2024.M29K
(deferred examination)

2024L003A1EK

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2024

Mathematics

Paper 1

Higher Level

2 hours 30 minutes

300 marks

Examination number					

Centre stamp

For the Examiner only					
		Section	Question	Mark
		A	1	
			2	
Disallowed			3	
A			4	
B			5	
Total Disall.			6	
		B	7	
Cumulative Check			8	
Running Total			9	
− Total Disall.			10	
= Total		↔	Total	

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- **any five** questions from Section A – Concepts and Skills
- **any three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A Concepts and Skills 150 marks

Answer **any five questions** from this section.

Question 1 **(30 marks)**

(a) Factorise fully $5x^2 - 45$.

(b) Factorise fully $6xy - 8x - 28 + 21y$.

(c) Solve the simultaneous equations for $x, y \in \mathbb{R}$:

$$\begin{aligned} x^2 + 4xy &= -168 \\ 4x + 2y &= -7 \end{aligned}$$

Question 2 **(30 marks)**

(a) The first two terms of an arithmetic sequence are 3 and 10.

(i) Find the sum of the first 100 terms of this sequence.

(ii) The first, fourth, and nth terms of this sequence are the first three terms of a geometric sequence.

Find the value of n.

(b) Use **induction** to prove that the sum of the first n natural numbers can be found using the formula:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Question 3 **(30 marks)**

(a) f and g are two functions of $x \in \mathbb{R}$, defined as follows:

$$f(x) = 6x - 5$$
$$g(x) = x^2 - 2x + 1$$

Find $g(f(x))$.
Give your answer in the form $ax^2 + bx + c$, where $a, b, c \in \mathbb{Z}$.

(b) Write $\frac{1}{81}$ in the form 3^r, where $r \in \mathbb{Z}$.

(c) The following is an expression in x, where $x > 0$, $x \in \mathbb{R}$:

$$\log_{100} x + \log_{100} 18 - \log_{100} 3$$

Write this expression in the form $p \log_{10} k\,x$, where $p, k \in \mathbb{R}$.

Question 4 **(30 marks)**

In this question, $i^2 = -1$.

(a) Write the following complex number in the form $a + bi$, where $a, b \in \mathbb{R}$:

$$(2 + 5i)(3 - 4i) - 2i(10 - i)$$

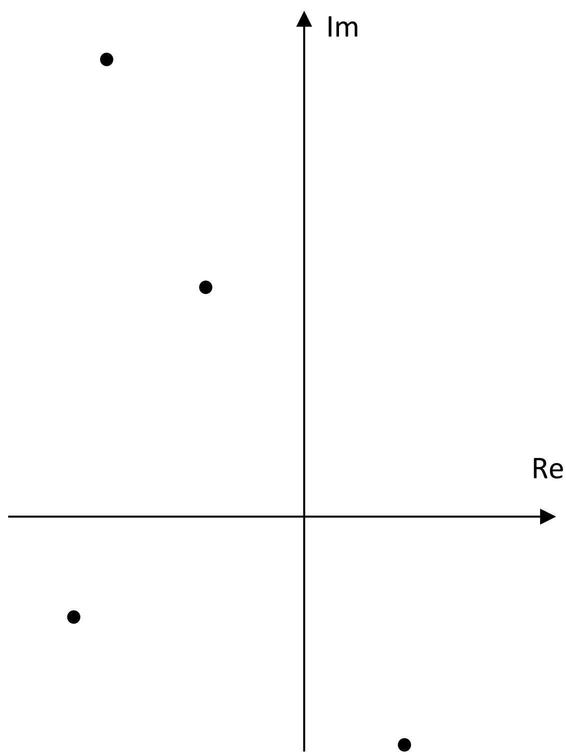
(b) Four complex numbers, w, x, y, and z, are represented by dots (●) on the Argand diagram on the right.

They satisfy the following:

$$y = 2z$$
$$x = -z$$
$$w = iz$$

Use this information to write w, x, y, and z next to the correct complex number on the diagram.

Use each letter only once.



(c) Use De Moivre's theorem to find the two solutions to the equation:

$$z^2 = 1 - \sqrt{3}\, i$$

Give each answer in the form $z = a + bi$, where $a, b \in \mathbb{R}$.

Question 5 **(30 marks)**

(a) $f(x)$ is a function of $x \in \mathbb{R}$.

The derivative of f is $f'(x) = 6x + 4$.
The point $(-1, 8)$ is on the graph of $y = f(x)$.

Use this information to find $f(x)$.

(b) $y = x\ \ln(x^3)$, for $x \in \mathbb{R}, x > 0$.

(i) Find the value of y when $x = 2$.
Give your answer correct to 4 decimal places.

(ii) Find $\dfrac{dy}{dx}$.

Give your answer in the form $a + b \ln x$, where $a, b \in \mathbb{N}$.

Question 6 **(30 marks)**

(a) The function $h(x)$ is defined for $x \in \mathbb{R}$ as:

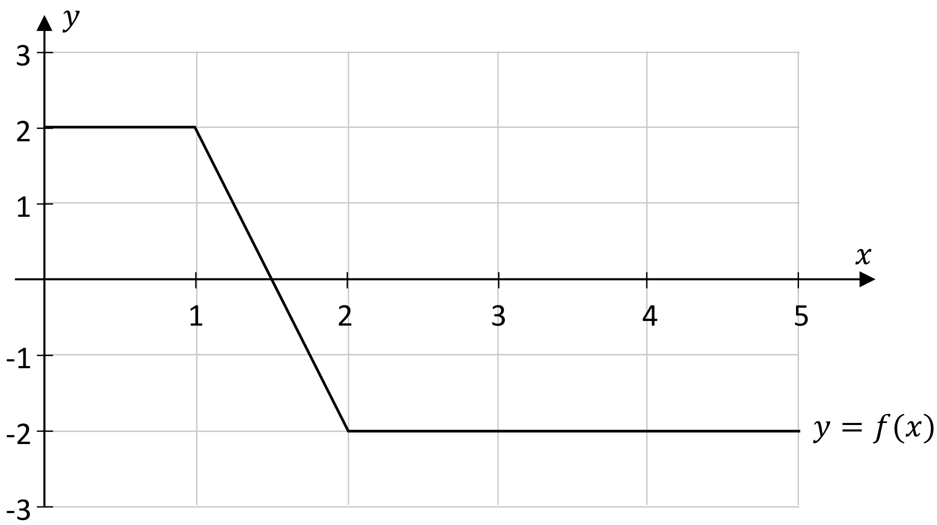
$$h(x) = \sin 2x$$

(i) Write down $h'(x)$, the derivative of $h(x)$.

(ii) Use integration to find the average value of $h(x)$ from $x = 0$ to $x = \frac{\pi}{6}$.

(b) The function $f(x)$ is defined for $0 \leq x \leq 5,\ x \in \mathbb{R}$.
The diagram below shows the graph of the function $f(x)$.
The graph is made up of three straight line segments, two of which are horizontal.

Use the graph to answer parts **(b)(i)** and **(b)(ii)**.



(i) For what range of values of x, if any, is $f'(x)$ **negative**?

Note that $f'(x)$ is the derivative of $f(x)$, and $0 \leq x \leq 5, x \in \mathbb{R}$.

(ii) a is a constant, with $0 \leq a \leq 5, a \in \mathbb{R}$.
Write down the range of values of a, if any, for which this expression is **positive**:

$$\int_0^a f(x)\, dx$$

Section B Contexts and Applications 150 marks

Answer **any three questions** from this section.

Question 7 **(50 marks)**

A company makes and sells different types of sheds. For each type of shed, they can model the profit they will make, based on the number of sheds they sell.

(a) The profit from **shed type F** can be modelled by:

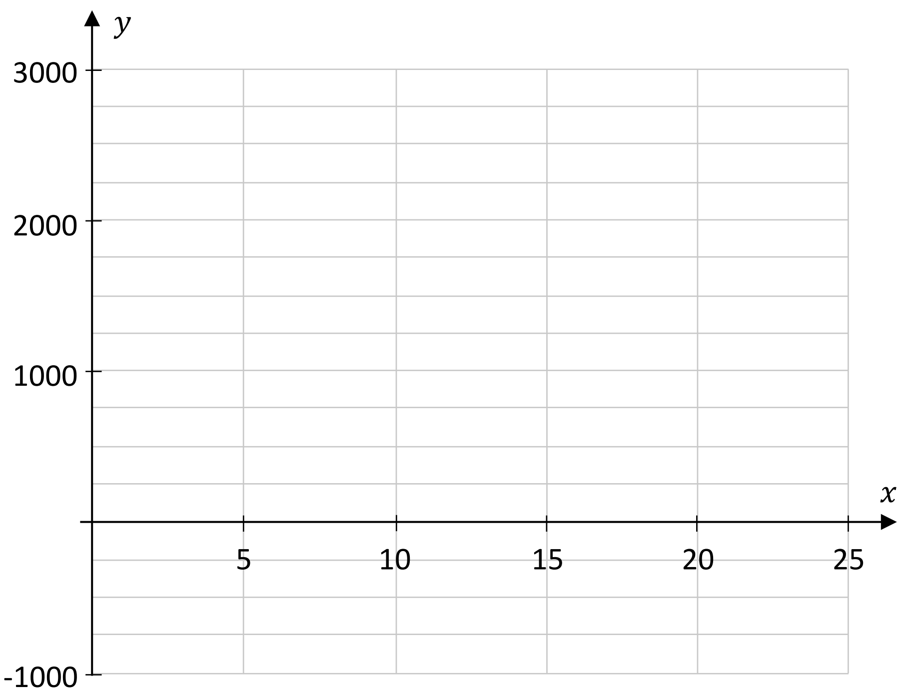
$$f(x) = 400x - 5x^2 - 0{\cdot}4\,x^3 - 900$$

where x is the number of sheds sold, and $f(x)$ is the profit, in euro.

(i) Fill in the table below, showing the value of $f(x)$ for each of the given values of x. Two values are already given.

x	0	5	10	15	20	25
$f(x)$		925			1900	

(ii) Draw the graph of the function $y = f(x)$ on the axes below, for $0 \leq x \leq 25$, $x \in \mathbb{R}$.



(b) The profit from **shed type G** can be modelled by the following function, $g(x)$:

$$g(x) = -2x^3 + 55x^2 + 974x - 6688$$

where x is the number of sheds sold, and $g(x)$ is the profit, in euro.

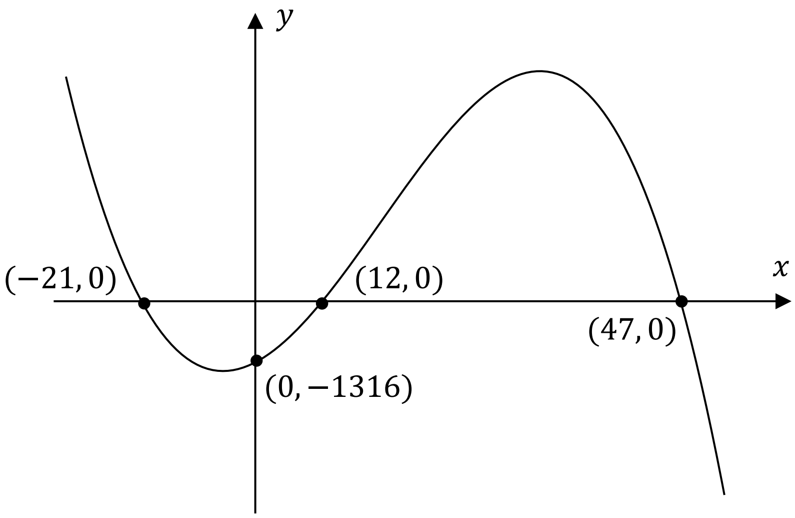
(i) Show that $x = -16$ is a root of $g(x)$.

(ii) Hence, or otherwise, find the other two values of $x \in \mathbb{R}$ for which $g(x) = 0$.

This question continues on the next page.

(c) The profit from **shed type H** can be modelled by a different function, $h(x)$, where x is the number of sheds sold, and $h(x)$ is the profit, in euro.

The diagram below (not to scale) shows the graph of $y = h(x)$ for some of $x \in \mathbb{R}$.
The co-ordinates of four points on the graph are given.



Use the information in the diagram above to write $h(x)$ in the form:

$$h(x) = a(x - b)(x - c)(x - d)$$

where $a, b, c, d \in \mathbb{R}$.

(d) Martha works for this company.

Martha's gross annual income is €x, where $x \in \mathbb{R}$ is a constant.

She pays tax at a rate of 20% on the first €$40\ 000$ that she earns, and at a rate of 40% on the balance. She has an annual tax credit of €3550.

Taking the tax credit into account, Martha pays $\frac{1}{6}$ of her gross annual income in tax.

Find the value of x, Martha's gross annual income.

Question 8 **(50 marks)**

A company produces firewood.

To make the firewood, the company cuts the wood and then leaves it in a warehouse to dry out.
The water content of the wood, $w(t)$, is the **percentage** of the firewood that is actually water.
The following equation describes the water content of the wood while it is drying out:

$$w(t) = 5 + 20\,e^{-0{\cdot}079\,t}$$

where t is the time, in **weeks**, from the start of the drying process, $t \in \mathbb{R}$, $t \geq 0$.

Use $w(t)$ to answer parts **(a)** to **(e)**.

(a) Work out the water content of the wood at the start of the drying process, **and** after 8 weeks.
Give each answer as a percentage, correct to the nearest percent where appropriate.

At the start of the drying process:	After 8 weeks:

(b) The company will sell the wood once the water content is 10% or less.

Work out the least number of **days** that the wood must dry out, to ensure that the water content will be below 10%.

(c) Work out the value of $w'(5)$, where $w'(t)$ is the derivative of $w(t)$.
Give your answer as a percentage per week, correct to 2 decimal places.

(d) $w'(t)$ is always negative, **and** $|w'(t)|$ gets smaller as t gets bigger.
Explain what this means, in the context of the water content in the wood.

This question continues on the next page.

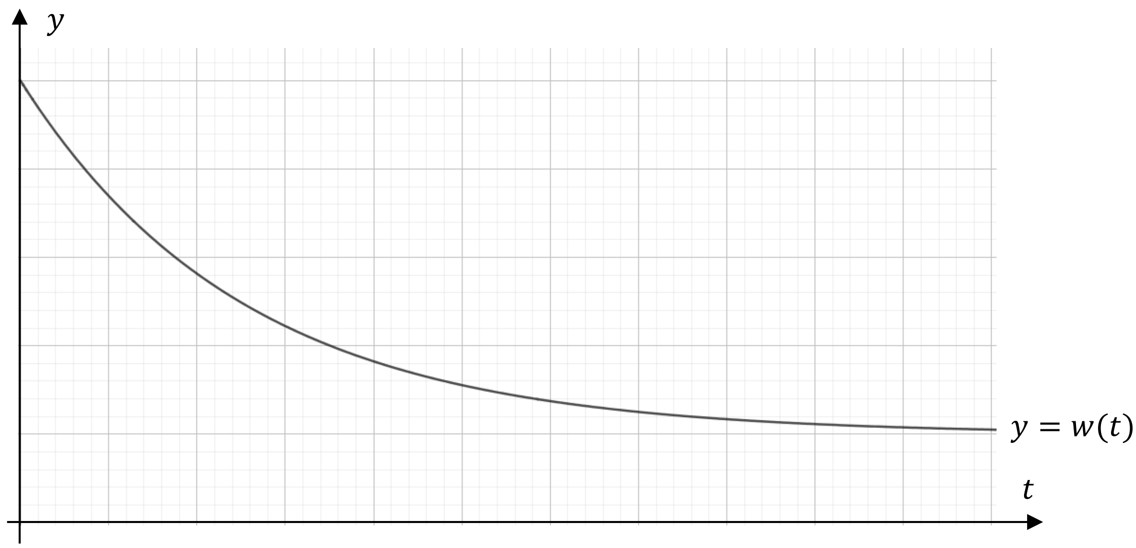
(e) The company starts using a new warehouse. In this warehouse, the wood **dries out faster** – that is, the water content of the wood decreases more quickly.

The following describes how the water content of the wood, $u(t)$, changes over time while it is drying out in the new warehouse:

$$u(t) = 5 + 20\, e^{\,k\,t}$$

Here, k is a constant and t is the time, in weeks, from the start of the drying process.

(i) The diagram below shows the graph of $y = w(t)$ for a given domain of $t \in \mathbb{R}$, $t \geq 0$. On the same domain, sketch a possible graph of $y = u(t)$.



(ii) Write down one possible value of k, so that $u(t)$ describes the wood drying out faster than $w(t)$.

Remember that $w(t) = 5 + 20\, e^{-0{\cdot}079\,t}$

$k =$ __________

(f) In three consecutive years, the amount of firewood the company produces relative to the previous year increases by 6%, 8%, and 17%, respectively.

The total increase over this time is equivalent to an increase of $r\ \%$ each year, compounded annually, where $r \in \mathbb{R}$ is a constant.

Work out the value of r. Give your answer correct to 1 decimal place.

Question 9 **(50 marks)**

The functions $f(x)$ and $g(x)$ are defined as follows, for $x \in \mathbb{R}$:

$$f(x) = -\frac{x^3}{4} + x^2$$

$$g(x) = -6x^2 + 25x - 24$$

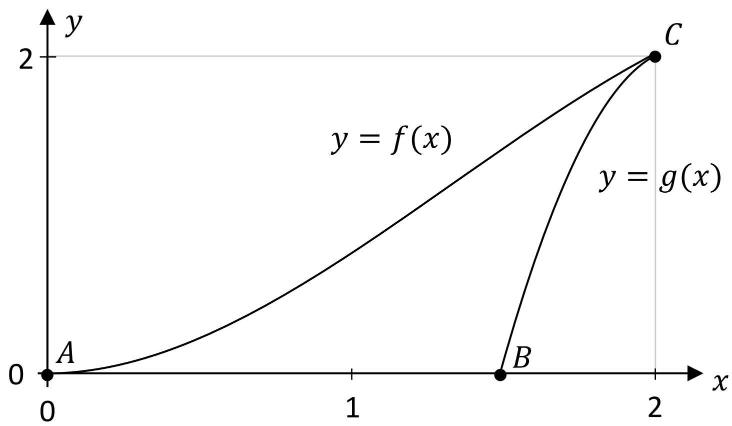
The graphs of f and g are used to make an image of a wave for a logo.
The wave is the region bounded by:

$f(x)$, for $0 \leq x \leq 2$

$g(x)$, for $1{\cdot}5 \leq x \leq 2$

the x-axis.

The vertices of the wave are labelled A, B, and C, as shown in the diagram below.



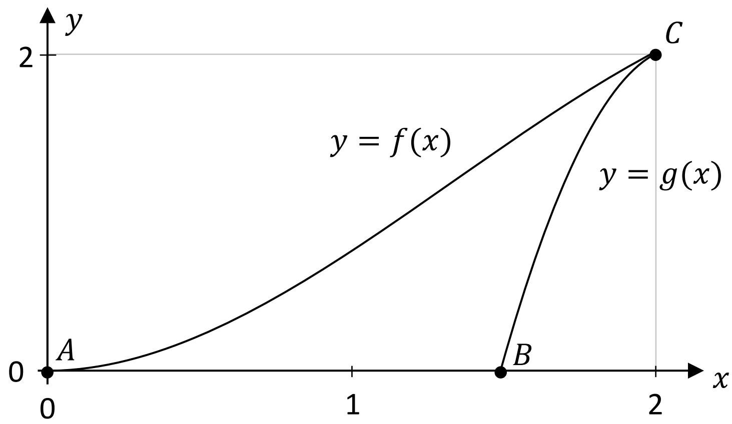
(a) Verify that g cuts the x-axis when $x = 1{\cdot}5$.

(b) Use calculus to find the local maximum point of $f(x)$.

(c) Use calculus to find the x co-ordinate of the steepest part of the graph of $f(x)$, for $0 \leq x \leq 2$.

This question continues on the next page.

(d) Use integration to work out the **area** of the image of the wave; that is, the area bounded by f, g, and the x-axis, for $0 \leq x \leq 2$.

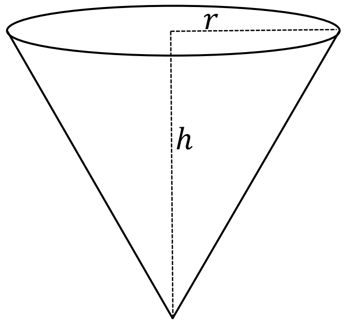


Remember that f and g are defined as follows:

$$f(x) = -\frac{x^3}{4} + x^2$$

$$g(x) = -6x^2 + 25x - 24$$

(e) A company makes candles in the shape of right circular cones. Each candle is made by pouring melted wax into a container.



As it fills the container, the melted wax is always in the shape of a cone. This cone has a radius of r cm.
Its height, h cm, is the same as its diameter.

(i) Show that V, the volume of the cone in cm^3, is given by:

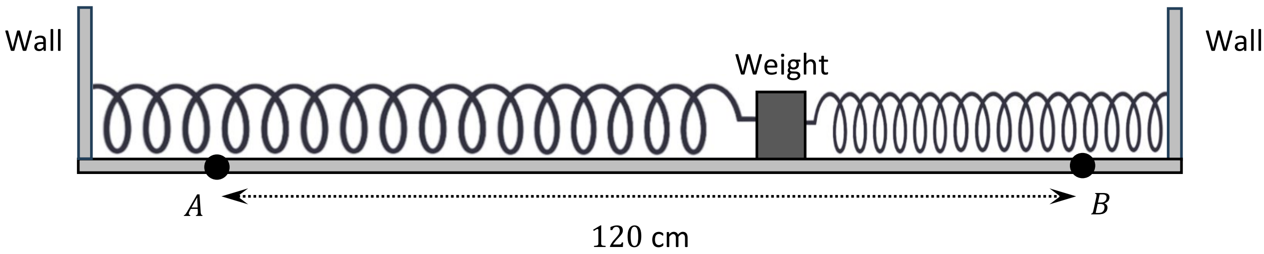
$$V = \frac{2}{3}\pi\, r^3$$

(ii) The volume of the cone of melted wax increases at a rate of $0{\cdot}5$ cm^3 per second.

Find the rate at which the radius of the cone of melted wax is increasing with respect to time when its radius is 4 cm. Give your answer in terms of π.

Question 10 **(50 marks)**

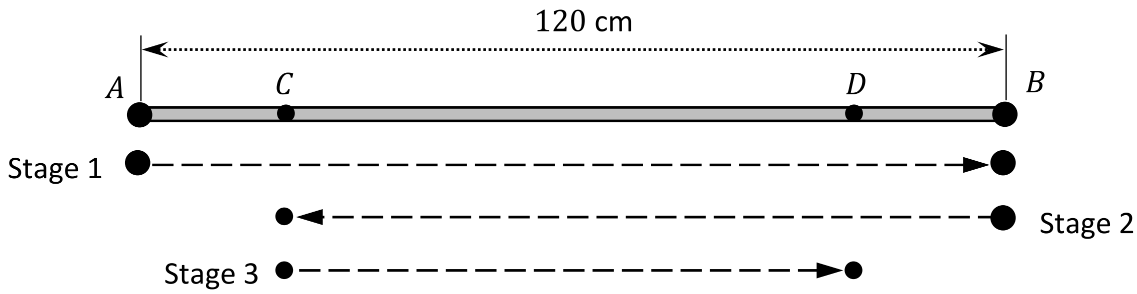
A weight is placed on a horizontal table with a vertical wall at either end.
A spring is attached to each of the walls, and to the weight, as shown below.
Two points, A and B, are marked on the table, with $|AB| = 120$ cm.



In an experiment, the weight is brought to the point A and then released.

It travels back and forth between A and B in stages so that, in each stage, it travels $\frac{4}{5}$ of the distance it had travelled in the previous stage, and in the opposite direction. The first three stages are:

- **Stage 1**: The weight travels 120 cm from A to B.
- **Stage 2**: From B, the weight travels back towards A, but only travels $\frac{4}{5}$ of the way to A, to a point C.
- **Stage 3**: From C, the weight travels back towards B, but only travels $\frac{4}{5}$ of the way to B, to a point D.



(a) Complete the table below to show the distance (in centimetres) that the weight travels in each of the first four stages. Where an answer is not a whole number, write it as a **fraction**.

Stage	**1**	**2**	**3**	**4**
Distance (cm)		96		

There is space for working out on the next page.

(b) Stage p is the first stage where the distance travelled by the weight is less than 10 cm, where $p \in \mathbb{N}$. Find the value of p.

(c) Work out the **total** distance that the weight travels during the first n stages, where $n \in \mathbb{N}$. Give your answer, in cm, in terms of $\boldsymbol{n}$.

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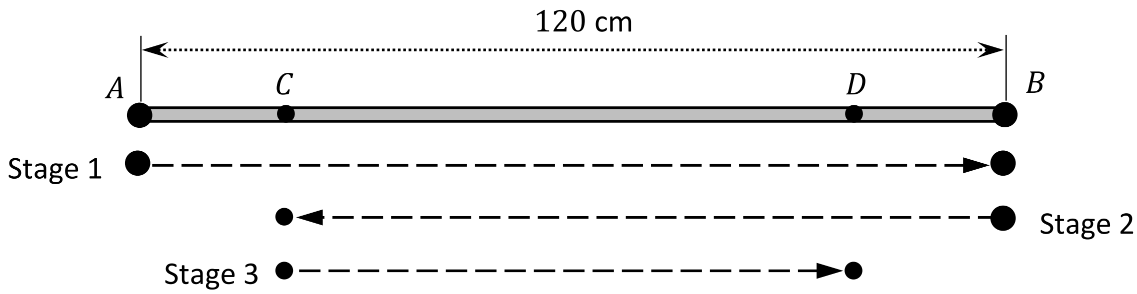

(d) **(i)** Work out the distances $|AC|$ and $|AD|$.

$|AC| =$ ____________ $|AD| =$ ____________

(ii) If the weight were to continue moving indefinitely, without stopping, it would tend towards the point X, between A and B.

Work out the distance $|AX|$.
Use the formula for S_∞ , the sum to infinity of a geometric series, in your solution.

(e) Water is pumped into a container through two pipes.
For each pipe, the water is pumped at a constant rate.

Pipe **A** can fill the container in 18 minutes.
Pipe **B** can fill the container in 15 minutes.

The container is empty.
Water is then pumped into the container through the two pipes.

After 5 minutes, Pipe **A** becomes blocked, so that water no longer comes in through Pipe **A**.
Water continues to come in through Pipe **B**, until the container is full.

Find the total length of time taken to fill the container.
Give your answer in minutes and seconds.

Hint: first find the fraction of the tank that each pipe fills in one minute.

Leaving Certificate 2024 – Higher Level

Mathematics – Paper 1

2 hours 30 minutes