Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2022
Deferred Examinations

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the marking schemes for the deferred examinations

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. However, it should be noted that the marking schemes for the deferred examinations may not necessarily be as detailed as the corresponding marking schemes for the main sitting of an examination, which serve to ensure consistency across a large team of examiners.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination, and the need to maintain consistency in standards between the main sitting and the deferred sitting and from year to year. In the case of the deferred examinations, this means that the level of detail may vary by question, as the marking scheme will only have been finalised for the questions attempted by the candidates who sat these examinations.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with a senior examiner when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes (whether for the main examinations or the deferred examinations) should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination concerned. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination will not necessarily be the same for the deferred sitting as for the main sitting or from one year to the next.

Leaving Certificate

Deferred Exam 2022

Marking Scheme

Mathematics

Higher Level

Paper 1

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales		0, 2, 5	0, 2, 3, 5		
10 mark scales			0, 3, 7, 10	0, 3, 5, 8, 10	
15 mark scales				0, 4, 8, 12, 15	
20 mark scales				0, 5, 10, 15, 20	
25 mark scales					

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	$(3-5i)(2+4i) = 3(2+4i) - 5i(2+4i)$ $= 6 + 12i - 10i - 20i^2$ $= 26 + 2i$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Work of merit, for example some correct multiplication *High Partial Credit:* • Correct expansion (second line of solution) • One error, otherwise correct
(b)	$r = 3\sqrt{2}$ or $\sqrt{18}$. $\theta = 315°$ $(3-3i)^6 = \left[\sqrt{18}(\cos 315° + i \sin 315°)\right]^6$ $= \left(\sqrt{18}\right)^6(\cos(6 \times 315) + i \sin(6 \times 315))$ $= 5832(0 + 1i)$ $= 0 + 5832i$	**Scale 10D (0, 3, 5, 8, 10)** Note: polar form must be used to achieve any credit **4 steps:** **1.** Finds r **2.** Finds θ **3.** Subs into de Moivre's Theorem **4.** Evaluates *Low Partial Credit* • Work of merit, for example, plots $3-3i$, or some correct substitution into de Moivre's Theorem *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit -1*: • Solution given as $5832i$

(c)	In the given Venn diagram: $\mathbb{R}\backslash X = \{\sqrt{17}\}$ $\mathbb{R} \cap X = \{\cos 180° + i \sin 180° \ , 2.5\}$ $X\backslash\mathbb{R} = \{3i,\ 3 + 2i\}$ $(\mathbb{R} \cup X)' = \{\pi + 5i\}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit* Work of merit One value in correct region *Mid Partial Credit:* 2 values in correct region *High Partial Credit:* 4 values in correct region *Full Credit –1:* 5 values in correct region

Q2	Model Solution – 30 Marks	Marking Notes
(a) (i)	$(2m + 1)^2 = 4m^2 + 4m + 1$	**Scale 5B (0, 2, 5)** Note: Accept correct answer without work *Partial Credit:* • Some correct multiplication • Brings down answer from (i) *Full Credit –1* • Multiplied out correctly but not simplified
(a) (ii)	$(2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, which is odd **OR** $(2k+1)^2 = 4k^2 + 4k + 1$ $=$ even + even + odd $=$ odd	**Scale 5B (0, 2, 5)** *Partial Credit:* • Tests two or more particular cases *Full Credit –1* • Expression in appropriate form, but no conclusion
(a) (iii)	$\boldsymbol{P(1)}$: $t^1 = 2m + 1$, for some $m \in \mathbb{N}$, which is odd. So $P(1)$ true. Assume $\boldsymbol{P(k)}$ true, for $k \in \mathbb{N}$. So $t^k = 2h + 1$, for some $h \in \mathbb{N}$ $\boldsymbol{P(k+1)}$**:** $t^{k+1} = t^k \times t^1$ $= (2h + 1)(2m + 1)$ $= 4hm + 2h + 2m + 1$ $= 2(2hm + h + m) + 1$, which is odd. So $\boldsymbol{P(k+1)}$ true. $P(n)$ true for $n = 1$, and is true for $n = k + 1$ whenever it is true for $n = k$. So it is true for all $n \in \mathbb{N}$.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Step $P(1)$, $P(k)$ or $P(k+1)$ • Shows some knowledge of proof by induction *Mid Partial Credit:* • Any two of $P(1)$, $P(k)$ or $P(k+1)$ *High Partial Credit:* • Uses *Step* $P(k)$ to prove *Step* $P(k+1)$ *Full Credit -1:* • Omits conclusion but otherwise correct **Note:** Accept *Step* $P(1)$*, Step* $P(k)$*, Step* $P(k+1)$ in any order

(b)	$\frac{2x}{(3x-5)(x)} - \frac{6(3x-5)}{x(3x-5)}$ $= \frac{2x-18x+30}{x(3x-5)} = \frac{-16x+30}{x(3x-5)}$	**Scale 10C(0, 3, 7, 10)** Accept with top line factorised, or bottom line multiplied out *Low Partial Credit:* • Finds common denominator • Some correct multiplication implied in the numerator *High Partial Credit:* • $\frac{\mathbf{2}(x)-\mathbf{6}(\mathbf{3}x-\mathbf{5})}{(\mathbf{3}x-\mathbf{5})(x)}$

Q3	Model Solution – 30 Marks	Marking Notes
(a)	$x^2 - x(sum) + Product = 0$ $x^2 - x\left(-\frac{7}{2}\right) + \left(-\frac{15}{2}\right) = 0$ $b = \frac{7}{2},\ c = -\frac{15}{2}$ **OR** $x = -5$: $25 - 5b + c = 0$ $5b - c = 25$ $x = \frac{3}{2}$: $\frac{9}{4} + \frac{3b}{2} + c = 0$ $6b + 4c = -9$ $20b - 4c = 100$ $6b + 4c = -9$ $26b = 91$ $b = \frac{7}{2}$ $c = -\frac{15}{2}$	**Scale 10C(0, 3, 7, 10)** Note: Award full credit for $x^2 + \frac{7}{2}x - \frac{15}{2}$ *Low Partial Credit:* • Some correct substitution into the given equation • One correct relevant factor, for example, $(x + 5)$ • Some correct multiplication of factors *High Partial Credit:* • One of b or c correct • Sum and product of roots found
(b)	$$\begin{array}{r} -3x^2 + 4x + 14 \\ 2x + 4 \quad \overline{\sqrt{-6x^3 - 4x^2 + 44x + 56}} \\ \underline{-6x^3 - 12x^2} \\ 8x^2 + 44x + 56 \\ \underline{8x^2 + 16x} \\ 28x + 56 \\ \underline{28x + 56} \\ 0 \end{array}$$ Answer $= -3x^2 + 4x + 14$	**Scale 10D(0, 3, 5, 8, 10)** *Low Partial Credit* • Division set up *Mid Partial Credit:* • One coefficient correct *High Partial Credit* • Two coefficients correct • One error, otherwise correct

(c)	$-4(2x-3)^2 \le (3x+5)(2x-3)$ $-4(4x^2-12x+9) \le 6x^2+x-15$ $-16x^2+48x-36 \le 6x^2+x-15$ $22x^2-47x+21 \ge 0$ $x = \frac{47 \pm \sqrt{(47)^2-4(22)(21)}}{2(22)}$ $x = \frac{7}{11}$ or $x = \frac{3}{2}$ Solution: $x \le \frac{7}{11}$ or $x > \frac{3}{2}$ **OR** **Case 1:** $x < \frac{3}{2}$ implies $2x-3 < 0$, so $-4(2x-3) \ge 3x+5$ so $x \le \frac{7}{11}$. So $x \le \frac{7}{11}$ **Case 2:** $x > \frac{3}{2}$ implies $2x-3 > 0$, so $-4(2x-3) \le 3x+5$ so $x \ge \frac{7}{11}$. So $x > \frac{3}{2}$ Answer: $x \le \frac{7}{11}$ or $x > \frac{3}{2}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Use of $(2x-3)^2$ • Relevant work but with linear inequality *Mid Partial Credit:* • Quadratic inequality involving 0 *High Partial Credit:* • Roots of quadratic found *Full Credit –1* Mishandles strictness of inequalities, otherwise correct.

Q4	Model Solution – 30 Marks	Marking Notes
(a)	$x - 3y - 1 = 0$ $x = 3y + 1$ $(3y + 1)^2 + 4y^2 = 1$ $13y^2 + 6y = 0$ $y(13y + 6) = 0$ $y = 0$ or $y = -\frac{6}{13}$ $x = 1$ or $x = -\frac{5}{13}$ $(1, 0)$ or $\left(-\frac{5}{13}, -\frac{6}{13}\right)$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • Work of merit, for example, $x = 3y + 1$ *Mid Partial Credit:* • Equation in one variable *High Partial Credit:* • Finds both x values or both y values • One error, otherwise correct *Full Credit -1:* • x and y values correct, but not listed as coordinates
(b)	$h\big(g(x)\big) = h(x(2x - 1) + 3)$ $= h(2x^2 - x + 3)$ $= 5(2x^2 - x + 3) + 7$ $= 10x^2 - 5x + 22$	**Scale 10C(0, 3, 7, 10)** *Low Partial Credit:* • Some correct substitution of $g(x)$ in $h(x)$ *High Partial Credit:* • $g(x)$ fully substituted for x in $h(x)$ • $g\big(h(x)\big)$ found correctly in the form $ax^2 + bx + c$
(c)	$k\big(j(2)\big) = k(3) = 5$	**Scale 5B(0, 2, 5)** *Partial Credit:* • Work of merit, for example $j(2)$ found or $j(2)$ indicated on the diagram

Q5	Model Solution – 30 Marks	Marking Notes
(a)	$g(4) = 2 - \frac{1}{e^4} = 1{\cdot}98168 \ldots = 1{\cdot}9817$ [4 DP]	**Scale 5B (0, 2, 5)** Note: Award full credit for the correct answer without work *Partial Credit:* • Work of merit, for example, some correct substitution
(b)	$\lim_{x\to\infty}\left(2 - \frac{1}{e^x}\right) = 2 - 0 = 2$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Limit formulated with some substitution *High Partial Credit:* • $2 - \frac{1}{e^\infty}$
(c)	$2 - \frac{1}{e^k} = \frac{1}{4}$ $\frac{1}{e^k} = \frac{7}{4}$ $e^k = \frac{4}{7}$ $k = \ln\frac{4}{7} = \ln 4 - \ln 7$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit, for example, sets up the equation *Mid Partial Credit:* • Isolates e^x *High Partial Credit:* • Finds $x = \ln\frac{4}{7}$
(d)	$\int_0^5 2 - e^{-x}\,dx$ $= 2x + e^{-x}\ \|_0^5$ $= 2(5) + e^{-5} - (2(0) + e^0)$ $= 10 + e^{-5} - 1$ $= 9 + e^{-5}$	**Scale 10D (0, 3, 5, 8, 10)** **4 steps** **1.** $\int_0^5 (2 - e^{-x})\,dx$ **2.** Integrates correctly **3.** Subs in limits **4.** Evaluates in the required form *Low Partial Credit:* • Work of merit, for example, some correct integration *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct

Q6	Model Solution – 30 Marks	Marking Notes
(a)	$f'(x) = 3x^2 - 12x = 0$ at local max/min $3x(x-4) = 0$ so $x = 0$ or $x = 4$ Positive coefficient of x^3, so local max at $x = 0$, local min at $x = 4$ $f''(x) = 6x - 12 = 0$ at point of inflection So $x = 2$ **(i)** $f(4) = -29$. Local min: $(4, -29)$ **(ii)** $f(0) = 3$. Local max: $(0, 3)$ **(iii)** $f(2) = -13$. Point of inflection: $(2, -13)$	**Scale 20D(0, 5, 10, 15, 20)** **4 steps** **1.** Finds $f'(x)$ **2.** Solves $f'(x) = 0$ **3.** Solves $f''(x) = 0$ **4.** Finds the coordinates of the 3 points *Low Partial Credit:* • Work of merit, for example, some correct differentiation *Mid Partial Credit:* • 2 steps correct • Finds the coordinates of any one of the local maximum, local minimum or point of inflection *High Partial Credit:* • 3 steps correct • Finds the coordinates of any two of the local maximum, local minimum or point of inflection
(b) (i)	$-1 < x < 1$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Indicates on the diagram the range of values for which $g(x)$ is increasing • Mentions -1 or 1
(b) (ii)	*Any valid reason, for example:* "Three different x-values from -2 to $2{\cdot}5$ give a y-value of 3"	**Scale 5B (0, 2, 5)** • Mentions injective or surjective

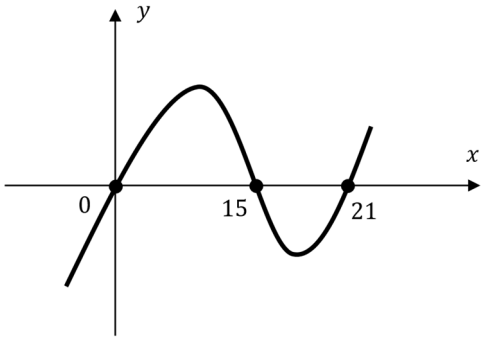
Q7	Model Solution – 50 Marks	Marking Notes
(a)(i)	Table below	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • 1 table entry correct *High Partial Credit:* • 2 table entries correct

Year	1	2	3	4	5
Price of car (€)	24 000	**18 000**	13 500	**10 125**	**7593·75**

(a)(ii)	$24000 \times (0{\cdot}75^{10})$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Some correct substitution into $a \times (b^n)$ • Not in the correct form, for example 1351.52 without work
(a)(iii)	$24000(0{\cdot}75)^p < 1000$ $\ln(0{\cdot}75)^p < \ln\left(\frac{1000}{24000}\right)$ $p \ln 0{\cdot}75 < \ln\left(\frac{1}{24}\right)$ So $p > \frac{\ln\left(\frac{1}{24}\right)}{\ln 0{\cdot}75}$ $p > 11 \cdot 047 \ldots$ so $p = 12$ [$p \in \mathbb{N}$]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Sets up the inequality *High Partial Credit:* • Inequality with *p* not written as an index
(a)(iv)	$S_{20} = \frac{24000(1-(0.75)^{20})}{1-0{\cdot}75} = €95\,695{\cdot}5 \ldots$ $= €95\,700$ [nearest multiple of 10]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some correct substitution into the relevant formula • a or r identified *High Partial Credit:* • Formula substituted correctly

Q7	Model Solution – 50 Marks	Marking Notes
(b)	$\frac{1}{10}\int_0^{10} 25\,000 \times (0\cdot 8^x)\,dx$ $= \frac{1}{10}\left[25000\frac{0\cdot 8^x}{\ln 0\cdot 8}\right]_0^{10}$ $= \frac{2500}{\ln 0\cdot 8}\ (0\cdot 8^{10} - 0\cdot 8^0)$ $= 10\,000\cdot 57 \ldots = [€]10\,001$ [nearest €]	**Scale 10D (0, 3, 5, 8, 10)** **1.** $\frac{1}{10}\int_0^{10} P(x)\,dx$ **2.** Integrates correctly **3.** Subs in limits **4.** Evaluates correctly *Low Partial Credit* • Work of merit, for example, some correct integration or integration indicated *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct
(c)	*Any valid explanation, for example:* The amount of money that would be equal to F after t years' compound interest, at interest rate i	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, explanation with one of F, t or i missing
(d) **(i)**	$0\cdot 85 \times 280\,000 = 238\,000$ $A = P\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$ $A = 238\,000\left[\frac{(0\cdot 0036)(1+0\cdot 0036)^{300}}{(1+0\cdot 0036)^{300} - 1}\right]$ $A = 12\,98\cdot 683 \ldots$ $= €1298\cdot 68$ [nearest cent]	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Formula with some substitution • 25 years = 300 months • A relevant present value *Mid Partial Credit:* • Finds €238,000 *High Partial Credit:* • Formula fully substituted
(d) **(ii)**	$\left(1+\frac{r}{100}\right)^{10} = \frac{350000}{280000} = 1\cdot 25$ $1 + \frac{r}{100} = \sqrt[10]{1\cdot 25} = 1\cdot 022565 \ldots$ $r = 2\cdot 26\%$ [2 DP]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some correct substitution into the formula, $F = P(1+i)^t$ *High Partial Credit:* • Formula fully substituted

Q8	Model Solution – 50 Marks	Marking Notes
(a)(i)	$h = x$ $l = 30 - 2x$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some work of merit, for example, $l = 30 - x$ *High Partial Credit:* • Either h or l correct
(a)(ii)	$x(20 - 2x)(30 - 2x)$ $= 4x^3 - 100x^2 + 600x$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some correct multiplication *High Partial Credit:* • One of $h(w)$, $h(l)$ or $w(l)$ evaluated correctly
(b)	$W'(x) = 12x^2 - 128x + 240 = 0$ at local max/min $x = \frac{128 \pm \sqrt{(-128)^2 - 4(12)(240)}}{2(12)} = 2{\cdot}42\ldots$ $= 2 \cdot 4$ cm	**Scale 10D (0, 3, 5, 8, 10)** Accept without checking endpoints of the given domain *Low Partial Credit:* • Some correct differentiation *Mid Partial Credit: nbnno* • $W'(x)$ correct *High Partial Credit:* • Formula fully substituted • Finds two roots
(c)(i)	$x = 0$, 15, and 21	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Finds one root • Writes $42 - 2x = 0$ or $30 - 2x = 0$ *High Partial Credit:* • Finds two roots

Q8	Model Solution – 50 Marks	Marking Notes
(c)(ii)		**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Correct shape • One root indicated on the $x-$ axis *High Partial Credit:* • $-U(x)$ plotted • Correct graph, but roots not indicated on the graph
(c)(iii)	$0 < x < 15$	**Scale 5B (0, 2, 5)** *Partial Credit:* • States one or more values for x from the correct range *Full Credit -1:* • $0 \leq x \leq 15$
(d)	$T(x) = 4x^3 - 4Ax^2 + A^2x$ $T'(x) = 12x^2 - 8Ax + A^2$ $T'\left(\frac{A}{6}\right) = 12\left(\frac{A}{6}\right)^2 - 8A\left(\frac{A}{6}\right) + A^2$ $= \frac{A^2}{3} - \frac{4A^2}{3} + A^2 = 0$ [so local max/min] $T''(x) = 24x - 8A$ $T''\left(\frac{A}{6}\right) = 24\left(\frac{A}{6}\right) - 8A = -4A < 0$ [as $A \in \mathbb{N}$] so local max.	**Scale 15D (0, 4, 8, 12, 15)** *Accept without checking endpoints of given domain* **4 Steps** **1.** Finds $T'(x)$ **2.** Shows $T'\left(\frac{A}{6}\right) = 0$ **3.** Finds $T''(x)$ **4.** Shows $T''\left(\frac{A}{6}\right) < 0$ *Low Partial Credit* • Any correct differentiation *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q9	Model Solution – 50 Marks	Marking Notes
(a)(i)	$V(0) = b = 2506$ $V(1) = a + 2506 = 2734$ so $a = 228$ $V(x) = 288x + 2506$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some relevant substitution into $V(x)$ • Relevant substitution into $T_n = a + (n-1)d$ *High Partial Credit:* • Finds b *Full Credit -1:* • a and b found, but $V(x)$ not stated
(a)(ii)	*Any valid quadratic*	**Scale 5B (0, 2, 5)** *Partial Credit:* • Some relevant substitution into $V(x)$
(a)(iii)	$V(0) = k = 2506$ $V(1) = 2506 \times e^p = 2734$ $e^p = \frac{2734}{2506}$ $p = \ln\frac{2734}{2506}$ $= 0 \cdot 0870 \ldots$ $p = 0.087$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Some relevant substitution into $V(x)$ • Relevant substitution into $T_n = ar^{n-1}$ *High Partial Credit:* • Isolates e^p
(b)	Time to work $= \frac{12}{40}$ hours Total time $= \frac{24}{25}$ hours Time from work $= \frac{24}{25} - \frac{12}{40} = \frac{33}{50}$ Speed from work $= 12 \div \frac{33}{50} = 18 \cdot 18 \ldots$ $= 18 \cdot 2$ km/hr [1 DP]	**Scale 10D (0, 3, 5, 8, 10)** **4 Steps** **1.** Finds the time to work **2.** Finds the total time **3.** Finds the time from work **4.** Finds speed from work *Low Partial Credit* • 1 step correct *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q9	Model Solution – 50 Marks	Marking Notes
(c)(i) & (ii)	**(i)** $47 \cdot 5$ km/hr **(ii)** $$115 \cdot 5 = \frac{D}{T}$$ $$D = 115.5T$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some relevant substitution into Distance, Speed & Time formula *High Partial Credit:* • One part correct
(c)(iii)	Distance travelled in 20mins: $120 \times \frac{20}{60} = 40$ [km] Total distance travelled: $D + 40$ Total time travelling: $T + \frac{1}{3}$ Average Speed = $\frac{\text{Total Distance}}{\text{Total Time}}$ $116 \cdot 5 = \frac{D+40}{T+\frac{1}{3}}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, $20\text{mins} = \frac{1}{3}\text{hrs}$ *High Partial Credit:* • Total distance and total time written in terms of D and T respectively
(c)(iv)	$90 + x = \frac{10485 + 120x}{117.5}$ $10575 + 117.5x = 10485 + 120x$ $2.5x = 90$ $x = 36$ Total distance $= 174 \cdot 75 + 120\left(\frac{36}{60}\right)$ $= 246 \cdot 75$ km.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit* • Work of merit, for example, one relevant and correct transposition *Mid Partial Credit* • An equation in x without denominators *High Partial Credit* • Finds x

Q10	Model Solution – 50 Marks	Marking Notes
(a)	Volume $= \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi \times 4^2 \times 12$ $= 201 \cdot 06 \ldots = 201$ [cm^3] [$\in \mathbb{N}$]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some substitution into the relevant formula *High Partial Credit:* • Fully substituted formula • $V = 64\pi$
(b)	$201 - 150 = 51$ cm^3 fallen $\frac{1}{3}\pi r^2(2r) = 51$ $r^3 = \frac{153}{2\pi}$ $r = \sqrt[3]{\frac{153}{2\pi}} = 2 \cdot 898 \ldots$ $h = 2.898 \ldots \times 2$ $h = 5 \cdot 80$ cm [2 DP]	**Scale 10 D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Some work of merit, for example finds the volume of the fallen sand *Mid Partial Credit:* • Finds the relevant equation in r *High Partial Credit:* • Finds r
(c)	$A = \pi r^2\sqrt{5}$ $\frac{dA}{dr} = 2\pi r\sqrt{5}$ At $r = 3$, $\frac{dA}{dr} = 6\pi\sqrt{5}$ [cm^2/cm]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Mentions the relevant derivative *High Partial Credit:* • Finds $\frac{dA}{dr}$
(d)	$\frac{H}{R} = \frac{12}{4}$ so $H = 3R$	Note: Accept correct answer without work **Scale 5 C (0, 2, 3, 5)** *Low Partial Credit:* • Mentions a relevant ratio, $\frac{H}{R}$, 4:12, 3 etc. *High Partial Credit:* • Equation correct, but H is not written in terms of R

(e)	$V = \frac{1}{3}\pi R^2 H = 8\pi$ $V = \frac{1}{3}\pi R^2 \times 3R = 8\pi$ $V = \pi R^3 = 8\pi$ $R = 2$ $\frac{dV}{dR} = 3\pi R^2$ At $R = 2$, $\frac{dV}{dR} = 3\pi 2^2 = 12\pi$ $\frac{dV}{dt} = \frac{dV}{dR} \times \frac{dR}{dt}$ $-3 = 12\pi \times \frac{dR}{dt}$ So $\frac{dR}{dt} = -\frac{1}{4\pi}$ [cm/sec]	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit*: • States relevant derivative, for example, $\frac{dV}{dr}$ • $\frac{1}{3}\pi R^2 H = 8\pi$ *Mid Partial Credit*: • Any **two** of the following: ○ $\frac{dV}{dR} = 3\pi R^2$ ○ $\frac{dV}{dR} = 12\pi$ ○ $\frac{dR}{dt} = \frac{dV}{dt} \times \frac{dR}{dV}$ ○ $R = 2$ *High Partial Credit:* • $\frac{dR}{dt} = \frac{dV}{dt} \times \frac{dR}{dV}$ **and** any **two** others from the MPC list above •
(f)	**(i)** $l^2 = 12^2 + 4^2 = 160$ $l = \sqrt{160}$ $l = 4\sqrt{10}$ [cm] **(ii)** Circumference of cone $= 2\pi(4) = 8\pi$ Circumference of disc $= 2\pi\sqrt{160}$ $\theta = \frac{8\pi}{2\pi\sqrt{160}} \times 360 = 113 \cdot 84 \ldots$ $= 114°$ [$\in \mathbb{N}$]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Some work of merit, for example, some substitution into $\boldsymbol{l^2 = h^2 + r^2}$ *High Partial Credit:* • One part correct

Leaving Certificate

Deferred Exam 2022

Marking Scheme

Mathematics

Higher Level

Paper 2

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	B	C	D	E
No of categories	3	4	5	6
5-mark scale	0, 2, 5	0, 2, 3, 5		
10-mark scale		0, 3, 7, 10	0, 3, 5, 8, 10	
15-mark scale			0, 4, 8, 12, 15	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded.

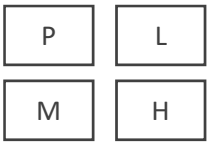
The only marks that may be awarded for a question are those on the scale above, or *Full Credit -1*.

A rounding penalty is applied only once in each section (a), (b), (c) etc. A penalty for an omitted unit is applied only once in each section (a), (b), (c) etc. There is no penalty for omitted units if the question specifies the unit to be used in the answer.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Unless otherwise specified, a correct answer without sufficient supporting work is generally awarded the highest level of partial credit (typically *Partial Credit* or *High Partial Credit*, as appropriate).

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
	Tick	Work of relevance	The work presented in the body of the script merits full credit
	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
	Star	Rounding / Unit / Arithmetic error / Misreading	
	Horizontal wavy	Error	
			The work presented in the body of the script merits the relevant level of partial credit (*Partial*, *Low Partial*, *Mid Partial*, and *High Partial* respectively)
	F star		The work presented in the body of the script merits *Full Credit – 1*
	Left Bracket		Another version of this solution is presented elsewhere, and it merits equal or higher credit
	Vertical wavy	No work on this page / portion of this page	
	Oversimplify	The candidate has oversimplified the work	
	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and  and  appear in the body of the work, then  should be placed in the right margin.

In the case of a **D scale** with the same annotations,  should be placed in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

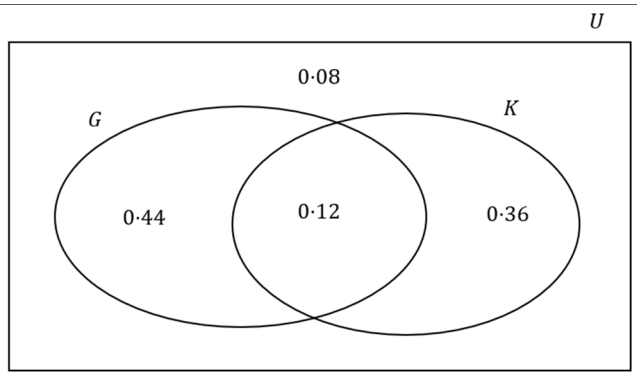
Where steps are listed in the Marking Notes, unless otherwise specified, it is to be taken that they can be independently correct / incorrect – that is, in a candidate's solution, step n can be considered correct even if previous step(s) have not been correctly presented, as long as the work done to arrive at step n from the previous step(s) has not been oversimplified. It is specifically noted where this does not hold. Note also that these steps may not need to be presented in the order specified in the Marking Notes.

Where "finishes correctly" is included in the Marking Notes, this is taken to mean: "finishes using the correct method, and the (incorrect) values the candidate has already found."

Q1	**Model Solution – 30 Marks**	**Marking Notes**
(a)	*Any suitable labelled graphical display.*	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Work of merit, for example, two or more plots correct. *High Partial Credit:* • At least 4 plots correct *Full Credit-1:* No labels or incorrect labelling

Q1	Model Solution – 30 Marks	Marking Notes
(b)	Mean $= \frac{\text{sum}}{\text{No. of boxes}} = \frac{2301}{53}$ $= 43{\cdot}4$ matches [1 DP] Mode $= 44$ matches Median is 27th box, so median $= 43$ matches	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit, for example, sum of two or more values towards finding mean, or states the mode is the value with greatest frequency, or indicates median is 27th box. *Mid Partial Credit:* • $\frac{\text{sum}}{\text{No. of boxes}}$ fully substituted • Mode correct • Median correct *High Partial Credit:* • Two of the following: $\frac{\text{sum}}{\text{No. of boxes}}$ fully substituted, mode correct, median correct
(c)	$\binom{6}{4}\left(\frac{45}{53}\right)^4\left(\frac{8}{53}\right)^2\frac{8}{53} = 0{\cdot}0268089\ldots$ $= 0{\cdot}0268$ [4 DP]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Work of merit, for example, relevant listing that in 7 'picks' that there are 4 in the first 6 that do not contain 42 matches. • $\frac{8}{53}$ or $\frac{45}{53}$ *High Partial Credit:* • $\binom{6}{4}\left(\frac{45}{53}\right)^4\left(\frac{8}{53}\right)^2$ and finishes. • $\binom{7}{3}\left(\frac{45}{53}\right)^4\left(\frac{8}{53}\right)^3$ and finishes.

Q2	Model Solution – 30 Marks	Marking Notes
(a) **(i)**	$$P(A) \times P(C) = P(A \cap C)$$ $$\frac{1}{5} \times P(C) = \frac{1}{20}$$ $$P(C) = \frac{1}{4}$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, identification of relevant formula *High Partial Credit:* • Formula fully substituted
(a) **(ii)**	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cap B) = \frac{1}{5} + \frac{1}{6} - \frac{1}{3} = \frac{1}{30}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Work of merit, for example, identifies correct formula, or substitutes into partially correct formula *High Partial Credit:* • Formula fully substituted
(a) **(iii)**	**Answer:** Not mutually exclusive Reason: *Any valid reason, for example:* $P(B \cup C) \neq P(B) + P(C)$, as $\frac{3}{8} \neq \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$ **Or** $$P(B \cup C) = P(B) + P(C) - P(B \cap C)$$ $$\frac{3}{8} = \frac{1}{6} + \frac{1}{4} - P(B \cap C)$$ $$P(B \cap C) = \frac{1}{24} \neq 0$$	**Scale 5C (0,2,3,5)** *Low Partial Credit:* • Work of merit, for example correct conclusion, or makes a relevant statement *High Partial Credit:* • Correct conclusion but reason not clearly related to given data
(b)	$P(G\|K) = \frac{P(G \cap K)}{P(K)}$ So, $P(G \cap K) = P(K).P(G\|K) = (0 \cdot 48).(0 \cdot 25)$ $= 0 \cdot 12$ $P(G \backslash K) = 0 \cdot 56 - 0 \cdot 12 = 0 \cdot 44$ etc	**Scale 10D (0,3,5,8,10)** *Low Partial Credit:* • Work of merit, for example, makes a relevant statement, or identifies relevant formula for $P(G \cap K)$ or writes the probability of the event in one of the other regions

		correctly but based on an incorrectly deduced $P(G \cap K)$ *Mid Partial Credit:* • Expression for $P(G \cap K)$ partially substituted • Probability of the event in the other 3 regions correctly written/calculated but based on an incorrectly deduced $P(G \cap K)$ *High Partial Credit:* • $P(G \cap K) = 0 \cdot 12$

Q3	**Model Solution – 30 Marks**	**Marking Notes**
(a) (i)	$x - 2y + 8 = 0$ $(k) - 2\left(\frac{k+8}{2}\right) + 8$ $k - k - 8 + 8 = 0$ $0 = 0$	**Scale 5C (0,2,3,5)** *Low Partial Credit:* • Work of merit, for example, some correct substitution into *l* *High Partial Credit:* • *P* fully substituted
(a) (ii)	$\frac{\left(\frac{k+8}{2} - 1\right)}{k+1}$ $= \frac{k+6}{2k+2}$	**Scale 5C** *Low Partial Credit:* • Work of merit, for example some correct substitution into slope formula *High Partial Credit:* • Slope formula *fully* substituted

Q3	Model Solution – 30 Marks	Marking Notes
(a) (iii)	Slope $(AB) = \frac{3-1}{3+1} = \frac{1}{2}$, so $\perp$ slope $= -2$ So $\frac{k+6}{2k+2} = -2$ **or** $\frac{k+6}{2k+2} . \frac{1}{2} = -1$ i.e., $k + 6 = -4k - 4$ So $k = -2$ **OR** $\left(\sqrt{4^2 + 2^2}\right)^2 + \left(\sqrt{(k+1)^2 + \left(\frac{k}{2} + 3\right)^2}\right)^2 =$ $\left(\sqrt{(k-3)^2 + \left(\frac{k}{2} + 1\right)^2}\right)^2$ So $20 + k^2 + 2k + 1 + \frac{k^2}{4} + 3k + 9 = k^2 -$ $6k + 9 + \frac{k^2}{4} + k + 1$ i.e. $10k = -20$, so $k = -2$	**Scale 10C (0,3,7,10)** *Low Partial Credit:* • Work of merit, for example some correct substitution into a relevant formula -slope formula, length of line segment etc. *High Partial Credit:* • Fully correct substituted equation
(b)	$(-2, 5) \rightarrow (0, 0)$ $(4, 11) \rightarrow (6, 6)$ $(7, -5) \rightarrow (9, -10)$ Area $= \frac{1}{2}\|6(-10) - (6)(9)\|$ $= \frac{1}{2}\|-114\| = 57$ [square units]	**Scale 10D (0,3, 5, 8, 10)** *Low Partial Credit:* • Work of merit, for example, indication of translation, or area of triangle formula with some substitution, or length of line segment formula with some substitution. *Mid Partial Credit:* • Two vertices translated or length of two sides calculated *High Partial Credit:* • Area formula fully substituted

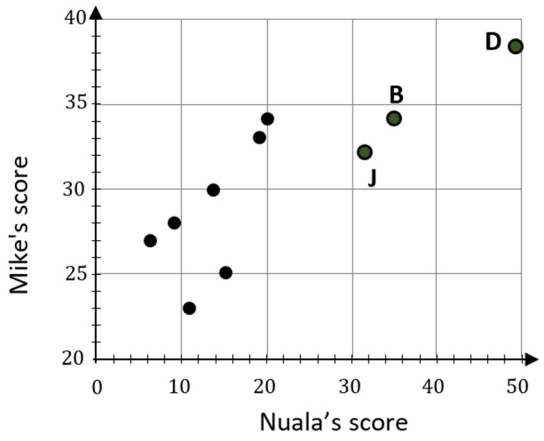
Q4	Model Solution – 30 Marks	Marking Notes
(a) **(i)**	*Large number of different possible approaches, for example:* $m_{BC} = \frac{6}{8} = \frac{3}{4}$ $m_{AB} = -\frac{4}{3}$ $\frac{3}{4} \times -\frac{4}{3} = -1$, so, triangle is right-angled, So, circumcentre lies on hypotenuse $[AC]$, and must be midpoint.	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Work of merit, for example, slope formula or length of line segment formula or midpoint formula or with some relevant substitution *High Partial Credit:* • $\frac{3}{4} \times -\frac{4}{3} = -1$ • $\|BC\|^2 + \|AB\|^2 = \|AC\|^2$ • Finds point of intersection of two perpendicular bisectors • Finds distance midpoint AC to vertex B equal to distance to either vertex A or to vertex C *Full Credit-1* • Correct answer but no or incorrect conclusion
(a) **(ii)**	Centre = midpoint $[AC] = (2 \cdot 5,\ 3)$ Radius $= \frac{1}{2}\sqrt{11^2 + 2^2} = \frac{\sqrt{125}}{2}$ Equation: $(x - 2 \cdot 5)^2 + (y - 3)^2 = \frac{125}{4}$ $x^2 - 5x + \frac{25}{4} + y^2 - 6y + 9 = \frac{125}{4}$ $x^2 + y^2 - 5x - 6y - 16 = 0$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Work of merit, for example, length of line segment formula or midpoint formula or equation of circle formula with some relevant substitution *High Partial Credit:* • Finds radius and centre or equivalent sufficient to form equation of circle (for example, g, f, c from simultaneous equations).

(b)	$j\colon\ y = 6$ or $y = -4$ $k\colon\ x = -8$ or $x = 2$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Plots centre and indicates radius on a set of axes • Indicates point on one of the relevant tangents *High Partial Credit:* • One equation of the line *j* or *k* • Both equations of *j* or *k*

Q5	Model Solution – 30 Marks	Marking Notes
(a) **(i)**	Triangle QRS: $7^2 = 7^2 + 4^2 - 2(7)(4)CosA$ $56\ CosA = 16$ $=>\ \ CosA = \frac{16}{56} = \frac{2}{7}$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Some correct substitution into Cosine formula *High Partial Credit:* • Fully substituted Cosine formula
(a) **(ii)**	Triangle SOR: $\|OR\|^2 = 3{\cdot}5^2 + 4^2 - 2(3{\cdot}5)(4)\frac{2}{7}$ $\|OR\|^2 = \frac{81}{4}$ $\|OR\| = \frac{9}{2}$ $\|PR\| = 2\left(\frac{9}{2}\right) = 9$ cm	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Some correct substitution into Cosine formula *High Partial Credit:* • Fully substituted Cosine formula for $\|OR\|^2$ or $\|PR\|^2$ *Full Credit -1:* $\|OR\| = \frac{9}{2}$ cm or $4{\cdot}5$cm
(b)	$\tan^{-1}\left(-\sqrt{3}\right) = -60°$ Quadrants 2 and 4: So $A = 120°, 300°, 480°, 660°$	**Scale 10D (0,3,5,8,10)** *Low Partial Credit:* • Work of merit, for example, quadrants 2 and/or 4 identified, or some reference to $60°$ or $= -60°$ *Mid Partial Credit:* • $120°$or $300°$ *High Partial Credit:* • $120°$ and $480°$ • $300°$and $660°$ • $120°$ or $480°$ **and** $300°$ or $660°$

Q6	Model Solution – 30 Marks	Marking Notes
(a)	*Appropriate construction*	**Scale 15D (0,4,8,12,15)** *Low Partial Credit:* • Work of merit, for example, indication of constructing a right angle at either end of line segment, or indication of construction of a relevant semi-circle /semicircle with centre on XY or relevant equilateral triangle with base XY, or equivalent *Mid Partial Credit:* Example: • Right angled triangle with hypotenuse $\sqrt{2}$ constructed • Circle with XY or multiple of XY as diameter constructed • Equilateral triangle of side length |XY| constructed *High Partial Credit:* • Right angled triangle with hypotenuse $\sqrt{\mathbf{2}}$ constructed , **and** further relevant work to construct a triangle with $\sqrt{\mathbf{2}}$ as base. • Circle with XY or multiple of XY as diameter constructed, **and** further work on constructing another relevant circle with either X or Y as centre**.** • Equilateral triangle of side |XY| **and** further work to construct another relevant triangle *Full Credit -1:* • Correct construction but $\sqrt{3}$ not clearly indicated

(b) **(i)**	$\angle CAD$ $\angle CBD$	**Scale 5B (0,2,5)** *Partial Credit:* • One correct Note: Accept for Full Credit if labelled correctly on diagram
(b) **(ii)**	$\frac{\lvert AD\rvert}{\lvert DC\rvert} = \frac{\lvert DC\rvert}{\lvert DB\rvert}$ and $\lvert DB\rvert = \frac{5}{2}\lvert AD\rvert$ $\frac{5}{2}\lvert AD\rvert^2 = 64$ $\lvert AD\rvert^2 = \frac{128}{5}$ $\lvert AD\rvert = \sqrt{\frac{128}{5}}$ $\lvert AB\rvert = \frac{7}{2}\sqrt{\frac{128}{5}} = 17 \cdot 70 \ldots = 17 \cdot 7$ cm [1 DP]	**Scale 10C (0,3,7,10)** *Low Partial Credit:* • Work of merit, for example, indicates two similar triangles, or a trigonometric ratio of a relevant angle or $\lvert DB\rvert$ in terms of $\lvert AD\rvert$ *High Partial Credit:* • States $\frac{\lvert AD\rvert}{\lvert DC\rvert} = \frac{\lvert DC\rvert}{\lvert DB\rvert}$ and $\lvert DB\rvert = \frac{5}{2}\lvert AD\rvert$ or equivalent

Q7	Model Solution – 50 Marks	Marking Notes
(a) **(b)**	 **(b)** 0·81 [2 DP]	**Scale 10C (0,3,7,10)** *Low Partial Credit:* • 1 correct plot and labelled • 2 correct plots *High Partial Credit:* • (a) or (b) correct • Work of merit in both parts *Full Credit -1:* • Points and correlation correct but no or incorrect labels on points
(c)	**(i)** *Appropriate line of best fit drawn* **(ii)** *Correct score estimated from line of best fit, with some work on diagram*	**Scale 5C (0,2,3,5)** In (i), accept line of best fit with some values on each side and with reasonable slope *Low Partial Credit:* • Work of merit in either part *High Partial Credit:* • (i) or (ii) correct • Work of merit in both parts

Q7	Model Solution – 50 Marks	Marking Notes
(d) **(i)**	*Appropriate point(s) and slope taken, and used to find equation of line* Example: $(20,20), (40,30)$ Slope = $\frac{1}{2}$ Line: $y - 20 = \frac{1}{2}(x - 20)$ $x - 2y + 20 = 0$	**Scale 10C (0,3,7,10)** *Low Partial Credit:* • Work of merit, for example, slope or line formula with some substitution, or any correct point on line indicated *High Partial Credit:* • Correct slope and further work towards finding equation of line
(d) **(ii)**	On average, how much Mike's score increases for an act, for a 1-point increase in Nuala's score, *or any other valid explanation*	**Scale 5B (0,2,5)** *Partial Credit:* • Makes a relevant but incomplete connection with the slope in the required context
(e) **(i)**	$\frac{32-31\cdot6}{\frac{3\cdot2}{\sqrt{12}}} = 0\cdot433\ldots$ $\frac{30-31\cdot6}{\frac{3\cdot2}{\sqrt{12}}} = -1\cdot732\ldots$ $P(30 < X_{mean} < 32) = P(-1\cdot73 < Z < 0\cdot43)$ $= P(Z < 0\cdot43) - [1 - P(Z < 1\cdot73)]$ $= 0\cdot6664 - [1 - 0\cdot9582]$ $= 0\cdot6664 - 0\cdot0418 = 0\cdot6246$	**Scale 10D (0,3,5,8,10)** **1.** Finds z-score for x=30 **2.** Finds z-score for x=32 **3.** Finds $0\cdot9582$ or $0\cdot6664$ **4.** Finds solution Accept rounding for z up or down limits *Low Partial Credit:* • Work of merit, for example, draws a relevant diagram, or indicates μ or σ *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct

Q7	Model Solution – 50 Marks	Marking Notes
(e) **(ii)**	$\frac{33\cdot5-31\cdot6}{\frac{3\cdot2}{\sqrt{20}}} = 2\cdot655\ldots$ $p = 2[1 - P(Z < 2\cdot66)]$ $= 2[1 - 0\cdot9961] = 0\cdot0078$	**Scale 5C (0,2,3,5)** *Low Partial Credit:* • Work of merit, for example, draws a relevant diagram, or indicates μ or σ *High Partial Credit:* • Finds z-score **and** further work, for example finds $0\cdot9961$ or indicates $[1 - P(Z < 2\cdot66)]$ or $2[1 - P(Z < 2\cdot66)]$
(e) **(iii)**	The website's claim has been substantiated / there is enough evidence to support the claim [at the 5% level of significance] Reason: p-value is less than 5%	**Scale 5B (0,2,5)** *Partial Credit:* • Work of merit, for example, correct conclusion with no reason

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$\binom{14}{5} = 2002$	**Scale 5B (0,2,5)** *Note:* Accept 2002 without work shown *Partial Credit:* • Work of merit, for example, some relevant work with 14 and 5.
(b)	$5 \times 1 \times 10 \times 10 \times 5 = 2500$	**Scale 10C (0,3,7,10)** *Low Partial Credit:* • Work of merit, for example, code with one correct element • A listing of 3 possible codes, other than the given example *High Partial Credit:* • $5 \times 1 \times 9 \times 9 \times 5$ • $5 \times 6 \times 9 \times 9 \times 5$ and finishes *Full Credit -1:* $5 \times 1 \times 10 \times 10 \times 5$
(c)	$(1 \cdot 2 \times 0 \cdot 16) + \cdots + (2 \cdot 35 \times 1 \cdot 19)$ $= 1 \cdot 7975 = [€]1 \cdot 80$ [nearest cent]	**Scale 10C (0,3,7,10)** *Low Partial Credit:* • Work of merit, for example, indicates one relevant operation *High Partial Credit:* • All individual operations completed but sum not found
(d) **(i)**	$P(\text{weight} < 53)$ $\bar{x} = 60 \quad \sigma = 15$ $z = \frac{53-60}{15} = \frac{-7}{15}$ $P(x < 53) = P\left(z < \frac{-7}{15}\right) = 1 - P\left(z < \frac{7}{15}\right)$ $= 1 - 0{\cdot}6808 = 0{\cdot}3192$ 32 [%]	**Scale 5C (0,2,3,5)** *Low Partial Credit:* • Work of merit, for example, draws a relevant diagram, or indicates μ or σ *High Partial Credit:* • Finds z-score **and** further work, for example finds $0 \cdot 6808$ or indicates $1 - P(z < \frac{7}{15})$ or equivalent

(d) **(ii)**	$\frac{63-60}{15} = 0 \cdot 2$ $P(x < 63) = P(z < 0 \cdot 2) = 0 \cdot 5793$ $P(x < N) = 0 \cdot 5793 + 0 \cdot 23 = 0 \cdot 8093$ $P(z < 0 \cdot 88) = 0 \cdot 8106$ $N = 60 + 0 \cdot 88 \times 15 = 73 \cdot 2 = 73$ g [$\in \mathbb{N}$]	**Scale 10D (0,3,5,8,10)** **1.** Finds z **2.** Finds $0 \cdot 5793$ **3.** Finds $0 \cdot 8093$ **4.** Finds z score, $0 \cdot 88$ or equivalent and finishes *Low Partial Credit:* • Work of merit, for example, draws a relevant diagram, or indicates μ or σ *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct
(d) **(iii)**	$P(at\ least\ 5) = P(5) + P(6)$ $= \binom{6}{5}\left(\frac{19}{42}\right)^5\left(\frac{23}{42}\right) + \left(\frac{19}{42}\right)^6$ $= 0 \cdot 06225 + 0 \cdot 00857$ $= 0 \cdot 0708 \ldots = 0 \cdot 071$ [3 DP]	**Scale 10D (0,3,5,8,10)** *Low Partial Credit:* • First line of solution • One correct probability *Mid Partial Credit:* • Fully substituted $P(5)$ or $P(6)$ *High Partial Credit:* • $P(5)$ and $P(6)$ fully substituted • $P(5)$ or $P(6)$ correctly calculated

Q9	Model Solution – 50 Marks	Marking Notes
(a) **(i)**	$3{\cdot}8^2 = 7{\cdot}6^2 + 8{\cdot}2^2 - 2(7{\cdot}6)(8{\cdot}2)\cos A$ $\cos A = \frac{7{\cdot}6^2+8{\cdot}2^2-3{\cdot}8^2}{2(7{\cdot}6)(8{\cdot}2)} = 0\cdot 8870\ldots$ $A = 27\cdot 49\ldots = 27\cdot 5[°]$ [1 DP]	**Scale 10C (0,3,7,10)** *Low Partial Credit:* • Some correct substitution into Cosine Rule formula *High Partial Credit:* • Fully substituted Cosine Rule formula
(a) **(ii)**	$\frac{1}{2}(7\cdot 6)(3\cdot 8)\sin 85°$ $= 14\cdot 38\ldots = 14\cdot 4$ cm^2 [1 DP]	**Scale 5C (0,2,3,5)** *Low Partial Credit:* • Some correct substitution into area formula *High Partial Credit:* • Area formula fully substituted
(a) **(iii)**	Actual area $= 152\times 50000^2 = 3\cdot 8\times 10^{11}$ cm^2 Area in km^2 $= \frac{3{\cdot}8\times 10^{11}}{(10^5)^2} = 3\cdot 8\times 10^1$	**Scale 10C (0,3,7,10)** *Low Partial Credit:* • Some work of merit, for example, indicates a connection between scale and given area, or attempts to convert the map area into m^2 or km^2 *High Partial Credit:* • Formulates actual area in cm^2
(b)	$\frac{x}{\sin 42} = \frac{x+8}{\sin 88}$ $x\sin 88 = x\sin 42 + 8\sin 42$ $x = \frac{8\sin 42}{\sin 88-\sin 42} = 16\cdot 20\ldots = 16\cdot 2$ [1 DP]	**Scale 10D (0,3,5,8,10)** *Low Partial Credit:* • Some correct substitution into Sine formula *Mid Partial Credit:* • Formula fully substituted *High Partial Credit:* • Eliminates fractions and expands linear equation in x

(c) **(i)**	$\tan 6 = \frac{h}{8 \cdot 3}$ $h = 8 \cdot 3 \tan 6 = 0 \cdot 8723 \ldots$ km $= 872$ [nearest m]	**Scale 5C (0,2,3,5)** *Low Partial Credit:* • Some work of merit, for example, identifies opposite or adjacent side in relation to given angle *High Partial Credit:* • Formula fully substituted
(c) **(ii)**	Extreme values of $\theta = 5 \cdot 5$ or $6 \cdot 5$ $\theta = 5 \cdot 5$: Error $= \frac{8 \cdot 3 \tan 6 - 8 \cdot 3 \tan 5 \cdot 5}{8 \cdot 3 \tan 5 \cdot 5} \times 100 = 9 \cdot 15 \ldots \%$ $\theta = 6 \cdot 5$: Error $= \frac{8 \cdot 3 \tan 6 \cdot 5 - 8 \cdot 3 \tan 6}{8 \cdot 3 \tan 6 \cdot 5} \times 100 = 7 \cdot 75 \ldots \%$ Max % error $= 9 \cdot 2\%$ [1 DP]	**Scale 10D (0,3,5,8,10)** Accept: $5 \cdot 5 \leq \theta \leq 5 \cdot 6$ and $6 \cdot 4 \leq \theta \leq 6 \cdot 5$ *Low Partial Credit:* • Some work of merit, for example, indication that error value of θ could be greater or less than 6^0, or some relevant work with *tan* function *Mid Partial Credit:* • Both extreme values of θ identified • Equation for one extreme error value of θ formulated *High Partial Credit:* • Equation for both extreme error values of θ formulated • Percentage error for one extreme error value found

Q10	Model Solution – 50 Marks	Marking Notes
(a)	$y = 31 + 16 = 47$ and $y = 16 - 31 = -15$	**Scale 5C (0,2,3,5)** *Low Partial Credit:* • Work of merit, for example, some relevant use of (31,16) on diagram, or substitution into slope or line formula or identifying slopes *High Partial Credit:* • One *y* intercept correct • Work of merit in finding both *y* values
(b)	l is $y = 12 - x$ Image is $y = -12 + x$	**Scale 5B (0,2,5)** *Partial Credit:* • Finds *y* in terms of *x* • Finds the coordinates of one point on line *l* and its image under axial symmetry
(c)	Midpoint of $(-1, 0)$ and $(5, 0)$ is $(2, 0)$. So, centre of circle is $(24, 22)$ Diameter is d: $d^2 + d^2 = 6^2$ $d^2 = 18$ $d = \sqrt{18}$ $r = \frac{\sqrt{18}}{2}$ or Perp distance (24,22) to $y = x + 1$ is $\frac{3}{\sqrt{2}}$ Equation: $(x - 24)^2 + (y - 22)^2 = \frac{9}{2}$ **Or** Eq of tangent through (-1,0): $y = x + 1$ $y = x + 1 \cap y = 22, \quad (21,22)$ Eq of tangent through (5,0): $y = x - 5$ $y = x - 5 \cap y = 22, \quad (27,22)$ Centre of circle (24,22) Radius: Perp distance (24,22) to $y = x + 1$ is $\frac{3}{\sqrt{2}}$ Equation: $(x - 24)^2 + (y - 22)^2 = \frac{9}{2}$	**Scale 15D (0,4,8,12,15)** *Low Partial Credit:* • Some work of merit, for example, indicates correct y ordinate of centre, or midpoint formula with some substitution, or equation of line formula with some substitution *Mid Partial Credit:* • Finds centre of circle • Finds diameter or radius *High Partial Credit:* • Centre and radius • Equation of a circle with correct centre, and **work** of merit for radius • Equation of a circle with correct radius **and** work of merit for centre

(d) **(i)**	50%	**Scale 5B (0,2,5)** Accept correct answer without work *Partial Credit:* • Finds area of square or area of any triangle
(d) **(ii)**	$\Delta\, HGF$ *Any valid proof, for example:* $\lvert\angle FHG\rvert = \lvert\angle CAB\rvert = 90°$ $\lvert HG\rvert = \lvert AB\rvert$ … opposite sides of parallelogram $\lvert\angle HGF\rvert = \lvert\angle ABC\rvert = 45°$ Congruent by ASA	**Scale 10D (0,3,5,8,10)** The proof requires properties for congruent triangles, for example, ASA, AAS, SSS, RHS *Low Partial Credit:* • Some work of merit, for example, identifies correct triangle or marks a pair of equal sides or angles triangles *Mid Partial Credit:* • Correct triangle and one of the properties necessary for congruence *High Partial Credit:* • Correct triangle and correct for congruence but reason not stated
(d) **(iii)**	Eq: $y = -1(x-1)$ then $x = k,\ y = 1-k$	**Scale 5C (0,2,5)** Accept correct answer without work *Partial Credit:* • Work of merit, for example, draws horizontal line through C or attempt to find equation of line slope -1 through B or (0,1)

(d) (iv)	$\|FH\| = \|AC\|$, as triangles are congruent Similarly, $\|FD\| = \|CD\|$. $\|FH\| + \|FD\| + \|CD\| + \|AC\| = 1$, so Shaded $= \|FD\| + \|AC\| = \frac{1}{2}$ **OR** Shaded length $= \|AC\| + \|FD\|$ $= 1 - k + \left(k - \frac{1}{2}\right) = \frac{1}{2}$	**Scale 5D (0,2,3,5)** *Low Partial Credit:* • Work of merit, for example, shows $\|FH\| = \|AC\|$ or $\|FD\| = \|CD\|$ or equivalent *High Partial Credit:* • $\|FH\| = \|AC\|$ and $\|FD\| = \|CD\|$. • $\|FD\|$ or $\|CD\| = \frac{1}{2} - (1 - k)$ or equivalent