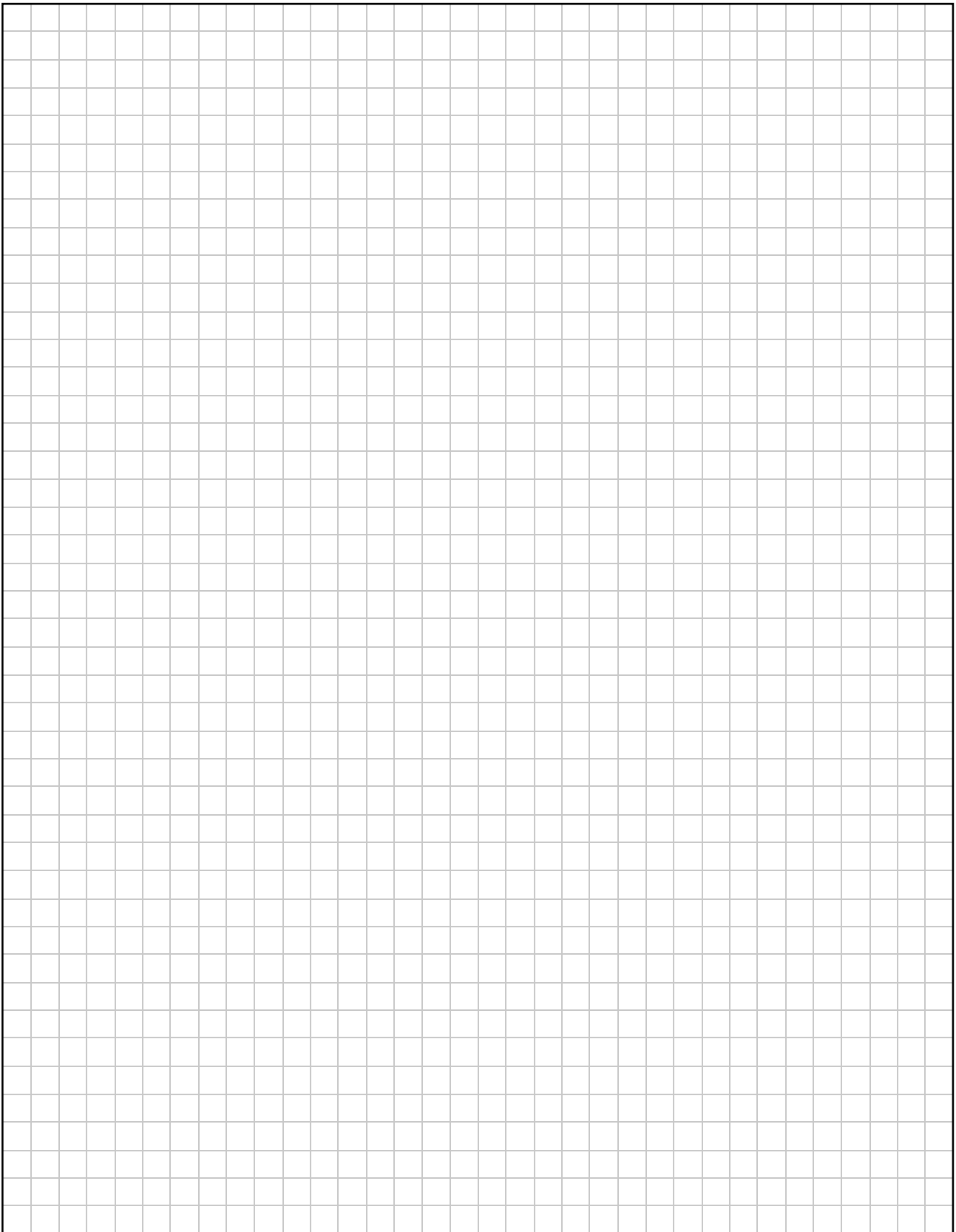
Working out:

Is this game fair?

Why?

Page for extra work.
Label any extra work clearly with the question number and part.

Page for extra work.
Label any extra work clearly with the question number and part.

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Leaving Certificate – Ordinary Level

Mathematics Paper 2

Monday 12 June
Morning 9:30 - 12:00